Final Exam:

4 problems-

1. linear op amp (find vo, find R values given specific input(s) and output, Find Rin=V/I)

2. comparator op amp
   choose R,L,C for boxes given output graph
   Graphs of signals in circuit
   Diodes(on wire, off open)

3. AC input with multiple time dependent elements
   Find V or I in frequency domain circuit. (Phasors)

4. thevenin equivalent circuit with dependent source and DC input.
   Find RL to give max power transfer and power value.

linear OpAmp

1. Assume 0V between +and - terminals
   assume 0A (redraw circuit)

2. Make sure to take a loop through the 0V at the input terminals.

comparator:

1. solve circuit at terminal with fixed inputs.

2. Label graph according to which terminal needs to be greater to achieve given output graph

3. Go through all possiblilities. Match possible values to see if graph can be achieved.
   Use the properties of L/C at t=0- and at infinity to make decisions.
   For each choice, write general form equation.

4. Graph each voltage desired

5. Look at diode circuit (line side of diode is negative)
   Look at voltages present at each side of diode ignoring Resistances.
   If diode is "on" replace with a wire. If "off" replace with an open.
   Look at all possibilities on output graph.

frequency domain circuit:

already in frequent domain. Will have dependent source. Use Kirchoffs laws, node V or mesh to solve.

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FINAL EXAM solution

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Ex:

$\frac{V_+}{R_1} - i_{s1} + \frac{V_+ - V_o}{R_2} = 0$

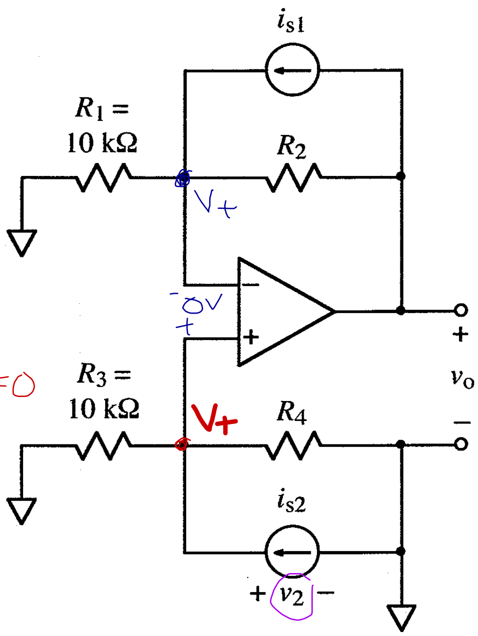
$V_+ = \frac{\left(i_{s1} + \frac{V_o}{R_2}\right)}{\frac{1}{R_1} + \frac{1}{R_2}}$

solve for V_o

$\frac{V_+}{R_3} + \frac{V_+}{R_4} - i_{s2} = 0$

$V_+ = i_{s2} \cdot R_3 \| R_4$

$V_+ = V_2$



a) The above circuit operates in linear mode. Derive a symbolic expression for v_o. The expression must contain not more than the parameters i_{s1}, i_{s2}, R_1, R_2, R_3, and R_4.

b) If $i_{s1} = 10\ \mu A$ and $i_{s2} = 0\ \mu A$, find the value of $R_2 = R_4$ that will yield an output voltage of $v_o = 1$ V.

c) Derive a symbolic expression for v_o in terms of common mode and differential input currents:

$$i_\Sigma \equiv \frac{i_{s1} + i_{s2}}{2} \quad \text{and} \quad i_\Delta \equiv \frac{i_{s1} - i_{s2}}{2}$$

The expression must contain not more than the parameters i_Σ, i_Δ, R_1, R_2, R_3, and R_4. Write the expression as i_Σ times a term plus i_Δ times a term. Hint: start by writing i_{s1} and i_{s2} in terms of i_Σ and i_Δ:

$$i_{s1} \equiv i_\Sigma + i_\Delta \quad \text{and} \quad i_{s2} \equiv i_\Sigma - i_\Delta$$

d) Write a formula for the circuit's input resistance, R_{in}, as seen by source i_{s2}. In other words, write a formula for voltage, v_2, across i_{s2} divided by i_{s2}:

$$R_{in} \equiv \frac{v_2}{i_{s2}}$$

Write R_{in} in terms of not more (and possibly less) than R_1, R_2, R_3, and R_4.

