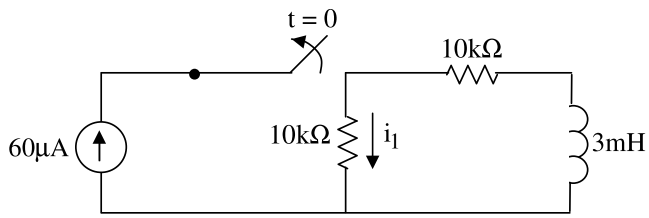
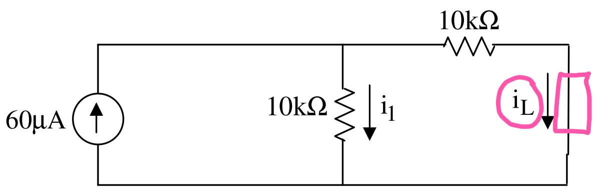
UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ECE 1270 **HOMEWORK #6 Solution** Summer 2009

1. After being closed a long time, the switch opens at t = 0. Find $i_1(t)$ for $t > 0$.



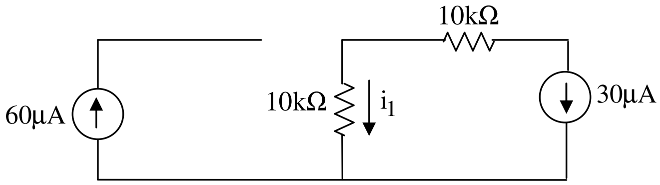
Step 1: (Redraw circuit at **t=0⁻** and solve for i_L. Inductor acts as a wire since it has sat for a long time)



This circuit is a current divider:

$$i_L = \frac{60\mu \cdot 10k}{10k + 10k} = 30\mu A$$

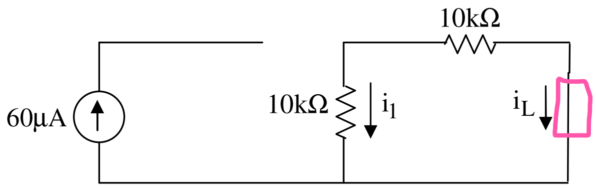
Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Inductor acts as a current source since the current in the inductor has to remain the same.)



Only one current in the branch:

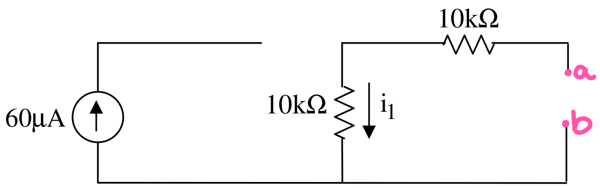
$$\boxed{i_1 = -30\mu}$$

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Inductor acts as a wire since it has sat for a long time in this position.)



There are no sources connected to the final circuit:

$\boxed{i_1=0A}$

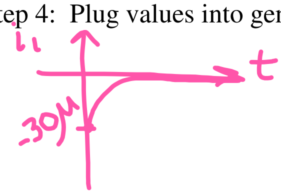


To find R_{eq} the inductor is removed from the final circuit to find path from top of inductor to bottom of inductor:

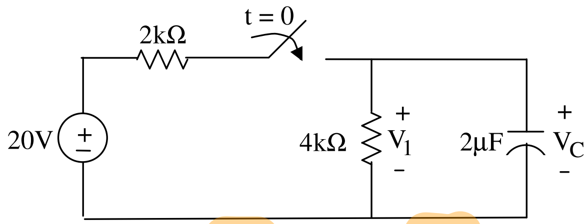
$$\tau = \frac{L}{R_{eq}} = \frac{3m}{10k + 10k} = \boxed{150n\sec}$$

Step 4: Plug values into general equation:

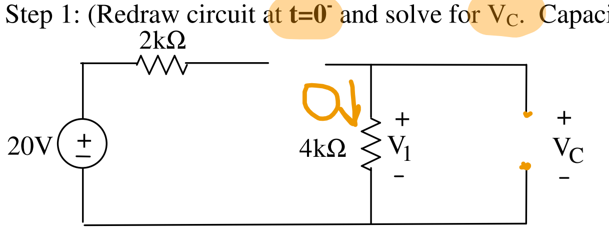
$$i_1(t) = 0 + [-30\mu - 0]e^{-t/150n\sec}A = -30\mu e^{-t/150n\sec}A$$



