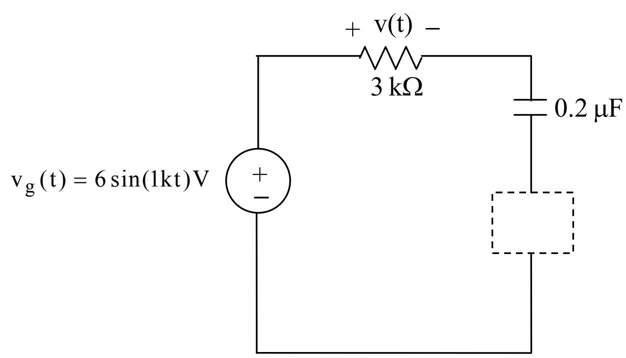
UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

ECE 1000 HOMEWORK #8 Spring 2005

1. Give numerical answers to each of the following questions:
   a. Rationalize $\frac{23 + j7}{15 - j8}$. Express your answer in rectangular form.
   b. Find the polar form of $(2 + j3)(3 + j2) + [3 + j16]^*$. Note the asterisk that means "conjugate".
   c. Find the following phasor: $P\left[-5\sin\left(100t - 30^\circ\right)\right]$.
   d. Find the magnitude of $\frac{100(3 + j4)(4 + j3)}{(7 + j)(7 - j)}$.
   e. Find the imaginary part of $(1 + j)e^{-j45^\circ}(j2)$.

2.



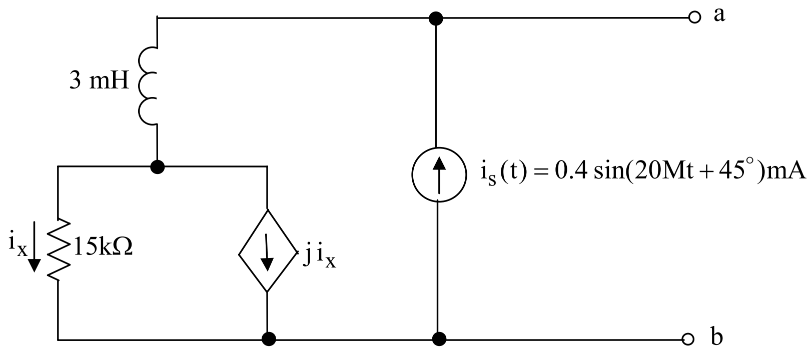
Choose an R, an L, or a C to be placed in the dashed-line box to make

$$v(t) = \mathbf{V}_o \cos(1kt - 45^\circ)V$$

where $\mathbf{V}_o$ is a real constant. State the value of the component you choose.

3. With your component from problem 2 in the circuit, calculate the resulting value of $\mathbf{V}_o$.

4.



Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

1. (25 points)

Give numerical answers to each of the following questions:

pts

(5) a. Rationalize $\frac{23+j7}{15-j8}$. Express your answer in rectangular form.

(5) b. Find the polar form of $(2+j3)(3+j2)+[+j16]^*$. Note the asterisk that means "conjugate".

(5) c. Find the following phasor: $P\left[-5\sin\left(100t-30^\circ\right)\right]$.

(5) d. Find the magnitude of $\frac{100(3+j4)(4+j3)}{(7+j)(7-j)}$.

(5) e. Find the imaginary part of $(1+j)e^{-j45^\circ}(j2)$.

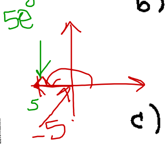
$\sin(xt) = \cos(xt - 90^\circ)$

$a + bj = \sqrt{a^2+b^2}\, e^{j\tan^{-1}(\frac{b}{a})}$

$re^{j\phi} = r\cos\phi + r\sin\phi\, j$

sol'n: 1. a) $\frac{23+j7}{15-j8}\cdot\frac{15+j8}{15+j8} = \frac{(23)15 - 7(8) + j[23(8)+7(15)]}{15^2+8^2} = \frac{289+j289}{289} = \boxed{1+j}$