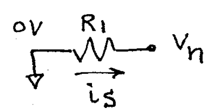
soln: a) First, we calculate v_p. No current flows into the op-amp, leaving a current divider. Equivalently, we have R_3 in parallel with R_4 driven by current source i_{s2}.

$$v_p = i_{s2} \cdot R_3 \| R_4$$

Since the op-amp is operating in linear mode, $v_n = v_p$.

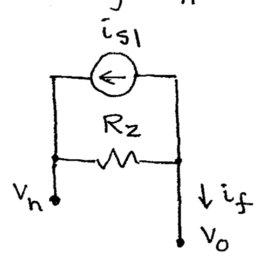
$$v_n = v_p = i_{s2} \cdot R_3 \| R_4$$

Now we the current flowing toward the − input from the left



$$i_s = \frac{0V - v_n}{R_1} = -\frac{v_n}{R_1}$$

Since no current flows into the op-amp, we have the same current flowing in the feedback. We calculate the feedback current, i_f, using v_n.



$$i_f = -i_{s1} + \frac{v_n - v_o}{R_2}$$

We set $i_s = i_f$ and solve for v_o.

$$-\frac{v_n}{R_1} = -i_{s1} + \frac{v_n - v_o}{R_2}$$

or

$$\frac{v_o}{R_2} = -i_{s1} + \frac{v_n}{R_2} + \frac{v_n}{R_1}$$

or

$$v_o = -i_{s1} R_2 + v_n \left(1 + \frac{R_2}{R_1}\right)$$

or

$$\boxed{v_o = -i_{s1} R_2 + i_{s2} \cdot R_3 \| R_4 \cdot \left(1 + \frac{R_2}{R_1}\right)}$$

b) If $i_{s2} = 0\,\mu A$, then $v_p = 0\,V$ and $v_n = v_p = 0\,V$. It follows that all of i_{s1} must flow thru R_2. Thus,

$$v_o = -i_{s1} R_2$$

or

$$R_2 = \frac{v_o}{-i_{s1}} = \frac{1V}{-10\,\mu A}$$

or

$$\boxed{R_2 = -100\,k\Omega}$$

c) Using the answer from part (a), we substitute for i_{s1} and i_{s2} using the formulas given.

$$v_o = -(i_\Sigma + i_\Delta) R_2 + (i_\Sigma - i_\Delta) R_3 \| R_4 \left(1 + \frac{R_2}{R_1}\right)$$

or

$$v_o = \left[(R_3 \| R_4)\left(1+\frac{R_2}{R_1}\right) - R_2\right] i_{\Sigma}$$

$$- \left[(R_3 \| R_4)\left(1+\frac{R_2}{R_1}\right) + R_2\right] i_{\Delta}$$

d) v_2 is the voltage across i_{s2}, as shown in the diagram. $v_2 = v_p$, (the voltage at the + input).

$$v_2 = i_{s2} \cdot R_3 \| R_4$$ (see answer to part (a))

Using this result, we find R_{in}:

$$R_{in} = \frac{v_2}{i_{s2}} = \frac{i_{s2} R_3 \| R_4}{i_{s2}}$$

or

$$R_{in} = R_3 \| R_4$$

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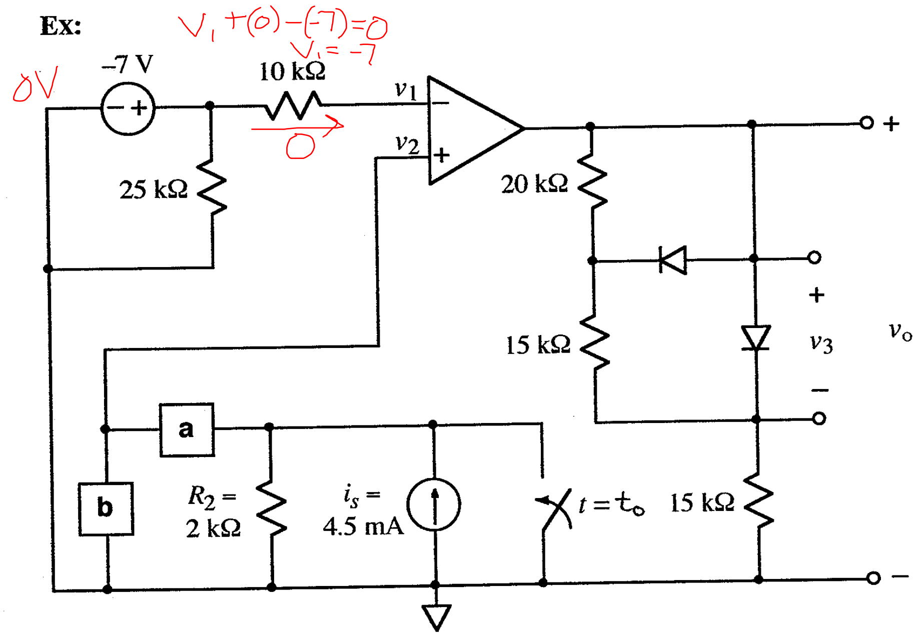
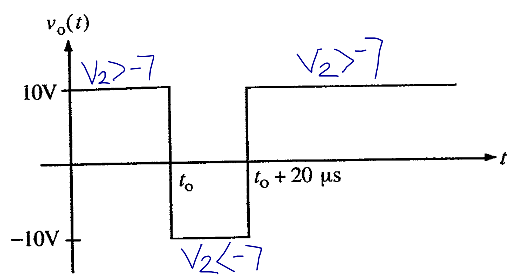
FINAL EXAM solution

Ex:



Rail voltages = ±10 V

After being open for a long time, the switch closes at time $t = t_o$.



a) Choose either an R or C to go in box **a** and either an R or L to go in box **b** to produce the v_o(t) shown above. Use an R value of 3 kΩ. Also, note that v_o stays high forever after $t_o + 20$ μs. Specify which element goes in each box and its value.

b) Sketch $v_1(t)$, showing numerical values appropriately.

c) Sketch $v_2(t)$, showing numerical values appropriately.

d) Sketch $v_3(t)$. Show numerical values for $t < t_o$, for $t_o < t < t_o + 20$ μs, and for $t > t_o + 20$ μs. Use the ideal model of the diode: when forward biased, its resistance is zero; when reverse biased, its resistance is infinite.

sol'n: a) We have a comparator, since the op-amp lacks negative feedback.

v_1 is a fixed voltage. Since the 25 kΩ is across the −7V source, it has no effect on v_1. Since no current flows into the op-amp, the voltage drop across the 10 kΩ resistor is zero, and the 10kΩ also has no effect on v_1.

$$\therefore\ v_1 = -7V \quad \text{(at all times)}$$

To obtain the waveform given in the problem for $v_o(t)$, the voltage for v_2 must be more positive than $v_1 = -7V$ for $t < t_o$ and $t > t_o + 20\ \mu s$. The voltage for v_2 must also be more negative than $v_1 = -7V$ for $t_o < t < t_o + 20\mu s$.

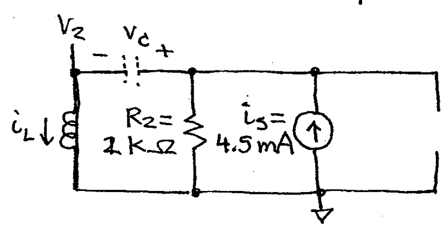
To determine what components to put in box **a** and box **b**, we consider each of the possibilities.

case I: $a = R \quad b = R$
This fails because v_2 would never be negative. Thus, v_2 would never be less than v_1.

case II: $a = C \quad b = L$
Although this type of circuit is beyond the scope of this course, we may

consider whether such a circuit might work.

