1. (75 points)

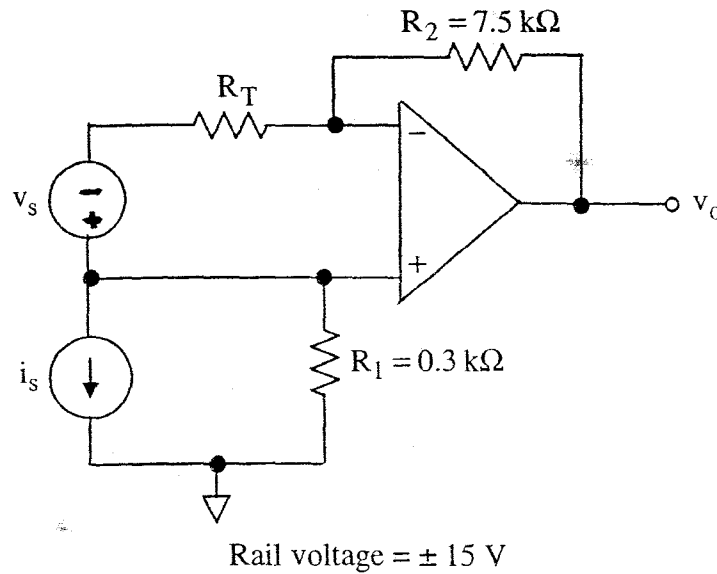$$R_2 = 7.5 \text{ k}\Omega$$

$$R_1 = 0.3 \text{ k}\Omega$$

Rail voltage = ± 15 V

Design an electronic thermometer using the circuit diagram shown above. The voltage v_0 is used to indicate temperature. Use a thermister with a resistance described by

$$R_T = R_o e^{\beta\left(\frac{1}{T} - \frac{1}{300}\right)}$$

where $R_o = 2.625 \text{ k}\Omega$, $\beta = 1200°\text{K}$, and T is temperature in °K.

Pts		
30	a.	Derive a symbolic expression for v_0. The expression must contain not more than the parameters i_s, V_s, R_1, R_2, and R_T. **Hint: Use superposition.**
10	b.	Calculate the numerical values of R_T (273°K) and R_T (373°K).
15	c.	Determine a value for v_s such that $v_0(T = 373°\text{K}) - v_0(T = 273°\text{K}) = 1\text{V}$
10	d.	Using your answer to (c), determine a value of i_s such that $v_0 (T = 273°\text{K}) = 0 \text{ V}$.
10	e.	Using the component values you chose above, calculate v_0 when T = 323°K. Make a rough sketch of v_0 vs. T on the basis of the values when T = 273°K, 323°K, and 373°K. On the same axes, sketch the ideal linear response.

Sol'n: 1. a) Use superposition

Case I: V_s on, i_s off = open
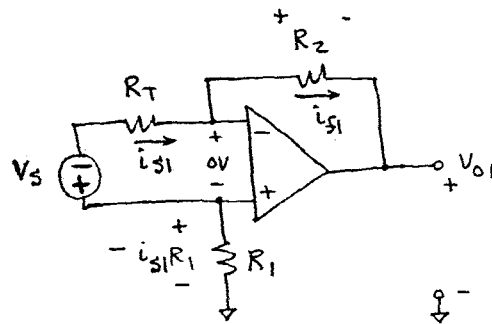
Negative feedback $\Rightarrow$ 0V across + and − terminals.