2. After being open for a long time, the switch closes at t = 0. Find $V_1(t)$ for t > 0.

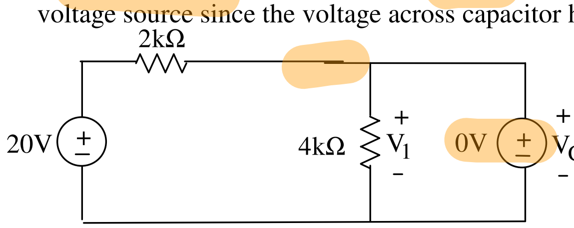


Step 1: (Redraw circuit at **t=0⁻** and solve for V_C. Capacitor acts as an open since it has been a long time)



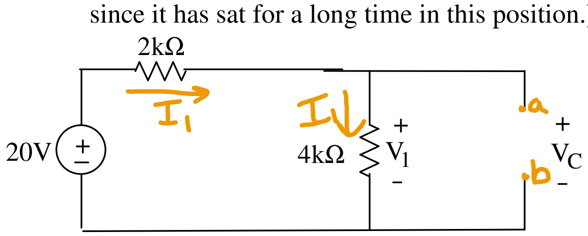
There is no source connected between V_c so $\boxed{V_1 = V_C = 0}$

Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Capacitor acts as a voltage source since the voltage across capacitor has to remain the same.)



$$\boxed{V_1 = 0V}$$

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Capacitor acts as an open since it has sat for a long time in this position.)



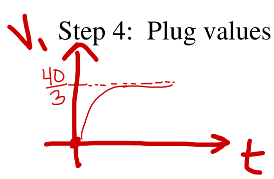
V1 is found by a voltage divider:

$$V_1 = \frac{20 \cdot 4k}{2k + 4k} = \frac{80k}{6k} = \boxed{\frac{40}{3}V}$$

To find R_{eq} the capacitor is removed from the final circuit(same circuit) to find path from top to bottom of capacitor. Independent sources are removed and the equivalent resistance is found:

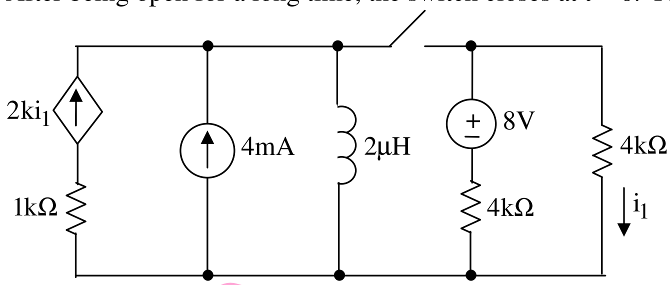
$$\tau = R_{eq} \cdot C = (4k \| 2k) \cdot 2\mu = \left(\frac{1}{\frac{1}{4k} + \frac{1}{2k}} \right) \cdot 2\mu = \boxed{\frac{16m}{6}\sec}$$

Step 4: Plug values into general equation:

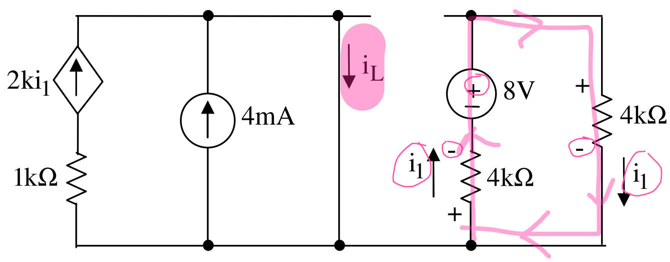


$$\boxed{V_1(t) = \frac{40}{3} + \left[0 - \frac{40}{3} \right] e^{-6t/16msec} V} = \frac{40}{3}\left(1 - e^{-6t/16msec}\right)V$$

3. After being open for a long time, the switch closes at t = 0. Find $i_1(t)$ for t > 0.



Step 1: (Redraw circuit at **t=0⁻** and solve for i_L. Inductor acts as a wire since it has sat for a long time)



