**Ex:** After being open for a long time, the switch closes at \( t = 0 \).

\[ \begin{array}{c}
\text{\( v_g = 2.3 \text{V} \)} \\
\text{\( R = 10 \text{k}\Omega \)} \\
\text{\( C = 2 \text{nF} \)} \\
\text{\( v_C(t=0) = 5 \text{V} \)}
\end{array} \]

(a) Find an expression for \( v_C(t) \) for \( t \geq 0 \).

(b) Find the energy stored in the capacitor at time \( t = 30 \mu\text{s} \).

**Sol’n:**

a) The following general form of solution applies to any RC circuit with a single capacitor:

\[
v_C(t \geq 0) = v_C(t \to \infty) + [v_C(t = 0^+) - v_C(t \to \infty)]e^{-t/R_{\text{Th}}C}
\]

The Thevenin resistance, \( R_{\text{Th}} \), is for the circuit after \( t = 0 \) (with the \( C \) removed) as seen from the terminals where the \( C \) is connected. In the present case, we have \( R_{\text{Th}} = 10 \text{k}\Omega \).

\[ R_{\text{Th}}C = 10 \text{k}\Omega \cdot 2 \text{nF} = 20 \mu\text{s} \]

The value of \( v_C(t=0) \) is given in the problem as \( 5 \text{V} \). Note that the \( C \) could have any voltage before \( t = 0 \) in this circuit if the value were not specified. The voltage would stay on the ideal \( C \) indefinitely prior to \( t = 0 \).

As time approaches infinity, the \( C \) will charge to its final value, and current will cease to flow in the \( C \). Thus, the \( C \) will become an open circuit. It follows that the current through the \( R \), which is the same as the current through the \( C \), will become zero. By Ohm's law, this in turn means that the voltage drop across the \( R \) will become zero, and the voltage across the \( C \) will be the same as the source voltage, \( 2.3 \text{V} \).

\[
v_C(t \to \infty) = 2.3 \text{ V}
\]

Substituting values, we have the following result:

\[
v_C(t \geq 0) = 2.3 \text{ V} + [5 \text{ V} - 2.3 \text{ V}]e^{-t/20\mu\text{s}} = 2.3 \text{ V} + 2.7 \text{ V} \cdot e^{-t/20\mu\text{s}}
\]
b) The energy in a capacitor is given by the following formula:

\[ w_C = \frac{1}{2} C v_C^2 \]

We use the solution to (a) to evaluate \( v_C(t) \) at \( t = 30 \, \mu\text{s} \).

\[ v_C(t = 30\mu\text{s}) = 2.3 \, \text{V} + 2.7 \, \text{V} \cdot e^{-30\mu\text{s}/20\mu\text{s}} = 2.90 \, \text{V} \]

Using this voltage, we evaluate the energy on the capacitor.

\[ w_C = \frac{1}{2} 2\text{nF} \cdot (2.90\text{V})^2 = 8.42 \, \text{nJ} \]