$5e^{j180^\circ}$



b) $(2+j3)(3+j2) + [j16]^* = (2+j3)(3+j2) + -j16 = 6-6+j(9+4)-j16$

$= -j3 = \boxed{3\angle -90^\circ \text{ or } 3e^{-j90^\circ}}$

$\cos(100t-30-90^\circ)$

c) $P[-5\sin(100t-30^\circ)] = 5\angle 180^\circ \cdot 1\angle(-30^\circ - 90^\circ) = \boxed{5\angle 60^\circ}$

$\frac{1}{e^{j30^\circ}} = e^{-j30^\circ}$

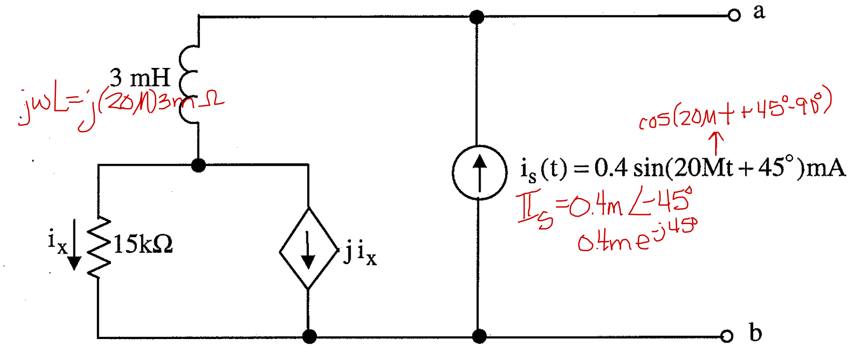
d) $\left|\frac{100(3+j4)(4+j3)}{(7+j)(7-j)}\right| = 100\frac{|3+j4||4+j3|}{|7+j||7-j|} = \frac{100\cdot\sqrt{3^2+4^2}\sqrt{4^2+3^2}}{\sqrt{7^2+1^2}\sqrt{7^2+1^2}} = \frac{100\cdot 5^2}{50} = \boxed{50}$

e) $\mathrm{Im}\left[(1+j)e^{-j45^\circ}(j2)\right] = \mathrm{Im}\left[\sqrt{2}\angle 45^\circ \cdot 1\angle -45^\circ \cdot 2\angle 90^\circ\right] = \mathrm{Im}\left[2\sqrt{2}\angle 90^\circ\right]$

$= \mathrm{Im}[j2\sqrt{2}] = \boxed{2\sqrt{2}}$

$\mathrm{Im}\left[2\sqrt{2}\angle 66^\circ\right] = \mathrm{Im}\left[2\sqrt{2}\cos(66^\circ) + \boxed{2\sqrt{2}\sin(66^\circ)}\, j\right]$

3. (35 points)



pts

(15) a. Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, and L. Label the dependent source appropriately.

(25) b. Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

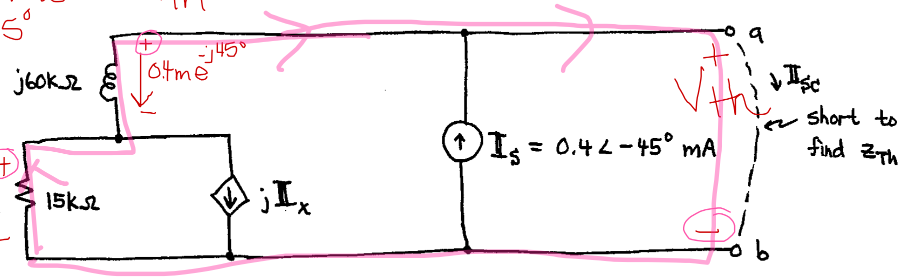
soln: 3. a) $\omega = 20M$ from $i_s(t)$ $\quad j\omega L = j\,20M\ 3m = j\,60k\Omega$

$+\mathbb{I}_x(15K) + 60Kj(.4m e^{-j45°}) = V_{th}$

$\mathbb{I}_x + j\mathbb{I}_x = .4m e^{-j45°}$

$\mathbb{I}_x = \dfrac{.4m e^{-j45°}}{(1+j)}$, $\quad (1+j) = \sqrt{2}e^{j45°}$

$\mathbb{I}_x = \dfrac{.4m}{\sqrt{2}} e^{j(-45°-45°)}$



b) Use $z_{eq} = \dfrac{\mathbb{I}_x\ 15k\Omega}{j\,\mathbb{I}_x} = -j15k\Omega$, (from $\frac{V}{\mathbb{I}} = z$), for dependent src.