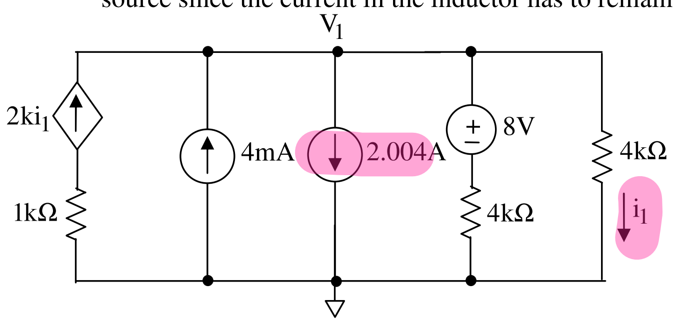
When the circuit contains a dependent source, an extra step is needed to determine the value of the dependent variable:

Note: $-i_1 \cdot 4k + 8 - i_1 \cdot 4k = 0 \Rightarrow i_1 = \frac{8}{8k} = 1mA$

Taking a current summation at the top node can be used to find i_L gives:

$$i_L = 2k \cdot i_1 + 4m \Rightarrow i_L = 2k \cdot 1m + 4m = 2.004A$$

Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Inductor acts as a current source since the current in the inductor has to remain the same.)



Solve the circuit for i_1. Mesh currents or node-voltage can be used. Node-voltage method:

- With dependent sources: solve for dependent variable in terms of either the mesh current or the node-voltage.

$$i_1 = \left(\frac{V_1}{4k}\right)$$

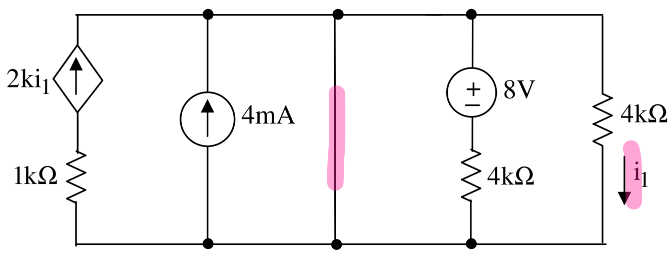
$$-2k \cdot i_1 - 4m + 2.004 + \left(\frac{V_1 - 8}{4k}\right) + i_1 = 0$$

$$-2k \cdot \left(\frac{V_1}{4k}\right) - 4m + 2.004 + \left(\frac{V_1 - 8}{4k}\right) + \left(\frac{V_1}{4k}\right) = 0$$

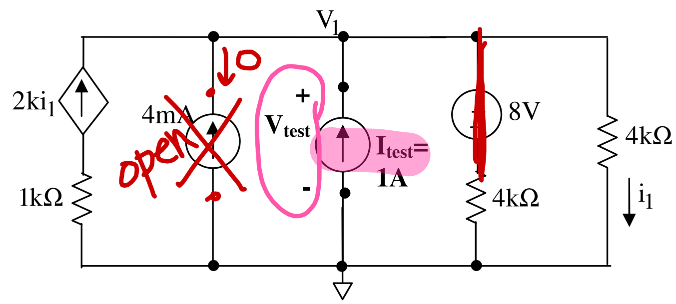
$$V_1\left(\frac{-2k}{4k} + \frac{1}{4k} + \frac{1}{4k}\right) = +4m - 2.004 + \frac{8}{4k} \quad \Rightarrow \quad V_1 = -1.998 \cdot \left(\frac{4k}{-2.002k}\right) \approx 4V$$

The desired variable is i_1: $i_1 = \left(\frac{V_1}{4k}\right) = \frac{4}{4k} = \boxed{1mA}$

Step 3: **Final Value**(Redraw circuit at t=∞ and solve for unknown variable. Inductor acts as a wire since it has sat for a long time in this position.)



This wire puts 0V across 4k ohm resistor so: $\boxed{i_1=0}$



To find R_{eq} the inductor is removed from the final circuit to find path from top of inductor to bottom of inductor(Thevenin Resistance):

Using a test source: $R_{th}=\frac{V_{test}}{I_{test}}$

Setting I_{test}=1A means that only V_{test} needs to be found.

