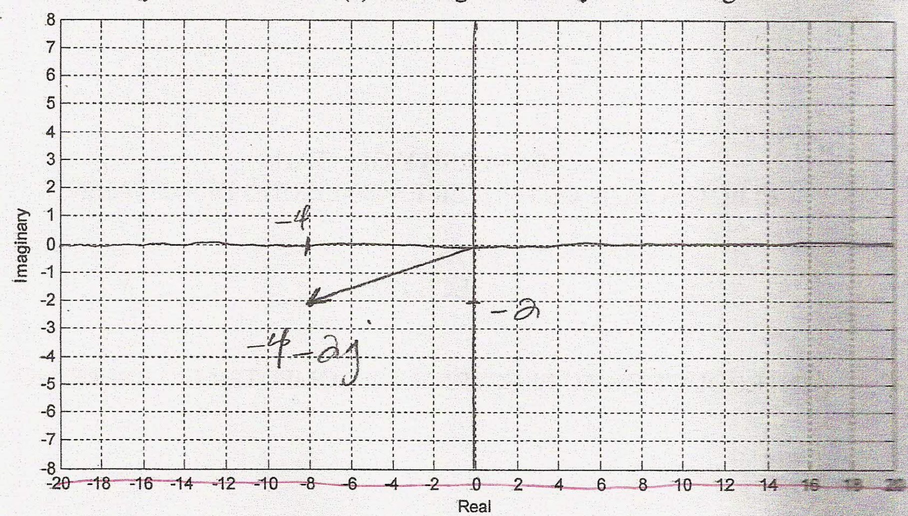
Exam 4 Solution

1. (25 points)

(5) a. Rationalize $\frac{5(20j)}{-10-20j}$. Express your answer in rectangular form. [Round your answer to the nearest whole integer.]

(5) b. Plot your answer from (a) showing the x and y values along with the vector.



(5) c. Write the answer from (a) in phasor form.

(5) d. Find the imaginary part of $\frac{e^{-j90^\circ}}{(1-j)}$.

(5) e. Given ω = 2k rad/s, what is the time domain expression for the phasor 50j.

a) $\frac{5(20j)}{-10-20j} = \frac{100j}{-10-20j} \cdot \frac{-10+20j}{-10+20j} = \frac{-2000-1000j}{100+400}$

$= \frac{\cancel{100}(-20-10j)}{\cancel{100}(5)} = \boxed{-4-2j}$

b) is on the graph

c) $-4-2j = \sqrt{4^2+2^2}\, e^{j\text{Arctan}\left(\frac{-2}{-4}\right)} = \sqrt{20}\, \cancel{\cancel{e}} e^{j(26.6^\circ+180^\circ)}$

$= 2\sqrt{5}\, e^{j206.6^\circ} = \boxed{2\sqrt{5} \angle 206.6^\circ}$

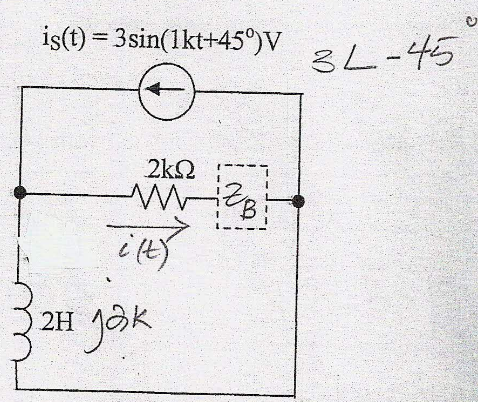
d) $\dfrac{e^{-j90^\circ}}{(1-j)} = \dfrac{\cos 90^\circ - j\sin 90^\circ}{1-j} = \dfrac{0 - j1}{1-j} \cdot \dfrac{1+j}{1+j}$

$= \dfrac{1 - j1}{1+1} = \frac{1}{2} - \frac{1}{2}j \Rightarrow Im\left[\dfrac{e^{-j90^\circ}}{(1-j)}\right] = \boxed{-\frac{1}{2}}$

e) $50j = \sqrt{50^2+0^2}\, e^{j90^\circ} = 50e^{j90} = 50\angle 90^\circ$

$= \boxed{50\cos(2Kt + 90^\circ)}$

2. (40 points)



(30) a. Choose an R, an L, or a C to be placed in the dashed-line box to make

$$i(t) = I_o \cos(1kt + 45^\circ)$$

where Io is a positive, (i.e., nonzero and non-negative), real constant with units of Amps. State the value of the component you choose.