For $t = 0^-$, (assume $t_0 = 0$), we have L = wire and C = open:



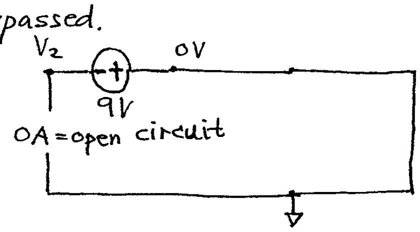
$$v_2(0^-) = 0V > v_1 = -7V \quad \checkmark \text{ OK}$$

$$i_L(0^-) = 0A$$

$$v_C(0^-) = i_s R_2 = 4.5mA \cdot 2k\Omega$$

$$= 9V$$

At $t = 0^+$, we have $i_L(0^+) = i_L(0^-) = 0A$ and $v_C(0^+) = v_C(0^-) = 9V$, whereas the voltage on the right side of the C will be 0V owing to the now-closed switch. i_{s2} and R_2 are bypassed.



$$v_2 = -9V < v_1 = -7V \quad \checkmark \text{ OK}$$

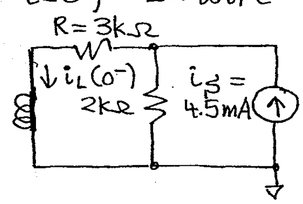
For $t \to \infty$, we have a situation similar to $t = 0^-$, except there is no R.

Without a resistor in the circuit, the energy stored in the circuit at $t=0^-$ will remain in the circuit forever. It back and forth from the C to the L and causes an oscillating voltage at V_2. This would cause $v_o(t)$ to repeatedly go high and low.

Thus, the L and C solution will not work.

case III: $a = R$ $b = L$

For $t=0^-$, L = wire and $v_2(0^-) = 0V$ ✓OK

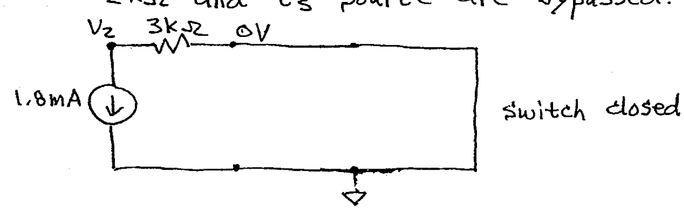


We have a current divider:

$$i_L(0^-) = i_s \cdot \frac{2k\Omega}{2k\Omega + 3k\Omega}$$

$$'' \quad = 4.5\,mA \cdot \frac{2}{5} = 1.8\,mA$$

For $t=0^+$, $i_L(0^+) = i_L(0^-) = 1.8mA$.
2kΩ and i_s source are bypassed.



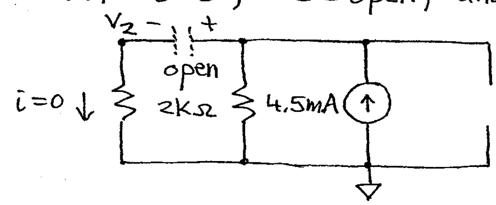
$$v_2(0^-) = 0V - 1.8mA \cdot 3k\Omega = -5.4V$$

But $-5.4V > -7V$ doesn't work!

The last possibility must be considered.

case IV: $a = C \quad b = R$

For $t=0^-$, C = open, and $v_2 = 0V$ ✓ok

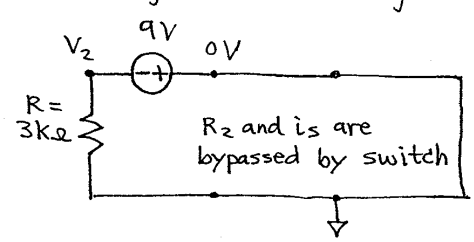


Since the C is open, v_2 is pulled down to ref by the R below it.

The voltage on C is $v_C(0^-) = 4.5mA \cdot 2k\Omega$.

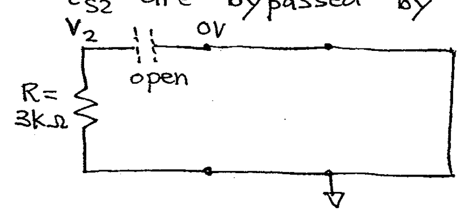
$$v_C(0^-) = 9V.$$

For $t=0^+$, $v_C(0^+) = v_C(0^-) = 9V$, and the closed switch makes the voltage on the right side of C 0V.