$V_{test}=V_1$

$$-2K\left[\frac{V_1}{4K}\right]-1+\frac{V_1}{4K}+\frac{V_1}{4K}=0$$

$$V_1\left[\frac{-2K+2}{4K}\right]=1 \rightarrow V_1\approx 1\left(\frac{4K}{2,002}\right)$$

$$V_1\approx -2V$$

$$i_1=\left(\frac{V_1}{4k}\right)$$

$$-2k\cdot i_1-4m-1+\left(\frac{V_1-8}{4k}\right)+i_1=0$$

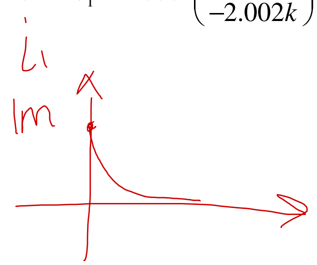
$$-2k\cdot\left(\frac{V_1}{4k}\right)-4m-1+\left(\frac{V_1-8}{4k}\right)+\left(\frac{V_1}{4k}\right)=0$$

$$V_1\left(\frac{-2k}{4k}+\frac{1}{4k}+\frac{1}{4k}\right)=4m+1+\frac{8}{4k} \quad\Rightarrow\quad V_1=1.006\cdot\left(\frac{4k}{-2.002k}\right)\approx -2V$$

$$V_{test}=-2V$$

$$R_{th}=\frac{V_{test}}{I_{test}}=\left|\frac{-2V}{1A}\right|=2\Omega$$

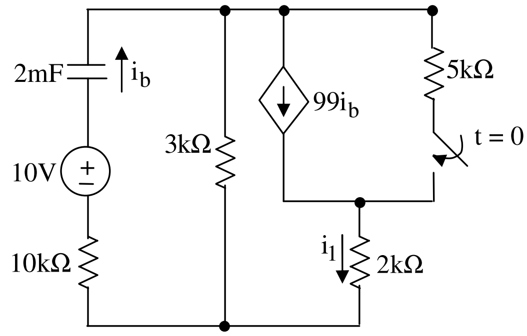
$$\tau=\frac{L}{R_{eq}}=\frac{2\mu}{2}=\boxed{1\mu\sec}$$



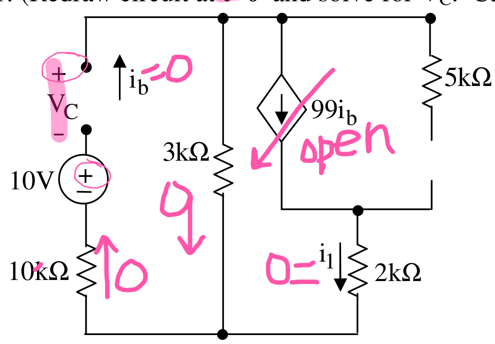
Step 4: Plug values into general equation:

$$i_1(t)=0+[1m-0]e^{-t/1\mu\sec}A=1me^{-t/1\mu\sec}A$$

4. After being open for a long time, the switch closes at t = 0. Find $i_1(t)$ for $t > 0$.



Step 1: (Redraw circuit at $t=0^-$ and solve for V_C. Capacitor acts as an open since it has been a long time)



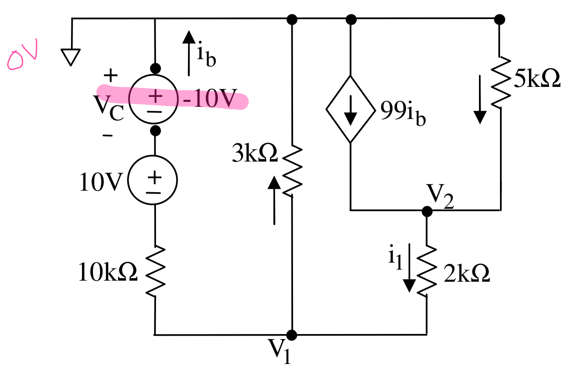
Solving for the dependent variable:

ib=0 which opens the dependent source => 99ib=0

Taking a V-loop to get V_c value(Be Careful- It is **not** 0 when there is a path with a V src.)

$$+0+10+V_C-0=0 \quad \Rightarrow \quad V_C=-10V$$

Step 2: **Initial Value** (Redraw circuit at $t=0^+$ and solve for unknown variable. Capacitor acts as a voltage source since the voltage across capacitor has to remain the same.)