(10) b. With your component from part (a) in the circuit, calculate the resulting value of Io.

a)

$$I_s = P[3\sin(1kt+45^\circ)A] = 3\cos(1kt+45^\circ-90^\circ)A$$

$$= 3\cos(1kt-45^\circ)A = 3\angle -45^\circ$$

$$I = P\{I_o\cos(1kt-45^\circ)\} = I_o\angle +45^\circ$$

$$Z_L = j\omega L = j(1K)(2H) = j2k$$

we have a current Divider

$$\Rightarrow I = I_s \frac{Z_L}{Z_L + (Z_R + Z_B)} = 3\angle -45^\circ \frac{j2K}{j2K + 2K + Z_B} = I_o\angle 45^\circ$$

$$\Rightarrow \angle \frac{3\angle{-45^\circ}(2k\angle 90^\circ)}{\angle\, j2k + 2k + Z_B} = \angle\left(I_0 \angle 45^\circ\right)$$

$$\Rightarrow \frac{\angle -45^\circ + 90^\circ}{\angle(j2k + 2k + Z_B)} = \angle 45^\circ \quad \Rightarrow \angle(j2k + 2k + Z_B) = \frac{\angle 45^\circ}{\angle 45^\circ} = \angle 0^\circ$$

we need $\angle 0^\circ$ and if we have $-j2k$ in the box

we are left with the resistor creating $\angle 0^\circ$

$$\Rightarrow j2k + 2k + \frac{-j}{wC} = 2k \quad \Rightarrow j2k = \frac{+j}{wC}$$

$$\Rightarrow 1/wC = 2k \quad \Rightarrow \frac{1}{(k)(C)} = 2k \Rightarrow 1/C = 2M$$

$$\Rightarrow C = 1/2M = 0.5\mu f = 500nf \Rightarrow \boxed{Box = C = 500nf}$$

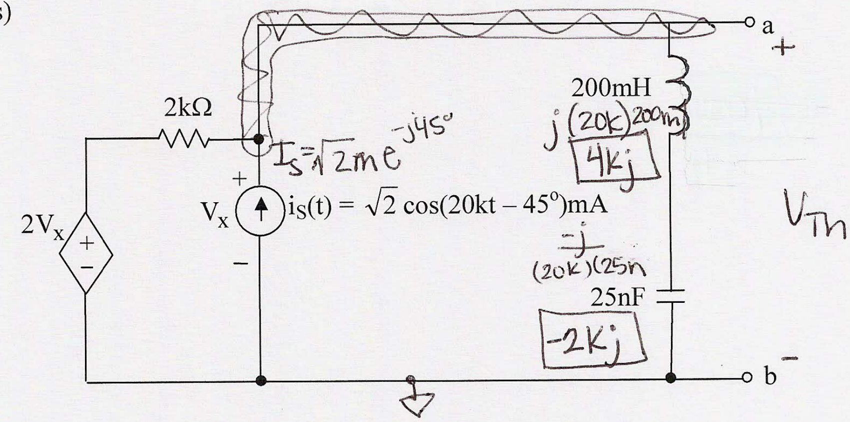
$$\Rightarrow Z_C = -j/wC = -j2k$$

$$b) \Rightarrow I = \frac{(3\angle -45^\circ)(-j2k)}{\cancel{j2k} + 2k \cancel{- j2k}} = \frac{(3\angle -45^\circ)(2k\angle -90^\circ)}{2k}$$

$$\Rightarrow I_0 = Re[I] = Re\left[\frac{6k \angle -135^\circ}{2k}\right] = \frac{6k}{2k} = \boxed{3A}$$

$$\Rightarrow \boxed{I_0 = 3A}$$

3. (35 points)



(15) a. Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_S(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

(20) b. Find the Thevenin equivalent (in the frequency domain) for the above circuit. Give the numerical phasor value for V_{Th} and the numerical impedance value of z_{Th}.