We have $v_2(0^+) = -9V < -7V$ ✓ok

For $t \to \infty$, the C is open and R_2 and i_{S2} are bypassed by the switch.



We $v_2(t \to \infty) = 0V$, (pulled down to ref by $3k\Omega$ R).

$$v(t \to \infty) = 0V > -7V \qquad \checkmark \text{ OK}$$

This circuit will work!

Using the general form of sol'n for RC problems, we have the following result:

$$v_2(t) = v_2(t \to \infty) + [v_2(0^+) - v_2(t \to \infty)]\, e^{-t/RC}$$

Here, the R is $3k\Omega$, as the $2k\Omega$ is bypassed for $t > 0$.

$$v_2(t) = 0V + [-9V - 0V]\, e^{-t/3k\Omega \cdot C}$$

or

$$v_2(t) = -9V e^{-t/3k\Omega \cdot C}$$

We want $v_o(t)$ to go high at $t = 20\mu s$. The transition of $v_o(t)$ from low to high occurs when $v_1 = v_2$. Using our expression for $v_2(t)$ with $t = 20\mu s$ and $v_2(20\mu s) = v_1 = -7V$ we have

$$-7V = -9V e^{-20\mu s/3k\Omega \cdot C}$$

or

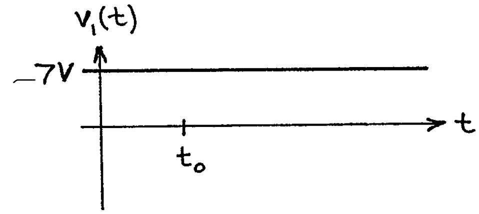
$$\frac{7}{9} = e^{-20\mu s/3k\Omega \cdot C}$$

or

$$\ln \frac{7}{9} = -20\mu s/3k\Omega \cdot C$$

or $$C = -20\mu s/[\ln(7/9) \cdot 3k] = 26.5\,nF$$

b) From part (a), $v_1(t) = -7V$ at all times.



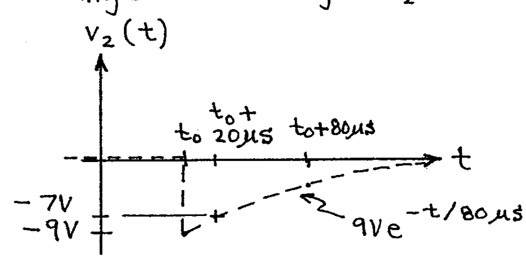
c) From part (a), $v_2(t) = -9V\, e^{-t/3k\Omega \cdot 26.5nF}$

where $t_o = 0$ is assumed, and $t > 0$

Our time constant is

$$\tau = 3k\Omega \cdot 26.5\,nF = 79.6\,\mu s \approx 80\,\mu s$$

In time τ, ≈ 2/3's of the total change in voltage $v_2(t)$ occurs.



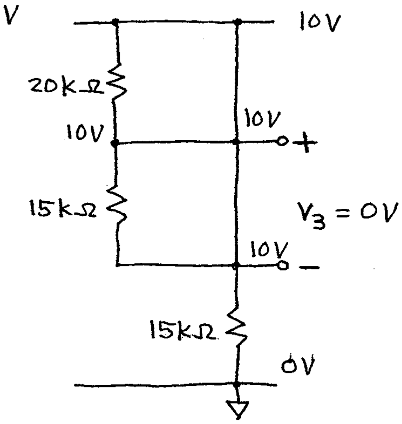
Note: From part (a), we have $v_2(t=0^-)$ is 0V because no current flows thru $R = 3k\Omega$.

We also have $v_2(t = 20\mu s) = -7V$.

d) When v_o is high ($10V = v_{rail}$), the two diodes will be forward biased. (Otherwise, they would be open circuits. But that would result in positive v-drops across the diodes, which is a contradiction.)