Using node-voltage to solve this circuit to find i_1:

First find dependent variable in terms of node-voltage variable, V_1.

$$i_b=\frac{V_1-(-10)-(-(-10))}{10k}=\frac{V_1}{10k}$$

Next, take current summation equation at V_1 node:

$$\frac{V_1}{10k}+\frac{V_1}{3k}-\frac{(V_1-V_2)}{2k}=0$$

Current summation equation at V_2 node:

$$\frac{(V_2-V_1)}{2k}-99\cdot\frac{V_1}{10k}-\frac{(0-V_2)}{5k}=0$$

Using the first equation and solving for V_1:

$$\frac{V_1}{10k}+\frac{V_1}{3k}-\frac{V_1}{2k}=\frac{V_2}{2k}$$

$$V_1\left(\frac{6}{60k}+\frac{20}{60k}-\frac{30}{60k}\right)=\frac{V_2}{2k}$$

$$V_1=\frac{V_2}{2k}\left(\frac{60k}{-4}\right)=\frac{-15V_2}{2}$$

Plugging into the second equation:

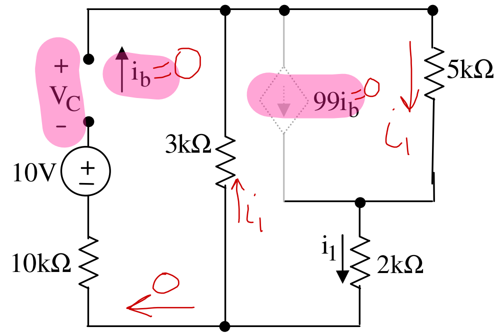
$$\frac{V_2}{2k}-\left(\frac{-15V_2}{4}\right)\left(\frac{1}{2k}\right)-99\cdot\frac{1}{10k}\left(\frac{-15V_2}{4}\right)+\frac{V_2}{5k}=0$$

$$V_2\left(\frac{1}{2k}+\frac{15}{8k}+\frac{99(15)}{40k}+\frac{1}{5k}\right)=0 \quad \Rightarrow \quad V_2=0$$

$$V_1=0$$

$$\boxed{i_1=0}$$

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Capacitor acts as an open since it has sat for a long time in this position.)

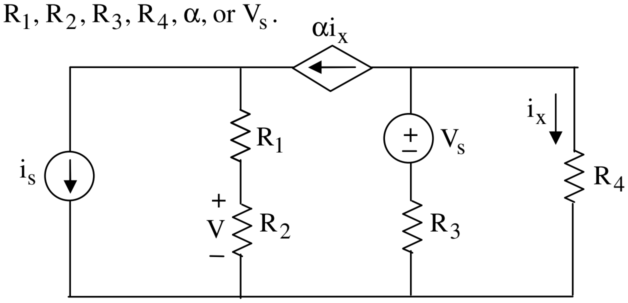


$i_b=0$ *this means that 99ib source becomes open.*

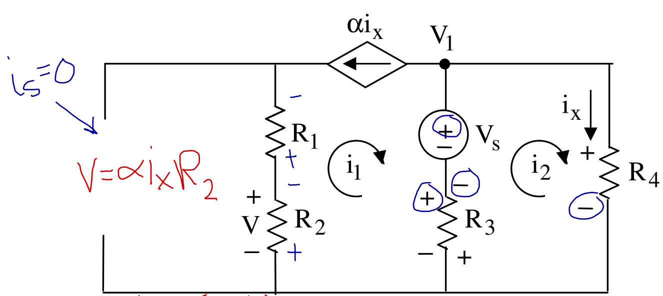
$\boxed{i_1=0}$

Since i_1 is always the same => $\boxed{i_1(t)=0.}$

5. Using superposition, derive an expression for V that contains no circuit quantities other than $i_s, R_1, R_2, R_3, R_4, \alpha$, or V_s.