① Find V_{Th}:

$$V_{Th} = V_x$$

Using node-voltage:

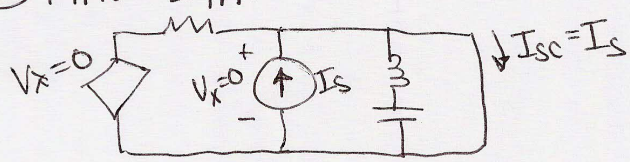
$$\frac{V_{Th} - 2V_{Th}}{2k} + \frac{V_{Th}}{(4kj - 2kj)} - \sqrt{2}me^{-j45°} = 0$$

$$V_{Th}\left(\frac{j}{2kj} - \frac{2j}{2kj} + \frac{1}{2kj}\right) = \sqrt{2}me^{-j45°}$$

$$V_{Th}\left(\cancel{\frac{1-j}{2kj}}\right) = \frac{\sqrt{2}me^{-j45°}\cdot(2ke^{j90°})}{(1-j)} = \frac{\cancel{\sqrt{2}}\cancel{m}(2k)e^{j45°}}{\cancel{\sqrt{2}}e^{-j45°}}$$

$$\boxed{V_{Th} = 2e^{j90°}}$$

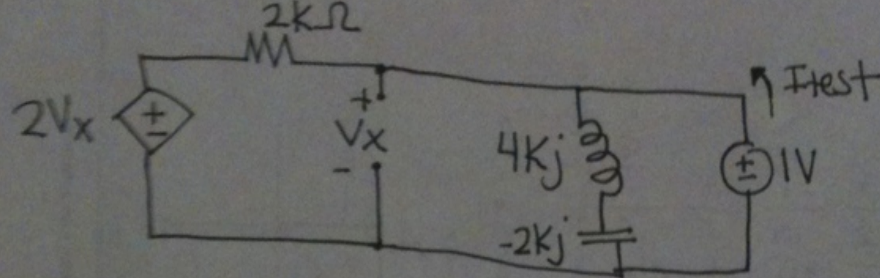
② Find Z_{Th}:



$$\therefore Z_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{2e^{j90°}}{\sqrt{2}me^{-j45°}}$$

$$\boxed{Z_{Th} = \frac{2k}{\sqrt{2}}e^{j135°}}$$

Using a test source:



$V_x = 1V$

$$\frac{+1-2(1)}{2k} + \frac{1}{2kj} - I_{test} = 0$$

$$I_{test} = -\frac{1}{2k} + \frac{1}{2kj} = \frac{1}{2k}(-1-j)$$

$$Z_{th} = \frac{1}{I_{test}} = \frac{2k}{(-1-j)} = \frac{2K}{\sqrt{2}\, e^{j225°}} = \frac{2K}{\sqrt{2}} e^{-j225°}$$

or

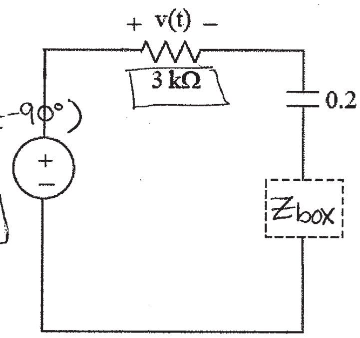
$$Z_{th} = \frac{2K}{\sqrt{2}} e^{j135°}$$

1.

$V_g(t) = 6\cos(1kt - 90°)$

$v_g(t) = 6\sin(1kt)$ V

$\boxed{V_g = 6e^{-j90°}}$



$\omega = 1K$

$0.2\,\mu F \Rightarrow \dfrac{-j}{\omega C} = \dfrac{-j}{1k(.2\mu)} = \boxed{-5{,}000j}$

Choose an R, an L, or a C to be placed in the dashed-line box to make $v(t) = V_0\cos(1kt - 45°)$ where I_0 is a positive, (i.e., nonzero), real constant. State the value of the component you choose.