Thus, we replace the diodes with short-circuits:

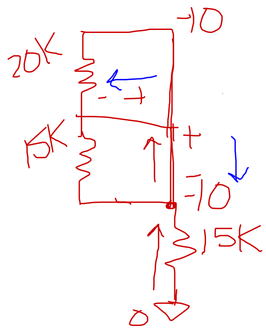
$v_o = +10V$

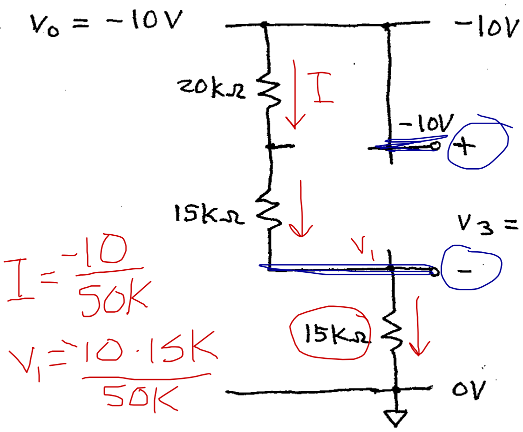


We have $v_3 = 0V$.

When v_o is low ($-10V = -v_{rail}$), the two diodes will be reverse biased. Otherwise, they would be short circuits. But that would result in negative v-drops across the diodes, which is a contradiction.)

Thus, we replace the diodes with open-circuits:





$$I = \frac{-10}{50K}$$

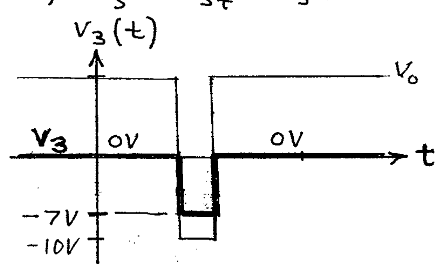
$$V_1 = \frac{-10 \cdot 15K}{50K}$$

We find voltages from V-divider eq'ns:

$$V_{3-} = -10V \cdot \frac{15k\Omega}{15k\Omega + 15k\Omega + 20k\Omega} = -3V$$

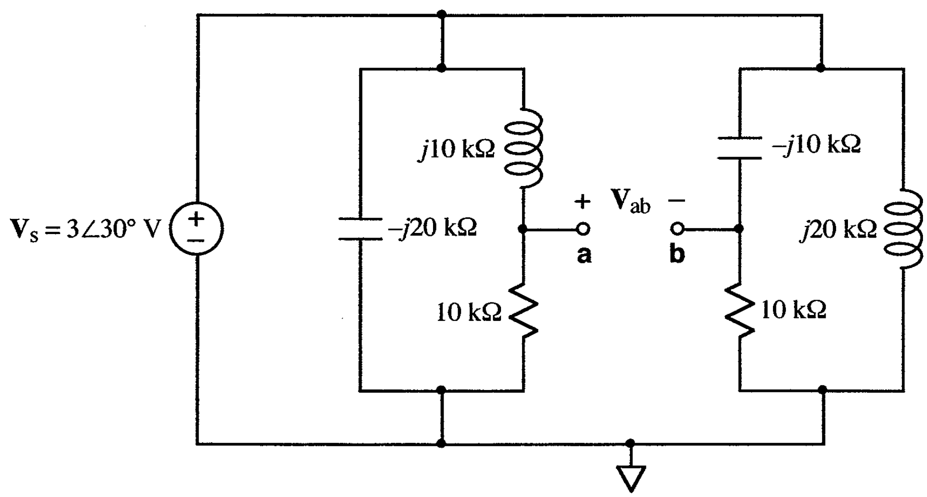
$$V_{3+} = -10V$$

Thus, $V_3 = V_{3+} - V_{3-} = -10V - (-3V) = -7V$



FINAL EXAM solution

Ex:



a) A frequency-domain circuit is shown above. Write the value of phasor voltage $\mathbf{V}_{ab}$ in polar form.

b) Given ω = 500k rad/s, write a numerical time-domain expression for $v_{ab}(t)$, the inverse phasor of $\mathbf{V}_{ab}$.