From v loop around V_s, R_T, and +− terminals,

we have $i_{s1} = -\dfrac{V_s}{R_T}$.
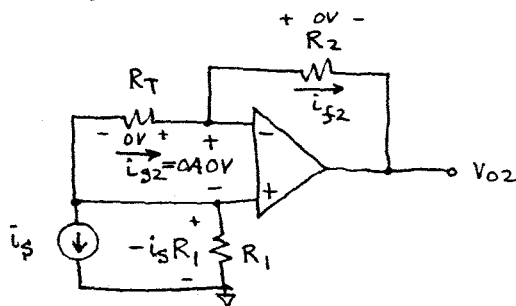
From v loop around R_1, +− terminals, R_2, and V_o,

we have $-i_{s1}R_1 + 0V - i_{s1}R_2 - V_{o1} = 0V$

Now use $i_{s1} = i_{f1}$ and solve for V_{o1}:

$$--\frac{V_s}{R_T} R_1 + 0V --\frac{V_s R_2}{R_T} - V_{o1} = 0V$$

$$V_{o1} = V_s \frac{R_1 + R_2}{R_T}$$

Case II: i_s on, V_s off = wire

We 0V across R_T $\Rightarrow$ $i_s = 0$.
Since $i_{s2} = i_{f2} = 0A$, we have $V_{o2} = v_- = V_+$

$$V_+ = -i_s R_1 \quad \Rightarrow \quad V_{o2} = -i_s R_1$$

Sum $\boxed{V_o = V_{o1} + V_{o2} = V_s \dfrac{R_1 + R_2}{R_T} - i_s R_1}$

soln: 1.b)

$$R_T(273°K) = 2.625k\Omega\, e^{1200°K\left(\frac{1}{273°K} - \frac{1}{300°K}\right)}$$

$$\boxed{R_T(273°K) \doteq 3.9k\Omega}$$

$$R_T(373°K) = 2.625\,k\Omega\, e^{1200°K\left(\frac{1}{373°K} - \frac{1}{300°K}\right)}$$

$$\boxed{R_T(373°K) \doteq 1.2k\Omega}$$

c) $1V = V_o(373°K) - V_o(273°K) = V_S\dfrac{(R_1 + R_2)}{R_T(373°K)} - i_S R_1$

$$- \left(V_S \dfrac{R_1 + R_2}{R_T(273°K)} - i_S R_1 \right)$$

$$= V_S (R_1 + R_2)\left(\dfrac{1}{R_T(373°K)} - \dfrac{1}{R_T(273°K)} \right)$$

$$= V_S (0.3k\Omega + 7.5k\Omega)\left(\dfrac{1}{1.2k\Omega} - \dfrac{1}{3.9k\Omega} \right)$$
$$\underbrace{}_{7.8k\Omega}$$

$$V_S = \dfrac{1.2k\Omega \| (3.9k\Omega)}{7.8k\Omega} V = \dfrac{0.3k\Omega}{0.3k\Omega} \cdot \dfrac{4\| -13V}{26} = \dfrac{-4+13}{(4-13)26}\,V$$

$$\boxed{V_S = \dfrac{2}{9}\,V}$$

d) $0V = V_o(273°K) = V_S\dfrac{R_1 + R_2}{R_T(273°K)} - i_S R_1 = \dfrac{2}{9}V\dfrac{7.8k\Omega}{3.9k\Omega} - i_S\,0.3k\Omega$

$$i_S\,0.3k\Omega = \dfrac{4}{9}V \Rightarrow i_S = \dfrac{4}{9}\dfrac{1V}{0.3k\Omega} = \dfrac{4}{2.7}\,mA$$

$$\boxed{i_S \doteq 1.48\,mA \approx 1.5\,mA}$$

e) $R_T(323°K) = 2.625\,k\Omega\, e^{1200°K\left(\frac{1}{323°K} - \frac{1}{300°K}\right)} = 1.97\,k\Omega \approx 2k\Omega$

$$V_o(323°K) \approx \dfrac{2}{9}V\dfrac{7.8k\Omega}{2k\Omega} - 1.5\,mA\,0.3k\Omega = 0.417\,V$$

Graph of $V_o(T)$ versus T showing linear response and actual response, with markings at $1V$, $0.417V$, $0V$ on vertical axis and $273°K$, $323°K$, $373°K$ on horizontal axis.