2. With your component from problem 1 in the circuit, calculate the resulting value of I_0.

① Note what the final angle needs to be.

$\angle V = -45°$

② Change circuit to frequency domain values & label box Z_{box}.

③ Write an equation for known variable (V(t) in this case).

- Use ohm's laws and Kirchoff's laws or node-voltage, current mesh techniques. (Also check for current or voltage divider)
- $V = \dfrac{V_g(3k)}{3k - 5kj + Z_{box}}$

④ Replace eq. with angles and plug in known value for V.

$\angle V = \dfrac{\angle V_g(3k)}{\angle(3k - 5kj + Z_{box})} \Rightarrow \angle V = \angle V_g(3k) - \angle(3k - 5kj + Z_{box})$

$\angle -45° = \angle -90° - \angle(3k - 5kj + Z_{box})$

$\angle\underbrace{(3k - 5kj + Z_{box})}_{} = \angle -90° - \angle -45° = \angle -45°$

To get this to be −45° which means $\left[\dfrac{Im}{Real} = -1\right]$

1. (cont.)

If $Z_{box} = R$ then $\frac{-5K}{3K+R} = -1$ and solving for R:

$$-5k = -3k - R$$

$$R = -3k + 5k = \boxed{2K\Omega}$$

Checking:

$$V = \frac{(6e^{-j90^\circ})(3k)}{3k - 5kj + 2K} = \frac{18Ke^{-j90^\circ}}{5k - 5kj} = \frac{18Ke^{-j90^\circ}}{\sqrt{5k^2+5k^2}\; e^{j\tan^{-1}\left(\frac{-5k}{5k}\right)}}$$

$$V = \frac{18K}{5k\sqrt{2}} \cdot e^{-j90^\circ + 45^\circ} = \frac{18}{5\sqrt{2}} e^{-j45^\circ}$$ ✓ correct form

$$\therefore \boxed{V_o = \frac{18}{5\sqrt{2}}}$$

If C, then $Z_{box} = \frac{-j}{1kC}$ and $\frac{\left(-5k - \frac{1}{1kC}\right)}{3k} = -1$

$$-5k - \frac{1}{1kC} = -3k$$

$$-5k + 3k = \frac{1}{1kC}$$

$$-2k(1kC) = 1 \implies C = -\frac{1}{2k(1k)}$$ ✗ Not possible! negative C?

If L, then $Z_{box} = j(1k)L$ and $\frac{-5k + 1kL}{3k} = -1$

$$-5k + 1kL = -3k$$

$$+2k = +1kL$$

$$\therefore \boxed{L = 2\,H}$$

checking: $V = \frac{18ke^{-j90^\circ}}{3k - 5kj + 2kj}$

$$V = \frac{18ke^{-j90^\circ}}{\sqrt{3k^2+3k^2}\; e^{j\tan^{-1}\left(\frac{-3k}{3k}\right)}} = \frac{18\cancel{k}}{3\cancel{k}\sqrt{2}} e^{-j90^\circ + 45^\circ} = \boxed{\frac{6}{\sqrt{2}}} e^{-j45^\circ}$$

$$\frac{6}{\sqrt{2}} = V_o$$

1a

2. Give numerical answers to each of the following questions:

a. Rationalize $\dfrac{-30k\cdot(j1k)}{30k+j1k}$. Express your answer in rectangular form.

b. Find the polar form of $\left(e^{j45^\circ}\right)^*\left(\sqrt{1+\frac{5}{4}}-j\sqrt{1-\frac{5}{4}}\right)^*$ (Note: The asterisk means conjugate.)

c. Find the following phasor: $\mathrm{P}\left[8\sin(3kt+115^\circ)\right]$.

d. Find the magnitude of $\dfrac{(1-4j)2e^{-j50^\circ}}{2+2e^{j90^\circ}}$.

e. Find the imaginary part of $\dfrac{1-5j}{e^{-j60^\circ}}$.

a. $\dfrac{-30k(1k)j(30k-j1k)}{(30k+j1k)(30k-j1k)} = \dfrac{(1k)(1k)[-30(30k)j+30k\,j^2]}{(1k)(1k)(30^2+1^2)} = \dfrac{-900kj-30k}{901}$