sol'n: a) We have four branches directly across V_s. We may solve each branch separately. For the voltages V_a and V_b, we have two voltage dividers:

$$V_a = V_s \cdot \frac{10\,k\Omega}{10\,k\Omega + j10\,k\Omega} \quad \text{and} \quad V_b = V_s \cdot \frac{10\,k\Omega}{10\,k\Omega - j10\,k\Omega}$$

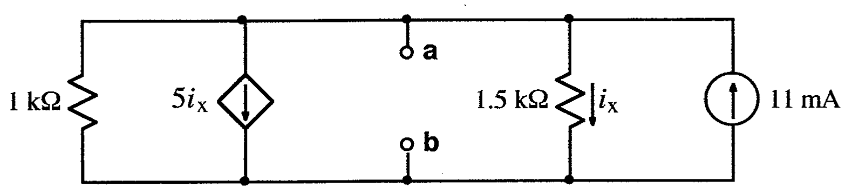
$$\therefore V_{ab} = V_a - V_b = V_s\left(\frac{1}{1+j} - \frac{1}{1-j}\right)$$

$$= 3\angle 30°\left(\frac{1-j}{2} - \frac{1+j}{2}\right) = 3\angle 30° \cdot \left(-j\frac{2}{2}\right)$$

$$V_{ab} = 3\angle 30° \cdot 1\angle -90° V = 3\angle -60° V$$

b) $v_{ab}(t) = 3\cos(500k\,t - 60°)$ V

Ex:



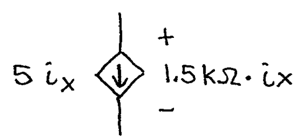
a) Find the Thevenin equivalent of the above circuit relative to terminals **a** and **b**.

b) If we attach R_L to terminals **a** and **b**, find the value of R_L that will absorb maximum power.

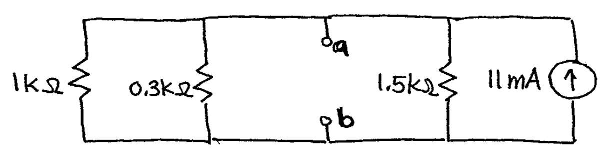
c) Calculate the value of that maximum power absorbed by R_L.

soln: a) We observe that the voltage across all the components is the same, and it is equal to $i_x \cdot 1.5\,k\Omega$.

For the dependent source, we may find an equivalent resistance:



$$R_{eq} = \frac{v}{i} = \frac{1.5\,k\Omega \cdot \cancel{i_x}}{5\,\cancel{i_x}} = 0.3\,k\Omega$$



The v_{Th} is the voltage across a, b and is equal to the 11 mA current times the combined parallel impedance.

$$V_{Th} = 11\,mA \cdot 1k\Omega \| 1.5k\Omega \| 0.3k\Omega$$

we have $1.5k\Omega \| 0.3k\Omega = 0.3k\Omega \cdot 5\|1$

$$\qquad = 0.3\,k\Omega \cdot \frac{5}{6}$$

$$\qquad = 250\Omega$$

and $1k\Omega \| 250\Omega = 250\Omega \cdot 4\|1$

$$\qquad = 250\Omega \cdot \frac{4}{5}$$

$$\qquad = 200\Omega$$

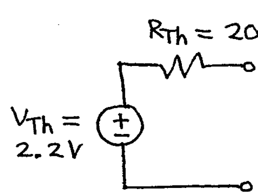
$$V_{Th} = 11\,mA \cdot 200\Omega = 2.2V$$

To find R_{Th}, we turn off the independent 11 mA source and look into a, b. We have

$$R_{Th} = 1k\Omega \| 1.5k\Omega \| 0.3k\Omega$$

or $R_{Th} = 200\,\Omega$

Thevenin equivalent:



b) For max pwr transfer, we use

$$R_L = R_{Th} = 200\Omega.$$

c) When $R_L = R_{Th}$, we achieve the max pwr transfer:

$$p_{max} = \frac{V_{Th}^2}{4 R_{Th}}$$

$$'' = \frac{(2.2V)^2}{4(200\Omega)}$$

$$p_{max} = 6.05 \text{ mW}$$