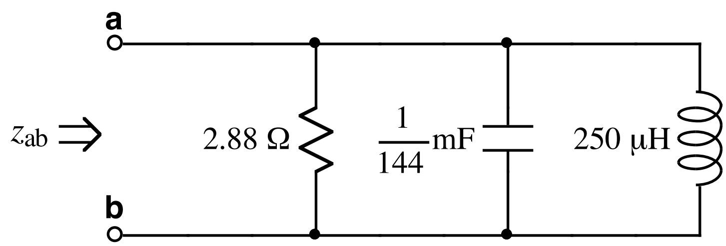
HOMEWORK #15 solution

Ex:



Find a frequency, ω, that causes z_{ab} to have a phase angle of –45°, (i.e., imaginary part is the negative of the real part). Hint: use admittance, (the reciprocal of impedance).

sol'n: For single components in parallel, using admittance = 1/z is helpful.

$$\frac{1}{z_{ab}} = \frac{1}{z_R} + \frac{1}{z_C} + \frac{1}{z_L}$$

Here, we have $z_C = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

$$= \frac{-j}{\omega \cdot \frac{1}{144m}} \Omega = \frac{-j}{\omega} 144m$$

$$z_L = j\omega L = j\omega \cdot 250\mu$$

If $\angle z_{ab} = -45°$, then $z_{ab} = k(1-j)$ where k is a positive real number.

Then $\frac{1}{z_{ab}} = \frac{1}{k(1-j)} = \frac{1+j}{k(1-j)(1+j)} = \frac{1+j}{2k}$.

Thus, $\angle \frac{1}{z_{ab}} = 45°$ and $\mathrm{Re}\left[\frac{1}{z_{ab}}\right] = \mathrm{Im}\left[\frac{1}{z_{ab}}\right]$.

We observe that the values of $\frac{1}{Z_C}$ and $\frac{1}{Z_L}$ are pure imaginary and constitute the entire imaginary part of $\frac{1}{Z_{ab}}$:

$$\mathrm{Im}\left[\frac{1}{Z_{ab}}\right] = \mathrm{Im}\left[\frac{1}{Z_C} + \frac{1}{Z_L}\right]$$

$$= \mathrm{Im}\left[j\omega C + \frac{1}{j\omega L}\right]$$

$$= \mathrm{Im}\left[j\omega C - \frac{j}{\omega L}\right]$$

$$= \omega C - \frac{1}{\omega L}$$

Note: Im[ ] has a <u>real</u> value. Im[a+jb] = b rather than jb.

The real part of Z_{ab} consists entirely of $\frac{1}{R}$:

$$\mathrm{Re}\left[\frac{1}{Z_{ab}}\right] = \mathrm{Re}\left[\frac{1}{R}\right] = \frac{1}{R}$$

Now we solve $\mathrm{Re}\left[\frac{1}{Z_{ab}}\right] = \mathrm{Im}\left[\frac{1}{Z_{ab}}\right]$

or $\quad \frac{1}{R} = \omega C - \frac{1}{\omega L}.$

or $$\frac{1}{RC}\omega = \omega^2 - \frac{1}{LC}$$

or $$\omega^2 - \frac{1}{RC}\omega - \frac{1}{LC} = 0$$

or $$\omega = \frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Note: since $\omega > 0$, we use only $+\sqrt{}$.

$$\omega = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Now we calculate values.

$$\frac{1}{2RC} = \frac{1}{2(2.88)\ \frac{1m}{144}\ s} = \frac{1k \cdot 1k}{2\,\frac{(2.88k)}{144}\ s}$$

$$\frac{1}{2RC} = \frac{1M}{40s} = 25k/s$$

$$\frac{1}{LC} = \frac{1}{250\mu \cdot \frac{1}{144} m\ s^2}$$

$$" = \frac{144}{250}\ G/s^2 \cdot \frac{4}{4}$$

$$" = 4(144)\ M/s^2$$

$$\frac{1}{LC} = (24\ k/s)^2$$

Using values, we have the following:

$$\omega = 25\,k/s + \sqrt{(25\,k/s)^2 + (24\,k/s)^2}$$

$$\omega \doteq 25\,k/s + 34.7\,k/s$$

$$\omega \doteq 59.7\,k/s$$