$\cong \boxed{-33-999j}$

b. $\left(e^{-j45^\circ}\right)\left(\sqrt{1+\frac{5}{4}}+j\sqrt{1-\frac{5}{4}}\right) = \left(e^{-j45^\circ}\right)\left(1.5+j(0.5j)\right) = e^{-j45^\circ}(1.5-0.5) = \boxed{e^{-j45^\circ}}$

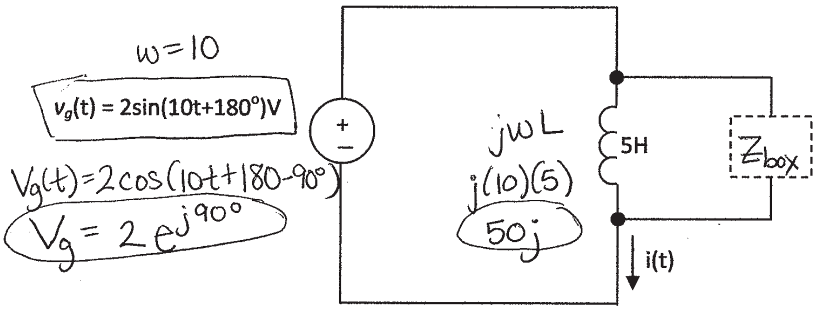
c. $8\sin(3kt+115^\circ) = 8\cos(3kt+115^\circ-90^\circ) = \boxed{8e^{j25^\circ}}$

d. $\dfrac{|(1-4j)|\,|2e^{j-50}|}{|2+\underbrace{2e^{j90}}_{2j}|} = \dfrac{\left[\sqrt{1^2+(-4)^2}\right][2]}{\sqrt{2^2+2^2}} = \boxed{\dfrac{2\sqrt{17}}{\sqrt{8}}}$

e. $\mathrm{Im}\left[\dfrac{1-5j}{e^{-j60^\circ}}\right] = \dfrac{\sqrt{1^2+(-5)^2}\,e^{j\tan^{-1}\left(\frac{-5}{1}\right)}}{e^{-j60^\circ}} \simeq \sqrt{26}\,e^{-j79^\circ+60^\circ} = \sqrt{26}\,e^{-j19^\circ}$

Im: $\sqrt{26}\sin(-19^\circ) \cong \boxed{-1.7}$

3.



Choose an R, an L, or a C to be placed in the dashed-line box to make $i(t) = I_o\cos(10t+45°)$A $\quad I = I_o e^{j45°}$

where I_O is a real constant. State the value of the component you choose.

4. With your component from problem 3 in the circuit, calculate the resulting value of I_o.

$$I = \frac{V}{Z_{eq}}$$

$$Z_{eq} = 50j \parallel Z_{box}$$

$$I = \frac{2e^{j90°}}{\frac{50j(Z_{box})}{50j + Z_{box}}} = \frac{2e^{j90°}(50j + Z_{box})}{50j\, Z_{box}}$$

(Try to remove as many Z_{box} as possible):

$$I = \frac{\cancel{Z_{box}}\,(2\cancel{e^{j90°}})\left(\frac{50j}{Z_{box}}+1\right)}{50j\,\cancel{Z_{box}}} = \frac{1}{25}\left(\frac{50j}{Z_{box}}+1\right)$$

