Ex: Derive a symbolic expression for the impedance of an \( R \), an \( L \), and a \( C \) in parallel at frequency \( \omega \). Rationalize the expression so the denominator is real.

Sol’N: When working with parallel impedances, it is typically easier to use the summation of conductance form of parallel impedance.

\[
\begin{align*}
\mathbf{z} &= \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} + \frac{1}{\frac{1}{j\omega L}}} \\
&= \frac{1}{\frac{1}{R} + \frac{j\omega C}{1} + \frac{1}{j\omega L}}
\end{align*}
\]

Keeping real parts and imaginary terms together reduces the complexity, so we sum the imaginary terms.

\[
\mathbf{z} = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)}
\]

At this point, we may choose from several different approaches. One choice would be to rationalize the expression. Another choice would be to write \( \mathbf{z} \) as a ratio of polynomials in \( \omega \). A third choice would be to make the denominator unitless by multiplying by \( R \). Their are more choices available as well, and which is chosen is a matter of personal preference. Here, we make the third choice and make the denominator unitless.

\[
\mathbf{z} = \frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)}
\]

Now we rationalize the expression to obtain our final answer.

\[
\begin{align*}
\mathbf{z} &= \frac{R}{1 + jR\left(\omega C - \frac{1}{\omega L}\right)} \cdot \frac{1 - jR\left(\omega C - \frac{1}{\omega L}\right)}{1 - jR\left(\omega C - \frac{1}{\omega L}\right)} \\
&= \frac{R - jR^2\left(\omega C - \frac{1}{\omega L}\right)}{1^2 + R^2\left(\omega C - \frac{1}{\omega L}\right)^2}
\end{align*}
\]