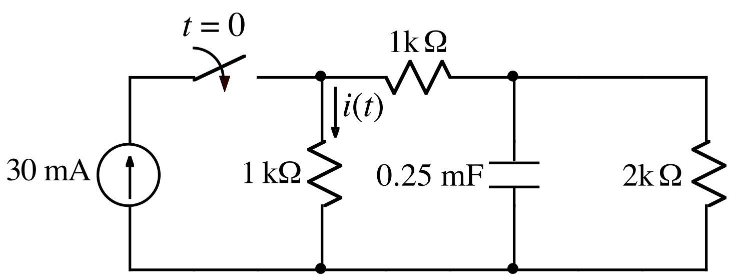
ECE 1270
Su 08

HOMEWORK #12 solution

Ex:



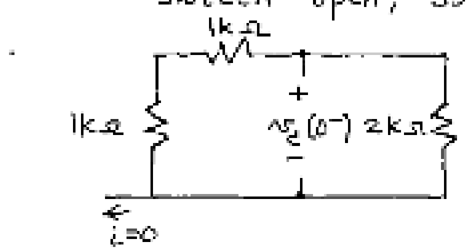
After being open for a long time, the switch closes at $t = 0$.

Write a numerical expression for $i(t)$ for $t > 0$.

sol'n: We use the general form of solution:

$$i(t) = i(t\to\infty) + [i(t=0^+) - i(t\to\infty)]\, e^{-t/R_{Th}C}, \quad t>0$$

$t=0^-$ model: C = open circuit, find $v_c(0^-)$
switch open, 30 mA disconnected

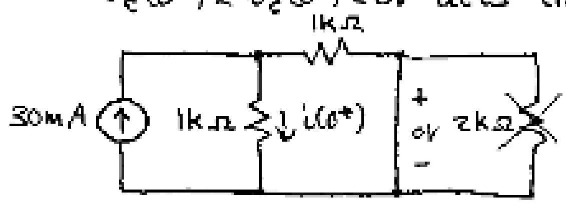


Since there is no power source and there is a resistor across C to discharge it, we must have

$$v_c(0^-) = 0\text{ V}$$

Note: No current can flow in the single wire to the left since that would cause charge to accumulate in that part of the circuit. There is no complete circuit.

$t=0^+$: $v_c(t=0^+) = v_c(t=0^-)$ since the energy variable v_c cannot change instantly. We model C as a v-src for the instant in time $t=0^+$.
$v_c(0^+) = v_c(0^-) = 0V$ acts like a wire.

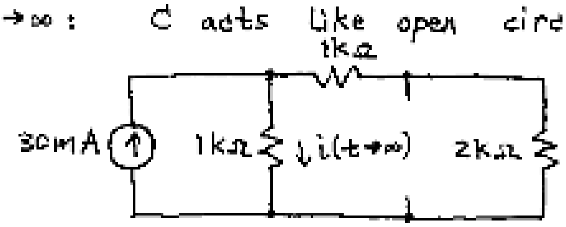


wire shorts out 2kΩ
∴ 0V across 2kΩ and no current flows in 2kΩ by Ohm's law.

We have a current divider with $R = 1k\Omega$ on both sides:

$$i(0^+) = 30mA \cdot \frac{1k\Omega}{1k\Omega + 1k\Omega} = 15\ mA$$

$t \to \infty$: C acts like open circuit

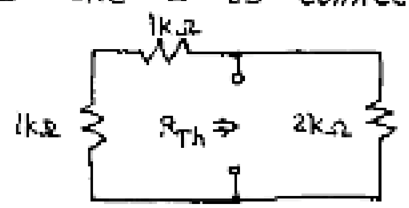


We have a current divider with 1kΩ on left and $1k\Omega + 2k\Omega = 3k\Omega$ on right.

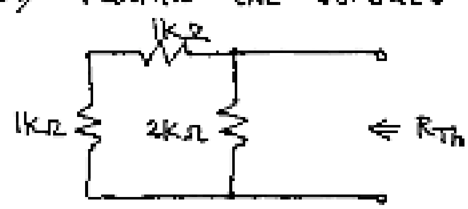
$$\therefore\ i(t\to\infty) = 30mA \cdot \frac{3k\Omega}{1k\Omega + 3k\Omega} = 22.5\ mA$$

$\tau = R_{Th} C$: We find R_{Th} looking into terminals where C is connected.

In this circuit, we need only turn off the independent source and look into the circuit from the terminals where the C is connected to find R_{Th}.



We may redraw the circuit as follows:



We have

$$R_{Th} = 2k\Omega \| (1k\Omega + 1k\Omega) = 1k\Omega$$

$$R_{Th}C = 1k\Omega \cdot 0.25\,mF = 0.25\,s$$

Combining results, we have our final answer:

$$i(t) = 22.5\,mA + (15\,mA - 22.5\,mA)\,e^{-t/0.25s}, \quad t>0$$