Ex:

\[ v_2(t) = 30 \sin(6kt + 45^\circ) \text{ V} \]

\[ i(t) = 0.5 \cos(6kt - 135^\circ) \text{ A} \]

a) Choose an R, an L, or a C to be placed in the dashed-line box to make

\[ i(t) = 0.5 \cos(6kt - 135^\circ) \text{ A}. \]

b) State the value of the component you chose for Problem 2. Note that the value of the component cannot be negative.

sol'n: We first transform the circuit to
the frequency domain.

\[ V_0 = 30 \cdot (-j) \cdot 1 \angle 45^\circ = 30 \angle 45^\circ \cdot -90^\circ = 30 \angle -45^\circ \]

Note: \(-j = 1 \angle -90^\circ\)

\[ j \omega L = j \cdot 6 \cdot 1 \text{mH} = j \cdot 6 \Omega \]

\[ I = \frac{1}{2} \angle -135^\circ \text{ A} \]

By Ohm's law, \( I = \frac{V_0}{\Xi + j \cdot 6 \Omega} \).
or \( z + j6\alpha = \frac{V_g}{I} \)

We could solve for \( z \) directly in this case, but it is instructive to solve the problem using phase and magnitude separately.

\[ \angle (z + j6\alpha) = \angle V_g - \angle I \]

Note: \( \angle \left( \frac{V_g}{I} \right) = \angle V_g - \angle I \)

\[ \angle (z + j6\alpha) = -45^\circ - 135^\circ = 90^\circ \]

We conclude that \( z + j6\alpha \) must be purely imaginary and positive.

This means that \( z \) might be an \( L \) or a \( C \). If it is a \( C \), however, it must be large enough that \( z \) is smaller in magnitude than \( j6\alpha \).

In either case, \((L \text{ or } C)\), we can write

\[ z + j6\alpha = jk \]

where \( k \) is a positive constant.

Now we consider magnitude.
\[ |z + j6| = \left| \frac{V_3}{I} \right| = \frac{|V_3|}{|I|} = \frac{30}{1/2} = 60 \]

Since we know \( z + j6 = j6 \), we have

\[ |z + j6| = |z| = 60 \]

\[ z = j60 - j6 = j54 \]

We must use \( L \) to get a positive imaginary \( z \):

\[ z = j\omega L = j \cdot 6k \cdot L = j54 \]

\[ L = \frac{54}{6k} = 9 \text{ mH} \]