$$50j \rightarrow \cancel{e^{j90°}}$$

In order to get $\angle 45°$, then Im. = Real

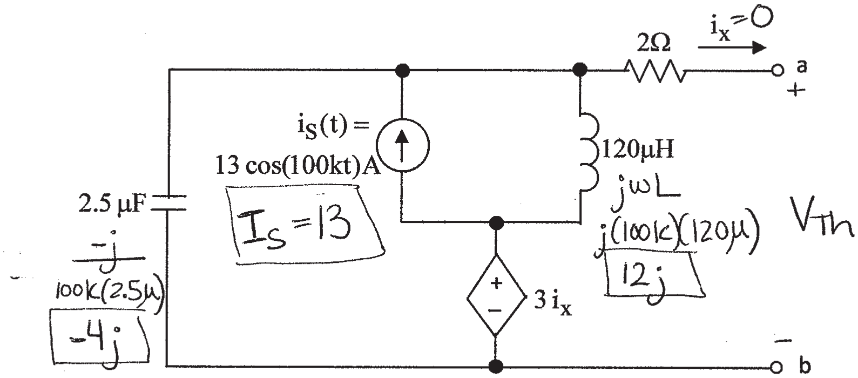
$\therefore\ Z_{box} = \boxed{R = 50\,\Omega}$ $\quad$ {Note: if $C \Rightarrow \frac{-j}{10C}: \Rightarrow \frac{1}{25}\left(\frac{50\cancel{j}\omega C}{-\cancel{j}}+1\right)$} No j

(If $L \Rightarrow j\omega L: \Rightarrow \frac{1}{25}\left(\frac{50\cancel{j}}{\cancel{j}\omega L}+1\right)$ again no $\cancel{j}$)

If $R = 50$ then $\frac{1}{25}(j+1) = \frac{1}{25}\sqrt{2}\,e^{j45°}$ and

$$I_o\cancel{e^{j45°}} = \frac{1}{25}\sqrt{2}\,\cancel{e^{j45°}} \Rightarrow \boxed{I_o = \frac{\sqrt{2}}{25}}$$

5.

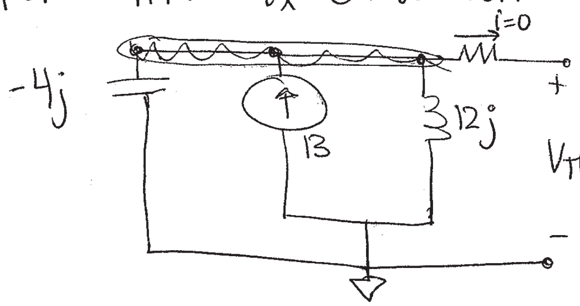


Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

6. Find the Thevenin equivalent (in the frequency domain) for the circuit from Problem 6. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical impedance value of z_{Th}.

① For V_{Th}: $i_x = 0$ which shorts the $3i_x$ source $= 0$

Using node-voltage:

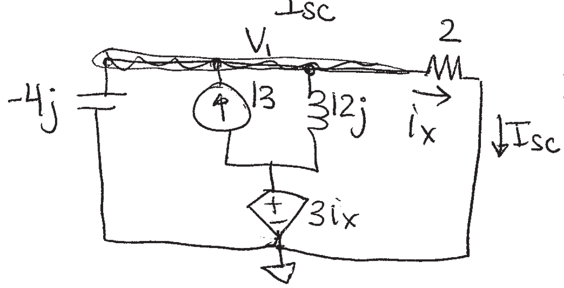


$$\frac{-V_{Th}(3)}{+4j(3)} - 13 + \frac{V_{Th}}{12j} = 0$$

$$\frac{-3V_{Th} + V_{Th}}{12j} = 13$$

$$V_{Th}(\cancel{-2}) = \frac{13(12j)}{-2} \Rightarrow \boxed{V_{Th} = -78j} = \boxed{+78e^{-j90}}$$

② Find $Z_{Th} = \frac{V_{Th}}{I_{sc}}$ (Redraw circuit)



$$I_{sc} = i_x = \frac{V_1}{2}$$

$$\frac{V_1}{-4j} - 13 + \frac{(V_1 - 3i_x)}{12j} + \frac{V_1}{2} = 0$$

$$V_1\left(\frac{-3(2)}{(2)4(3)j} + \frac{1(2)}{12j(2)} - \frac{3}{2(12j)} + \frac{12j}{2(12j)}\right) = 13$$

5. (cont.)