2. (65 points)

v_1

v_2

$3\,k\Omega$ $2\,k\Omega$ $12\,k\Omega$ $10\,k\Omega$

$6V$ $1V$

a

b

$t = t_o$ $30\,k\Omega$ v_3

v_o
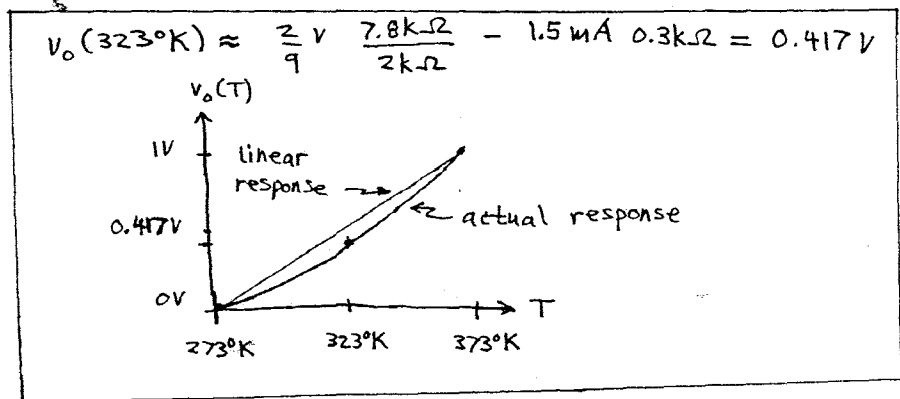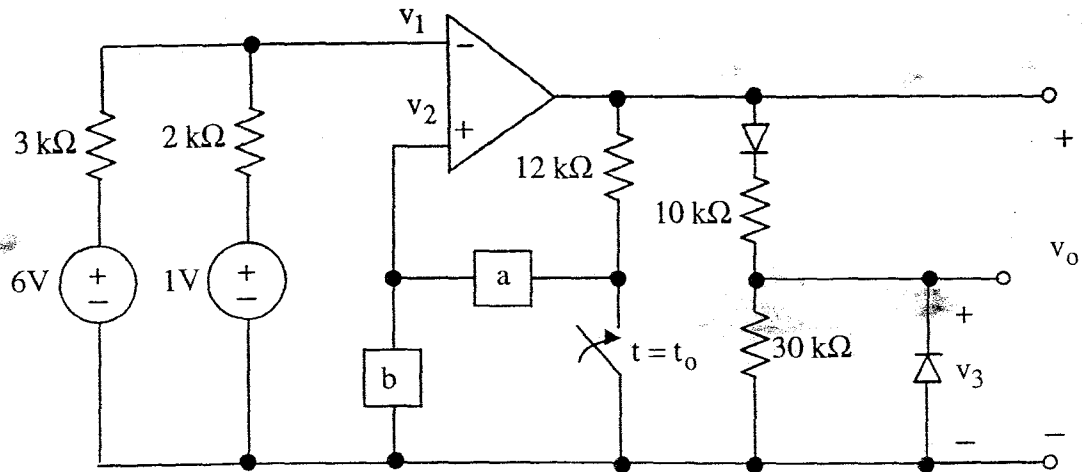
Rail voltages $= \pm 12$ V

After being open for a long time, the switch closes at $t = t_o$.

$v_o(t)$

12 V

t_o $t_o + 10\,\mu s$ t

-12 V
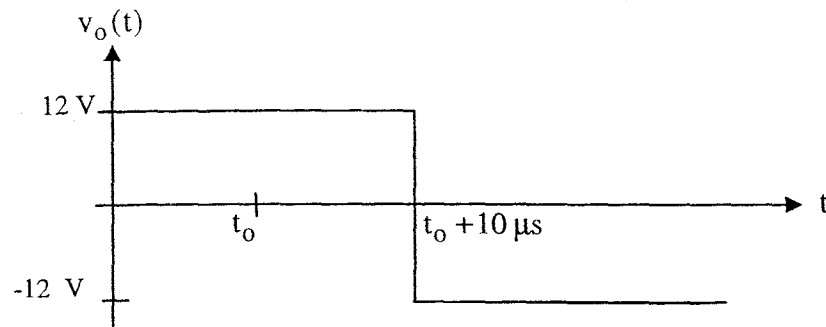
Pts

35 a. Choose either an R or L to go in box a and either an R or L to go in box b to produce the $v_o(t)$ shown above. Specify which element goes in each box and its value.

