Ex:

\[ \begin{align*}
  v_a & = +\frac{v_2}{\alpha v_2 - R_2} \\
  R_1 & = \frac{v_2}{i_1} \\
  v_2 & = -\frac{v_2}{R_1 + \alpha v_2} \\
  i_1 & = \frac{v_2}{R_1} \\
  \end{align*} \]

a) Derive an expression for \( i_1 \). The expression must not contain more than the circuit parameters \( \alpha, v_a, R_1, \) and \( R_2 \). **Note:** \( \alpha \neq 0 \).

b) Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

**Sol'n:**

\( a) \) we have two inner \( v \)-loops:

- \( v_a + v_2 + i_1 R_1 = 0V \) (top \( v \)-loop).
  - Using Ohm's law here.

- \( -i_1 R_1 + \alpha v_2 = 0V \) (bottom \( v \)-loop).

From the 2nd eq'n, \( \alpha v_2 = i_1 R_1 \)

or \( v_2 = \frac{i_1 R_1}{\alpha} \).

From 1st eq'n, \( -v_a - \frac{i_1 R_1}{\alpha} + i_1 R_1 = 0V \)

or \( i_1 \left( R_1 - \frac{R_1}{\alpha} \right) = v_a \)

or \( i_1 = \frac{v_a}{R_1 \left( 1 - 1/\alpha \right)} \)
b) For the consistency check, we choose values of sources and Rs that yield a simpler circuit for which solution is obvious. Many checks may be possible. Only one is required here.

\[ v_a = 0 \text{V}, \quad R_1 = 1 \Omega, \quad R_2 = 2 \Omega, \quad \kappa = 3. \]

The circuit has no independent power source. Thus, all currents and voltages \( = 0 \). So \( i_1 = 0 \text{A} \).

Now we try our formula from (a):

\[ i_1 = \frac{0}{1 \Omega (1 - 1/3)} = 0 \quad \checkmark \quad \text{(consistent)} \]

\[ v_a = 12 \text{V} \]

\[ R_1 = \infty \quad \text{(open circuit)}, \quad R_2 = 2 \Omega, \quad \kappa = 3 \]

If \( R_1 \) is open circuit, then \( i_1 = 0 \text{A} \).

Now we try our formula from (a):

\[ i_1 = \frac{12 \text{V}}{\infty \Omega (1 - 1/3)} = \frac{12 \text{V}}{\infty \Omega} = 0 \text{A} \]

Note: Some consistency checks might lead to invalid circuits such as \( v \)-sources shorted out. Avoid these. This particular circuit is prone to that problem.