Ex:

After being in position e for a long time, the switch moves from e to d at $t = t_0$.

Rail voltages = $\pm 12$ V

a) Choose either an R or C to go in box a and either an R or C to go in box b to produce the $v_0(t)$ shown above. (Note that $v_0$ stays high forever after $t_0 + 2$ ms.) Specify which element goes in each box and its value.

b) Sketch $v_1(t)$, showing numerical values appropriately.

c) Sketch $v_2(t)$, showing numerical values appropriately.

d) Sketch $v_3(t)$. Show numerical values for $t < t_0$, for $t_0 < t < t_0 + 2$ ms, and for $t_0 + 2$ ms $< t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.
SOLN: a) For \( v_0 \) to be low, (i.e., \(-12V\)), we must have \( v_2 < v_1 \).

To find \( v_1 \), we slide the 4V source through the 6kΩ resistor and find that we have the equivalent of a \(-15V\) source and a voltage divider formed by the 3kΩ and 6kΩ resistors.

\[
v_1 = -15V \cdot \frac{3k\Omega}{3k\Omega + 6k\Omega} = -5V
\]

At \( t=0^- \), we must have \( v_2 < -5V \).

This is possible only if box a contains a resistor and box b contains a capacitor. If a is an R and b is a C, then the C will charge until \( v_2 = -10V < v_1 \).

When the switch moves from c to d, the capacitor voltage starts charging toward 0V, but it will still be \(-10V\) initially. This gives the desired waveform for \( v_0(t) \): \( v_0 \) will go high when \( v_2 = v_1 = -5V \).

Note: The reasons why other components in boxes a and b fail to yield the desired \( v_0(t) \) are as follows:
\( a = R \) and \( b = R \) cannot give a waveform that changes after a delay. \( v_o \) would have to change instantly at \( t = t_a \).

\( a = C \) and \( b = R \) would result in \( C \) charging until no current flows in \( R \). This means \( v_2 = 0V \), or \( v_2 > v_1 \), causing \( v_o \) to be high before \( t = t_a \).

\( a = C \) and \( b = C \) would result in an arbitrary voltage at \( v_2 \). The total voltage drop across the two \( C \)’s would be 10V. When the switch changes from \( c \) to \( d \), the capacitors would charge until the total voltage drop across them was 0V. The same current would flow in both \( C \)'s, causing a voltage change that would be inversely proportional to the \( C \) values. The waveform shown for \( v_o(t) \) could be produced, but there is a lack of control over the initial value of \( v_2 \). This would make the timing of the \( v_o(t) \) waveform uncertain. Thus, we reject this solution.
Now we find possible values for $R$ and $C$. We have the following circuit model for $t > t_0$:

\[ V_c(t > t_0) = V_c(t \to \infty) + \left[ V_c(t_0^+) - V_c(t_\to \infty) \right] e^{-t/\tau} \]

\[ V_c(t_0^+) = -10 \text{V} \]

\[ V_c(t > t_0) = -10 e^{-t/\tau} \text{V} \] (where we take $t_0 = 0$)

where \( \tau = (R + 1 \text{ k}\Omega) C \)

We want \( V_c(t = 2 \text{ ms}) = V_1 = -5 \text{V} \)

or \( -10 e^{-2 \text{ ms} / \tau} = -5 \text{V} \)

\[ e^{-2 \text{ ms} / \tau} = \frac{1}{2} \]

\[ -2 \text{ ms} = \tau \ln \frac{1}{2} \]

\[ \tau = \frac{2 \text{ ms}}{\ln 2} \approx 2.9 \text{ ms} \]

One solution is $R = 1.9 \text{ k}\Omega$ and $C = 1 \mu\text{F}$.

Note: $R = 0 \Omega$ is min $R$, $C = 2.9 \mu\text{F}$ is max $C$. 
b) \( v_1(t) = -5V \) as shown earlier.

\[
v_3 = -12V \cdot \frac{5 \text{k}\Omega}{2 \text{k}\Omega + 5 \text{k}\Omega} = -\frac{60}{7} \text{V}.
\]
When $v_0$ is high, the top diode will act like an open circuit, leaving the bottom part of the circuit disconnected from $v_0$, (or any other power source).

Thus $v_3 = 0V$ when $v_0$ is high.