5 b. Sketch $v_1(t)$, showing numerical values appropriately.

15 c. Sketch $v_2(t)$, showing numerical values appropriately.

10 d. Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 10\,\mu s$, and for $t_o + 10\,\mu s < t$. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

Explain your work carefully.

Sol'n: 2.a) Ignore diode and resistor network on output (since it doesn't affect v_0).

For $t < t_0$, $v_0 = +12V \Rightarrow v_2 > v_1$.

Calculate v_1 using node-v method: $\dfrac{v_1 - 6V}{3k\Omega} + \dfrac{v_1 - 1V}{2k\Omega} = 0A$
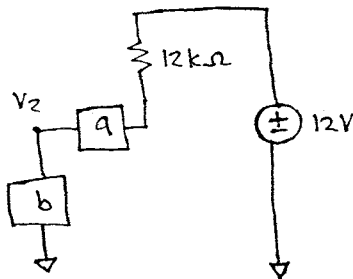
$v_1 \left(\dfrac{1}{3k\Omega} + \dfrac{1}{2k\Omega} \right) = \dfrac{6V}{3k\Omega} + \dfrac{1V}{2k\Omega}$

mult both sides by $6k\Omega$

$v_1 (2+3) = 6V \cdot 2 + 1V \cdot 3 = 15V$, $v_1 = \dfrac{15V}{5} = 3V$

So we need $\underline{\underline{v_2 > 3V.}}$

Circuit model: $v_0 = 12V$, switch open

If we have an L, it will be equivalent to a wire.

Consider possibilities:

case I: $a = L$ and $b = L$ $L = $ wire

$\qquad v_2 = 0V$ from v divider

$\qquad$ Doesn't work.

case II: $a = R$ and $b = L = $ wire

$\qquad v_2 = 0V$ from v divider

$\qquad$ Doesn't work.

case III: $a = R_1$ and $b = R_2$

$\qquad$ We can choose R_1 and R_2 to achieve $v_2 > 3V$, but we cannot get a delay in v_0 dropping from $+12V$ to $-12V$.

case IV: $a = L$ and $b = R$

$\qquad$ Since $L = $ wire and we can pick R, we can achieve $v_2 > 3V$. When switch moves, the L continues to carry same current initially. Thus, $v_2 > v_1$ is sustained for delay. Should work.

soln: 2.a) cont.

When switch closes, we have RL circuit that determines V_2. Time constant $\tau = L/R$.
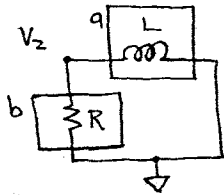
Output V_0 drops when V_2 drops below 3V.

As $t \to \infty$, the L in 'a' acts like a wire and the switch is closed $\Rightarrow V_2(t \to \infty) = 0V$

Without additional constraints, we may choose any V_2 between 3V and 12V. One choice is $V_2(0^-) = 6V$. Using v-divider of 12kΩ and 'b', $\boxed{b = 12k\Omega}$.

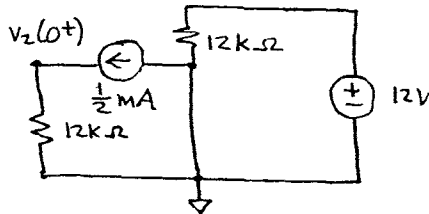we want $V_2(t = 10\mu s) = 3V$ so V_0 drops at time $t_0 + 10\mu s$. (Assume $t_0 = 0s$)

$$v_2(t>0) = v_2(t \to \infty) + [V_2(0^+) - v_2(t \to \infty)]e^{-t/\tau}$$
$$\text{"} = 0V + [V_2(0^+) - 0V]e^{-t/\tau}$$
$$\text{"} = V_2(0^+)e^{-t/\tau}, \quad \text{Now find } V_2(0^+).$$