$V_{th} = \frac{.4m}{\sqrt{2}} \cdot 15K(-j) + 60Ke^{j90°}(.4me^{-j45°})$

$V_{Th} = V_{ab}$ no load $= \mathbb{I}_s \cdot (j60k + 15k \| -j15k)\Omega \qquad 15k\Omega \cdot 1 \| -j = 15k\Omega\dfrac{(-j)}{1-j}$

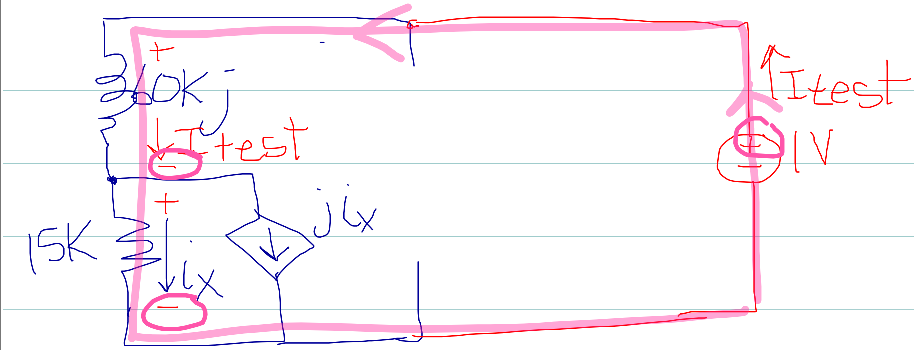
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad " = 15k\Omega\dfrac{(-j)(1+j)}{2} = 15k\Omega \cdot \dfrac{1-j}{2}$

$" = \frac{2}{5}\angle{-45°}$ mA $\cdot\left(\frac{15}{2}k + j\frac{15}{2}k \cdot 7\right)\Omega = \frac{2}{5}\frac{(1-j)}{\sqrt{2}}\frac{15}{2}(1+j7)$ V

$" = \frac{3}{\sqrt{2}}[1+7+j(7-1)]$ V $= \dfrac{3(8+j6)}{\sqrt{2}} = \boxed{\frac{24}{\sqrt{2}} + j\frac{18}{\sqrt{2}}}$ V or $\boxed{\frac{30}{\sqrt{2}}\angle 37°}$ $V = V_{Th}$

$z_{Th} = \dfrac{V_{Th}}{\mathbb{I}_{sc}}$, $\quad \mathbb{I}_{sc} = \mathbb{I}_s$ since 0V across z's on left, $\mathbb{I}_x = 0$ is soln.

$\boxed{z_{Th} = j60k + 15k \| -j15k\ \Omega = \frac{15}{2}k + j\frac{105}{2}k\Omega \text{ or } \frac{15k\sqrt{50}}{2}\angle 81°\Omega}$



$$I_{test} = i_x + j i_x$$

$$-I_{test}(60Kj) + 1 - 15K i_x = 0$$

$$i_x = \frac{-60Kj\, I_{test} + 1}{15K}$$

$$I_{test} = \left[\frac{-60Kj\, I_{test} + 1}{15K}\right](1+j)$$

$$15K\, I_{test} = -60Kj\, I_{test} - 60Kj^2 I_{test} + 1 + j$$

$j^2 = -1$

$$I_{test}(15K + \cancel{60Kj} - 60K) = \frac{(1+j)}{(-45K + 60Kj)}$$

$$Z_{th} = \frac{1}{I_{test}} = \frac{(-45K + 60Kj)}{(1+j)}$$

2. (40 points)

