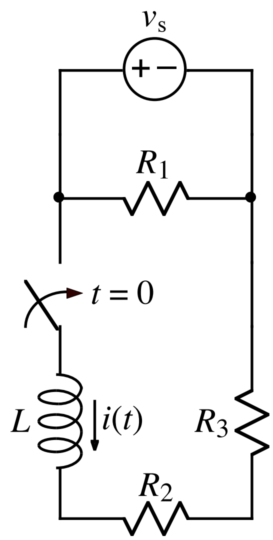
HOMEWORK #12 solution

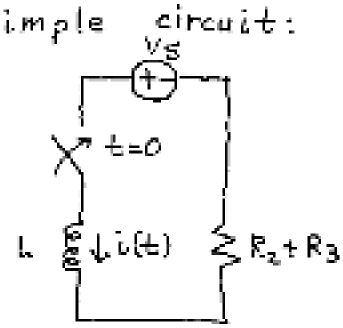
Ex:



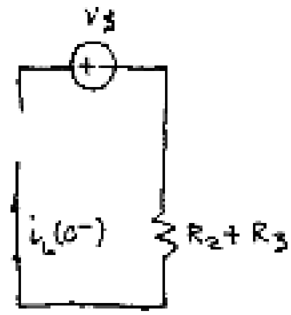
After being open for a long time, the switch closes at $t = 0$. Write an expression for $i_L(t > 0)$ in terms of no circuit quantities other than R_1, R_2, R_3, v_s, and L.

sol'n: Since R_1 is directly across v_s, it is a 2nd circuit in parallel with the 1st circuit consisting of all components below it that is directly across v_s. Consequently, we can solve the circuits separately, as though each had its own v_s source.
$\therefore$ we ignore R_1.

We also combine R_2 and R_3, yielding a simple circuit:

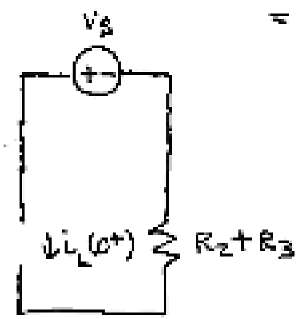


$t=0^-$ model: L = wire, find $i_L(0^-)$



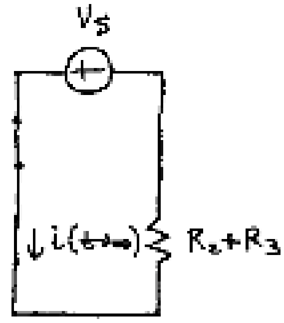
No current flows, owing to the open circuit.

$t=0^+$ model: $i_L(0^+)=i_L(0^-)$, L = i source
= 0 A = open circuit



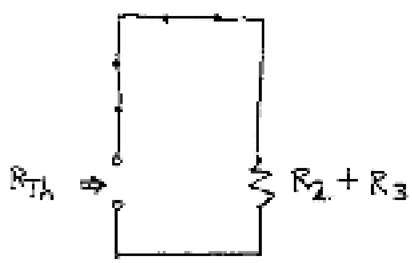
We have $i(0^+) = i_L(0^+) = i_L(0^-) = 0A$

$t\to\infty$ model: L = wire, find $i(t\to\infty)$



By Ohm's law, $i(t \to \infty) = \dfrac{V_S}{R_2 + R_3}$

$\tau = \dfrac{L}{R_{Th}}$: We find R_{Th} by turning off independent source v_S and looking into circuit from terminals where L is attached.



Clearly, $R_{Th} = R_2 + R_3$.

Now we use the general form of solution:

$$i(t) = i(t \to \infty) + \left[i(0^+) - i(t \to \infty) \right] e^{-t/\tau}, \quad t > 0$$

or
$$i(t) = \frac{V_S}{R_2 + R_3} + \left[0 - \frac{V_S}{R_2 + R_3} \right] e^{-t/[L/(R_2+R_3)]}, \quad t > 0$$

or
$$i(t) = \frac{V_S}{R_2 + R_3} \left[1 - e^{-t(R_2+R_3)/L} \right], \quad t > 0$$