Consider $t = 0^-$: $\quad L = $ wire $\quad R = 12k\Omega$
$$i_L(0^-) = \frac{12V}{12k\Omega + 12k\Omega} = \frac{1}{2}mA$$

$t = 0^+$: $\quad L = i$ src $\quad$ where $i_L(0^+) = i_L(0^-) = \frac{1}{2}mA$

$$V_2(0^+) = \frac{1}{2}mA \cdot 12k\Omega$$
$$V_2(0^+) = 6V$$

$$v_2(t>0) = 6Ve^{-t/\tau}$$

We want $v_2(10\mu s) = 3V = 6Ve^{-10\mu s/\tau}$

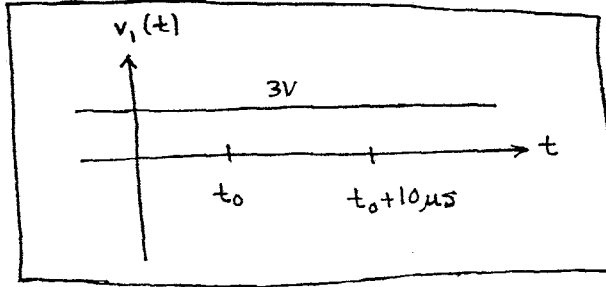$$\frac{3V}{6V} = \frac{1}{2} = e^{-10\mu s/\tau}, \qquad \ln\frac{1}{2} = -10\mu s/\tau$$

$$\tau = \frac{-10\mu s}{\ln\frac{1}{2}} = \frac{10\mu s}{\ln 2} = 14.4\mu s = \frac{L}{R} = \frac{L}{12k\Omega}$$
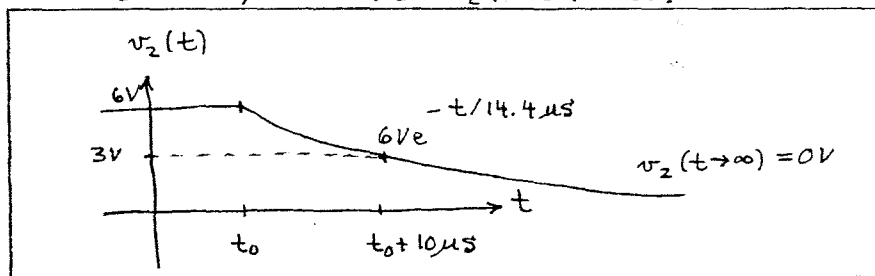
$$L = 14.4\mu s \cdot 12k\Omega = 173\ mH$$

Summary: $\boxed{\begin{array}{l} a = L = 173\ mH \\ b = R = 12k\Omega \end{array}}$

soln: 2.b) As shown in sol'n for (a), $V_1(t) = 3V$. It never changes.

$v_1(t)$
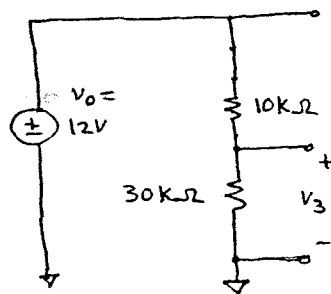
$3V$

t

t_0 $t_0 + 10\mu s$

c) From sol'n to (a), we have $V_2(t>0) = 6V e^{-t/14.4\mu s}$.

For $V_2(t<0)$, we have $V_2(t<0) = 6V$.

$v_2(t)$

$6V$

$-t/14.4\mu s$

$6V e$

$3V$

$v_2(t\to\infty) = 0V$

t

t_0 $t_0 + 10\mu s$

d) When $v_0 > 0V$, top diode = wire, bottom diode = open.

