

Ex: Write phasors (as both $Ae^{j\phi}$ and $A\angle\phi$) for each of the following signals:

a) $v(t) = 4 \cos(100t + 30^\circ) \text{ V}$

b) $i(t) = 7 \sin(\omega t - 45^\circ) \text{ mA}$

c) $i(t) = 50 \text{ nF} \cdot \frac{d}{dt} 4 \cos(100t + 30^\circ) \text{ V}$

d) $v(t) = 17 \text{ } \mu\text{H} \cdot \frac{d}{dt} 7 \sin(60t - 45^\circ) \text{ mA}$

e) $v(t) = 4 \cos(100t + 30^\circ) \text{ V} + 3 \sin(100t - 150^\circ) \text{ V}$

SOL'N: a) The magnitude of the phasor is the magnitude of the sinusoid, and the phase angle in the exponent of the phasor is the phase shift of the cosine waveform.

$$P[v(t) = 4 \cos(100t + 30^\circ) \text{ V}] = 4e^{j30^\circ} \text{ V}$$

b) The phasor of $\sin(\omega t)$ is $-j$.

$$P[i(t) = 7 \sin(\omega t - 45^\circ) \text{ mA}] = 7(-j)e^{-j45^\circ} \text{ mA}$$

or

$$P[i(t)] = 7e^{-j90^\circ} e^{-j45^\circ} \text{ mA} = 7e^{-j135^\circ} \text{ mA}$$

or

$$P[i(t)] = 7\angle -135^\circ \text{ mA}$$

c) When we take a derivative, we multiply by $j\omega$.

$$P\left[i(t) = 50 \text{ nF} \cdot \frac{d}{dt} 4 \cos(100t + 30^\circ) \text{ V}\right] = 50 \text{ nF} \cdot j\omega \cdot 4e^{j30^\circ} \text{ V}$$

or

$$P[i(t)] = 50 \text{ nF} \cdot j100 \text{ s}^{-1} \cdot 4e^{j30^\circ} \text{ V} = j20e^{j30^\circ} \mu\text{A} = 20e^{j90^\circ} e^{j30^\circ} \mu\text{A}$$

or

$$P[i(t)] = 20e^{j120^\circ} \mu\text{A}$$

d) Here, we have multiplication by $j\omega$ for the derivative and $-j$ for $\sin()$.

$$P\left[v(t) = 17 \mu\text{H} \cdot \frac{d}{dt} 7 \sin(60t - 45^\circ) \text{ mA}\right] = 17\mu\text{H} \cdot j60\text{s}^{-1} \cdot (-j7)e^{-j45^\circ} \text{ mA}$$

$$P[v(t)] = 7.14e^{-j45^\circ} \mu\text{V}$$

e) We convert the two waveforms to phasors before adding.

$$\begin{aligned} P[v(t) = 4 \cos(100t + 30^\circ) \text{ V} + 3 \sin(100t - 150^\circ) \text{ V}] \\ = 4e^{j30^\circ} + 3(-j)e^{-j150^\circ} \text{ V} \end{aligned}$$

or

$$P[v(t)] = 4e^{j30^\circ} + 3e^{-j90^\circ} e^{-j150^\circ} \text{ V} = 4e^{j30^\circ} + 3e^{-j240^\circ} \text{ V}$$

We could convert each term to rectangular form and sum, but a more efficient approach is to observe that the vectors are perpendicular. The first phasor is a vector of length 4 at an angle of 30° to the real axis. Adding the second phasor to the first creates a 4, 3, 5 triangle. The hypotenuse is the total phasor and has length 5. The angle of the phasor is 30° from the 1st phasor plus the angle in the 4, 3, 5 triangle between the sides of length 4 and 5.

$$P[v(t)] = 5\angle[30^\circ + \tan^{-1}(3/4)]\text{V} = 5\angle30^\circ + 36.9^\circ\text{V} = 5\angle66.9^\circ\text{V}$$