$$V_1\left(\frac{-6+2-3+12j}{24j}\right)=13$$

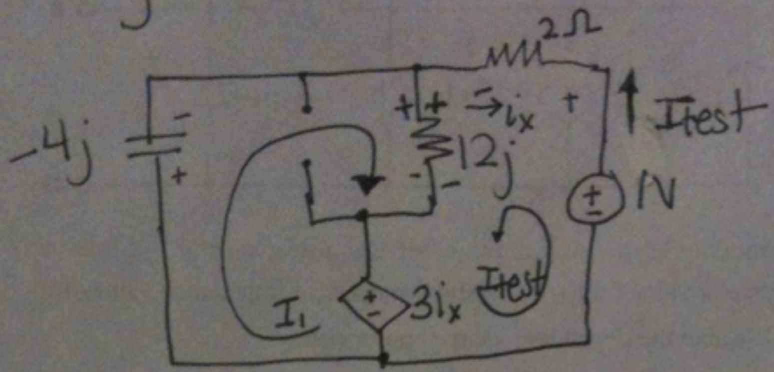
$$V_1=\frac{13(24j)}{-7+12j}$$

$$I_{sc}=\frac{V_1}{2}=\frac{312j}{2(-7+12j)}\cdot\frac{(-7-12j)}{(-7-12j)}=\frac{312j(-7)-312j(12j)}{2(7^2+12^2)}=\frac{-2184j+3744}{386}$$

$$Z_{Th}=\frac{V_{Th}}{I_{sc}}=\frac{-78j(386)}{3744-2184j}=\frac{-30{,}108j(3744+2184j)}{3744^2+2184^2}=-1.6mj(3744+2184j)$$

$$Z_{Th}\approx-6j-3.5j^2\approx+3.5-6j$$

Using a test source:



$I_{test} = -i_x \longrightarrow 3i_x = -3I_{test}$

① $-I_1(-4j) - I_1(12j) - I_{test}(12j) + 3I_{test} = 0$

$I_1(\underbrace{+4j - 12j}_{-8j}) + I_{test}(3 - 12j) = 0$

$I_1 = \frac{I_{test}(12j - 3)}{-8j} = \frac{I_{test}(+j)(12j - 3)}{8} = \frac{I_{test}}{8}(-12 - 3j)$

② $+3(-I_{test}) + I_{test}(12j) + I_1(12j) + I_{test}(2) - 1 = 0$

$I_1(12j) + I_{test}(-3 + 12j + 2) - 1 = 0$

$8 \cdot \left[\frac{I_{test}}{8}[-12-3j]12j + I_{test}(-1 + 12j) - 1\right] = 0 \cdot 8$

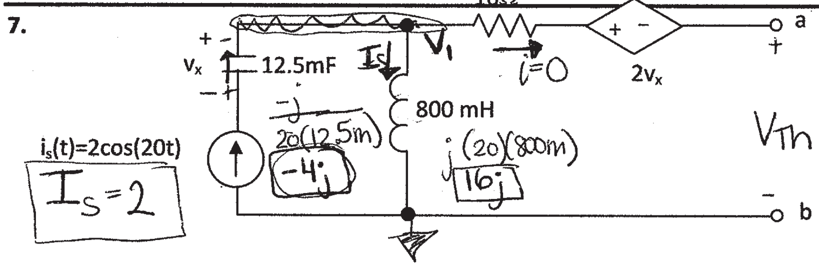
$I_{test}[-12(12j) - 3j(12j) + (-8) + 8(12j)] = +8$

$I_{test}[-144j + 36 - 8 + 96j] = +8$

$I_{test} = \frac{+8}{(-48j + 28)}$

$$Z_{th} = \frac{1}{I_{test}} = \frac{-48j+28}{8} = -6j+3.5$$

7.



Draw a frequency-domain equivalent of the above circuit. Show a numerical phasor value for $i_s(t)$, and show numerical impedance values for R, L, and C. Label the dependent source appropriately.

8. Find the Thevenin equivalent (in the frequency domain) for the circuit from Problem 8. Give the numerical phasor value for $\mathbf{V}_{Th}$ and the numerical impedance value of z_{Th}.