Step 1: is=0(off), Vs=on



$+\alpha i_x + \frac{(V_1-V_S)}{R_3} + i_x = 0$

$i_x = \frac{V_1}{R_4}$

Using mesh currents:

$i_1 = -\alpha i_x$

$i_2 = i_x$

Using a voltage loop and then substituting above eq:

$(+i_1 - i_2)R_3 + V_S - i_2 R_4 = 0$

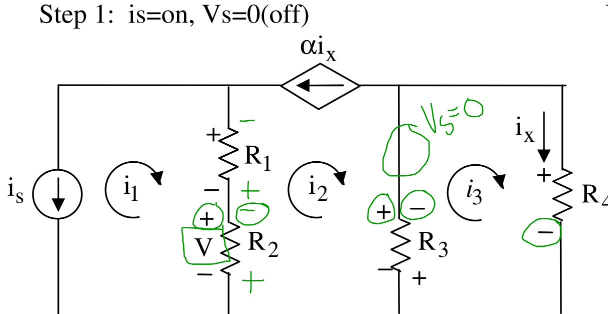
$(-\alpha i_x - i_x)R_3 + V_S - i_x R_4 = 0$

$i_x(+\alpha R_3 + R_3 + R_4) = V_S$

$$i_x = \frac{V_S}{(+\alpha R_3 + R_3 + R_4)}$$

$$V = \alpha i_x R_2 = \boxed{\frac{\alpha V_S R_2}{(+\alpha R_3 + R_3 + R_4)}}$$

Step 1: is=on, Vs=0(off)



$V=(+i_1-i_2)R_2$

Using mesh currents:

$\cancel{i_1 = i_s}$ $i_1=-i_s$

$i_2 = -\alpha i_x$

$i_3 = i_x$

Using a voltage loop and then substituting above eq:

$$(+i_2 - i_3)R_3 - i_3 R_4 = 0$$

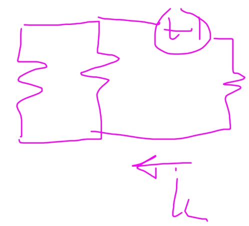
$$(-\alpha i_x - i_x)R_3 - i_x R_4 = 0$$

$$i_x(+\alpha R_3 + R_3 + R_4) = 0$$

$$i_x = 0$$

$$i_2 = 0$$

$$V = (+i_1 - i_2)R_2 = \cancel{i_s R_2}\ \boxed{-i_s R_2}$$



The total V is the sum of both solutions:

$$\boxed{V = i_s R_2 + \frac{\alpha V_s R_2}{(+\alpha R_3 + R_3 + R_4)}}$$

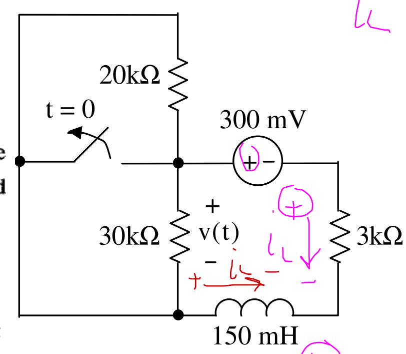
6. After being closed for a long time, the switch opens at t=0.

$W_L = \frac{1}{2} L (i_L)^2$

$i_L(t \to \infty)$

a) Calculate the energy stored on the inductor as t -> ∞.

b) Write a numerical expression for $v(t)$ for t> 0.



SOL'N: a) As t->∞, the switch is open and the L acts like a wire. The 20 kΩ and the 30 kΩ are in parallel, (which is 12 kΩ), and we use Ohm's law to find i_L(t->∞):

$$i_L(t \to \infty) = \frac{300 \text{ mV}}{12 \text{ k}\Omega + 3 \text{ k}\Omega} = \frac{300 \text{ mV}}{15 \text{ k}\Omega} = \boxed{20\ \mu\text{A}}$$

The stored energy is a function of the square of the current in the inductor:

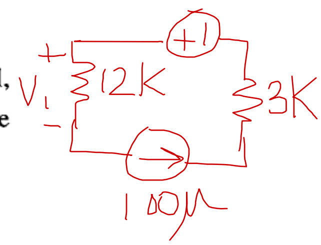
$$w_L(t \to \infty) = \frac{1}{2} L i_L^2(t \to \infty) = \boxed{\frac{1}{2} 150\text{m} \cdot (20\mu)^2} \text{ J} = \boxed{30 \text{ pJ}}$$

b) We first find the current in the inductor for $t = 0^-$. The switch is closed, shorting out both the 20 kΩ and 30 kΩ resistors. The inductor looks like a wire. This leaves only the 300 mV source and 3 kΩ resistor:

$+i_L(3K) + 300mV = 0$

$i_L = -100\mu A$

$$i_L(0^-) = \frac{300 \text{ mV}}{3 \text{ k}\Omega} = \boxed{100\ \mu\text{A}}$$