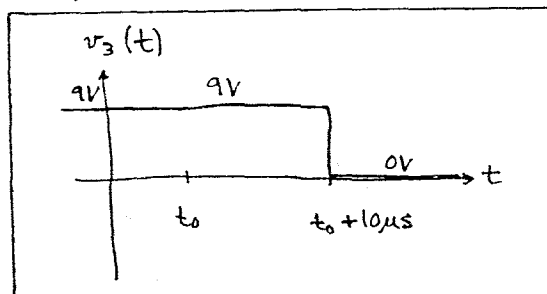$v_0 = 12V$ $\pm$

$10k\Omega$

$+$

$30k\Omega$ V_3

$-$

$V_3 = 12V \cdot \dfrac{30k\Omega}{30k\Omega + 10k\Omega} = 9V$

When $v_0 < 0V$, top diode = open, bottom diode doesn't matter since no current

$10k\Omega$

$+$

$30k\Omega$ V_3

$-$
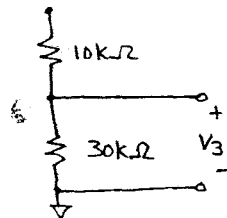
$V_3 = 0V$ since no current flowing

$v_3(t)$

$9V$ $9V$

$0V$ t

t_0 $t_0 + 10\mu s$

3. (35 points)

$v_1 = 28V$

$3 k\Omega$ $1 k\Omega$

$11 i_x$

i_x

$10 i_x$

28V

$6 k\Omega$

a

b

$\downarrow i_{sc}$ for finding R_{Thev}
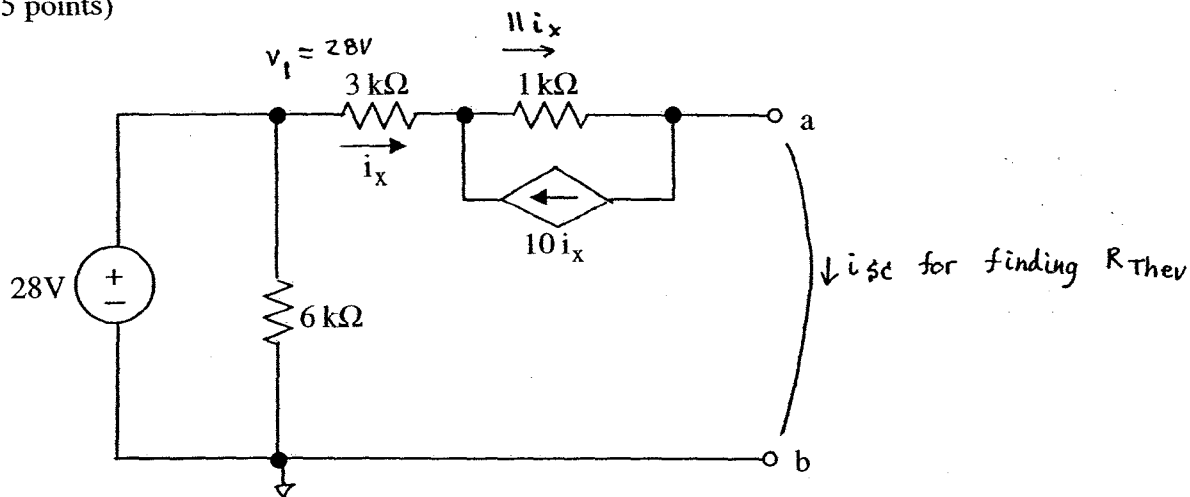
Pts

25 a. Find the Thevenin equivalent of the above circuit relative to terminals a and b.

5 b. If we attach R_L to terminals a and b, find the value of R_L that will absorb maximum power.

5 c. Calculate the value of that maximum power absorbed by R_L.

Sol'n: a) The $6k\Omega$ resistor is across the $28V$ source, so it may be ignored.

For V_{Thev} we use $V_{a,b}$ with no load. Since no current flows out of the 'a' terminal, $i_x = 0$.

∴ $10 i_x = 0A$ and v drop across $3k\Omega$ and $1k\Omega$ is zero.

∴ $V_{a,b} = 28V$ from V src   ∴ $\boxed{V_{Thev} = 28V}$

Now find i_{sc} flowing in wire connected from a to b.