① Find V_{Th}

Taking a voltage loop: $+V_1 - 2V_x - V_{Th} = 0$

$\therefore V_{Th} = V_1 - 2V_x$

where $V_x = -2(-4j) = +8j$

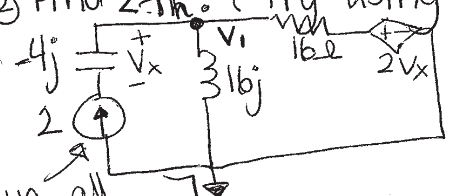
Using the fact that at V_1, the current is zero towards V_{Th}

So $+2(16j) - 2V_x - V_{Th} = 0$

$2(16j) - 2(8j) = V_{Th}$

$\therefore V_{Th} = 32j - 16j = \boxed{16j} = \boxed{16e^{j90}}$

② Find Z_{Th}: (Try using I_{sc} - short circuit current) $\Rightarrow Z_{Th} = \frac{V_{Th}}{I_{sc}}$



Leave all sources for this method

$-2 + \frac{V_1}{16j} + \frac{V_1 - 2V_x}{16} = 0$

$V_1\left(\frac{1}{16j} + \frac{j}{16j}\right) = 2 + \frac{2(8j)}{16}$

$V_1 = \left(\frac{32 + 16j}{\cancel{16}}\right)\frac{16j}{(1+j)} = \frac{(32j - 16)(1-j)}{(1+j)\,(1-j)}$

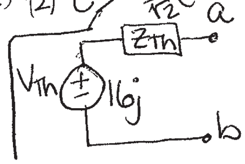
$V_1 = \frac{32j - 32j^2 - 16 + 16j}{1^2 + 1^2} = \frac{48j + 16}{2}$

$I_{sc} = \frac{V_1 - 2V_x}{16} = \frac{(48j+16)}{2(16)} - \frac{2(8j)}{16}$

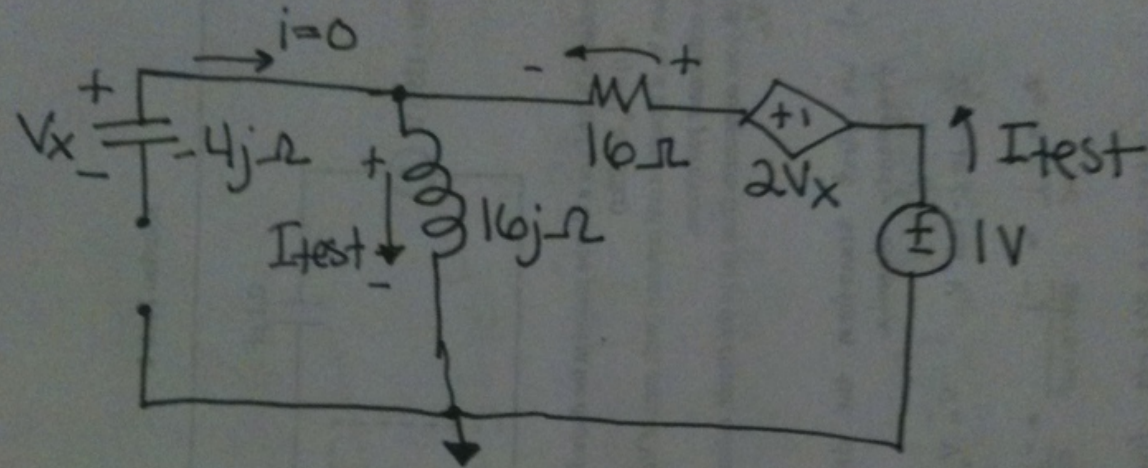
$I_{sc} = \frac{3}{2}j + \frac{1}{2} - \frac{2}{2}j = \frac{1}{2}j + \frac{1}{2}$

$Z_{Th} = \frac{16j}{\frac{1}{2}j + \frac{1}{2}} = \frac{16e^{j90°}}{\sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2}\, e^{j45°}} = \frac{32}{\sqrt{2}} e^{j45°}$

$\boxed{Z_{Th} = \frac{32}{\sqrt{2}} e^{j45°}}$



Using a test source:



$V_x = 0$

$+1 + 2\cancel{V_x}^{\,0} - I_{test}(16+16j) = 0$

$I_{test} = \dfrac{1}{16+16j}$

$Z_{th} = \dfrac{1}{I_{test}} = 16+16j = \underline{\underline{16\sqrt{2}\,e^{+j45°}}}$