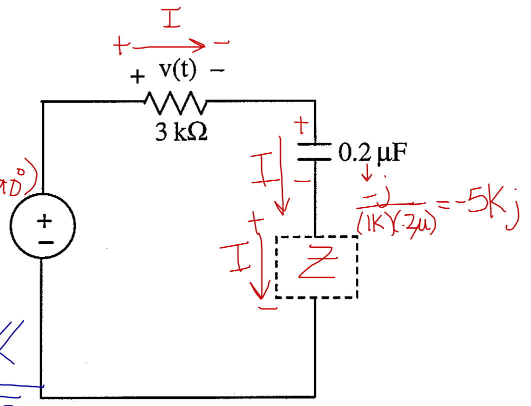
$I = \frac{-6j}{3K - 5Kj + Z}$

$V = I \cdot 3K$

$V = \frac{-6j \cdot 3K}{3K - 5Kj + Z}$

$\cos(1kt - 90^\circ) = 6\angle -90^\circ = 6(-j)$

$-j \over (1K)(.2\mu) = -5Kj$



$R = 2K$: $V = \left| \frac{+18K e^{-j90}}{\underbrace{5K - 5Kj}_{\sqrt{(5K)^2 + (5K)^2}\, e^{-j45^\circ}}} \right| = \frac{18K}{5K\sqrt{2}} = V_o$

pts

(25) a. Choose an R, an L, or a C to be placed in the dashed-line box to make

$V_o e^{-j45^\circ} \leftarrow$ $v(t) = \mathbf{V}_o \cos(1kt - 45^\circ)V$

where $\mathbf{V}_o$ is a real constant. State the value of the component you choose.

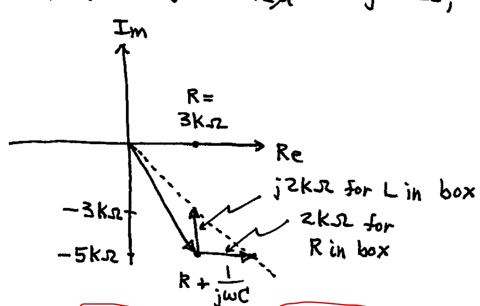
(10) b. With your component from (a) in the circuit, calculate the resulting value of $\mathbf{V}_o$.

sol'n: 2. a) $V = V_o \angle -45^\circ\, V$, $V_g = 6\angle -90^\circ\, V$, $V = V_g \dfrac{R}{R + \frac{1}{j\omega C} + Z_{box}}$ v-divider

$\angle V = \angle V_g + \angle R - \angle\left(R + \frac{1}{j\omega C} + Z_{box}\right)$

$-45^\circ = -90^\circ + 0^\circ - \angle\, R + \frac{1}{j\omega C} + Z_{box} \Rightarrow \angle\, R + \frac{1}{j\omega C} + Z_{box} = -45^\circ$

$R = 3k\Omega$, $\frac{1}{j\omega C} = \frac{1}{j \cdot 1k \cdot 0.2\mu} = -j5k\Omega$, $\omega = 1k$ r/s



$\boxed{\text{if L in box, } j\omega L = j2k\Omega \quad L = 2H}$

$\boxed{\text{if R in box, } R = 2k\Omega}$

b) $V_o = |V| = \boxed{|V_g| \cdot \dfrac{|R|}{\left|R + \frac{1}{j\omega C} + Z_{box}\right|}\, V} = 6 \dfrac{3k\Omega}{\left|R + \frac{1}{j\omega C} + Z_{box}\right|}$ $= \boxed{\frac{6}{\sqrt{2}} \text{ for L}}$ $= \boxed{\frac{18}{5\sqrt{2}} \text{ for R}}$

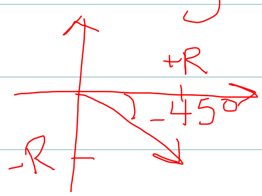
$\sqrt{2} \cdot 3k\Omega$ for L

$\sqrt{2} \cdot 5k\Omega$ for R

$$V_o e^{-j45^\circ} = \frac{-18Kj}{3K-5Kj+Z}$$

$$\angle -45^\circ = \frac{\angle -90^\circ}{\boxed{\text{need } \angle -45^\circ}}$$

$$\angle(3K-5Kj+Z) = \angle -45^\circ$$



Need Real = -Im for -45°

so $\boxed{R=2K} \rightarrow$ Real $=5K$, Im $=-5K$

$L \rightarrow 3K\underbrace{-5Kj+j(1K)L}_{=3K?}$

Yes if $\boxed{L=2H}$

Cap $\rightarrow 3K - 5Kj - \frac{j}{1K(C)}$ Not possible