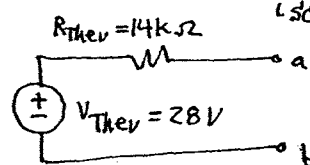From current sum at node on left end of $1k\Omega$, we have current $11 i_x$ flowing in $1k\Omega$ resistor.

Using v drops for $3k\Omega$ and $1k\Omega$, we must have

$i_x \cdot 3k\Omega + 11 i_x \cdot 1 k\Omega = 28V$ or $14 k\Omega \cdot i_x = 28V$

or $i_x = 2 mA$.   Since $i_{sc} = i_x$ and $R_{Th} = \dfrac{V_{Th}}{i_{sc}}$,

$\boxed{R_{Th} = \dfrac{28V}{2mA} = 14 k\Omega}$

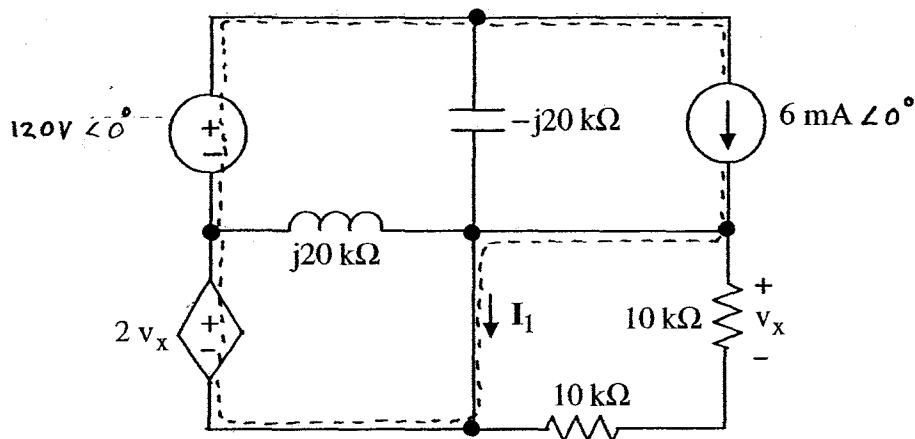$R_{Thev} = 14 k\Omega$

$V_{Thev} = 28V$

a

b

Sol'n: 3.b)   Max pwr when $\boxed{R_L = R_{Thev} = 14k\Omega}$

3.c)   Max pwr $= \dfrac{V_{Thev}^2}{4 R_{Thev}} = \dfrac{(28V)^2}{4 \cdot 14k\Omega} = 7(2)W$

$\boxed{Max \ pwr = 14}$ units; mW

4. (25 points)

Pts

20 a.   A frequency-domain circuit is shown above. Write the value of phasor I_1 in polar form.

5   b.   Given $\omega = \pi$ rad/s, write a numerical time-domain expression for $i_1(t)$, the inverse phasor of I_1.

Sol'n: a)   Since the two $10k\Omega$ resistors are shorted by wires.

∴ There is no v drop across the $10k\Omega$ resistors, and $V_x = 0V$.

Thus, the $2v_x$ dependent source $= 0V =$ wire

Superposition   Case I: 6mA on, 120V off = wire.

It follows that all of the 6 mA from the independent current source flows in the wires (shown as dashed lines above).

∴ $I_{11} = 6\,mA \angle 0°$

Case II: 120V on, 6mA off = open circuit
we observe that the $-j20k\Omega$ is directly across the 120V source, given the wires shown as dashed lines.

∴ $I_{12} = \dfrac{120V\angle 0°}{-j\,20k\Omega} = j\,6mA = 6mA \angle 90°$

Thus, $I_1 = I_{11} + I_{12} = 6\,mA \cdot (1+j)$

or $\boxed{I_1 = \sqrt{2} \cdot 6\,mA \angle 45°}$

b) $\boxed{i_1(t) = \sqrt{2} \cdot 6\,mA\,\cos(\pi t + 45°)}$