At $t = 0^+$, the switch is open, the 20 kΩ and the 30 kΩ are in parallel, (which is 12 kΩ), and the inductor acts like a current source with the same current as the inductor had at $t = 0^-$. The voltage, $v(t = 0^+)$, is given by the inductor current times the parallel resistance of 12 kΩ.

$$v(t = 0^+) = 100\ \mu\text{A} \cdot 12 \text{ k}\Omega = \boxed{1.2 \text{ V}}$$

From earlier, we have that the inductor current as t approaches infinity is 10 μA. The voltage, $v(t\text{->}\infty)$, is given by this inductor current times the parallel resistance of 12 kΩ.

$$v(t \to \infty) = 20\ \mu\text{A} \cdot 12 \text{ k}\Omega = \boxed{0.24 \text{ V}}$$

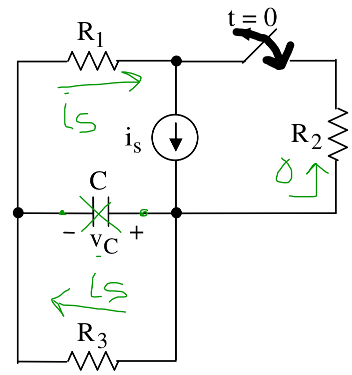
Now we use the general form of solution for RL problems:

$$v(t > 0) = v(t \to \infty) + [v(0^+) - v(t \to \infty)]e^{-t/(L/R_{\text{Th}})}$$

or, with L/R_{Th} = 150 mH/(12 kΩ + 3 kΩ) = $\boxed{10\ \mu\text{s}}$:

$$v(t > 0) = 0.24 + [1.2 - 0.24]e^{-t/10\ \mu\text{s}} \text{ V} = \boxed{0.24 + 0.96e^{-t/10\ \mu\text{s}} \text{ V}}$$

7. After being open for a long time, the switch closes at $t=0$.
   a) Write an expression for $v_c(t=0^+)$.
   b) Write an expression for $v_c(t>0)$ in terms of i_s, R_1, R_2, R_3, and C.

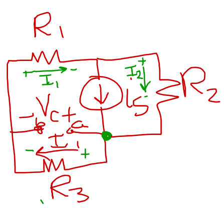


SOL'N: a) We first find the voltage across the capacitor for $t = 0^-$. The switch is open, eliminating R_2 from consideration. The capacitor looks like an open. This leaves only the i_s source driving R_1 and R_3 in series. The capacitor is directly across R_3 and so has the same voltage as R_3:

$$\boxed{v_C(0^-) = i_s R_3}$$

At $t = 0^+$, the capacitor has the same voltage as at $t = 0^-$.

$$\boxed{v_C(0^+) = i_s R_3}$$



b) As $t \to \infty$, the switch is closed, the C acts like an open, and we have a current divider with R_2 on one side and $R_1 + R_3$ on the other side. The current through R_3 is

$$-i_s + I_1 - I_2 = 0$$

$$-I_1(R_3+R_1) - I_2 R_2 = 0$$

$$I_2 = I_1 - i_s$$

$$-I_1(R_1+R_3) - R_2(I_1 - i_s) = 0$$

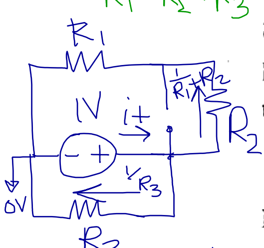
$$I_1 = +\frac{i_s R_2}{R_1+R_2+R_3}$$

$$i = i_s \frac{R_2}{R_1 + R_2 + R_3}.$$

The voltage, v_C, as $t \to \infty$ is the same as the voltage across R_3, and is given by i times R_3:

$$v_C(t \to \infty) = i_s \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

For the time constant of the circuit, we take the Thevenin resistance from the terminals where the C is attached with the switch closed for $t > 0$. Since we have only an independent source, we turn off the source, i_s, and look into the circuit from the terminals where C is attached (but without the C). We see R_3 in parallel with $R_1 + R_2$.



$$R_{Th} = R_3 \parallel (R_1 + R_2)$$

Now we use the general formula for RC circuit solutions:

$$-i_t + \frac{1}{R_3} + \frac{1}{R_1+R_2} = 0$$

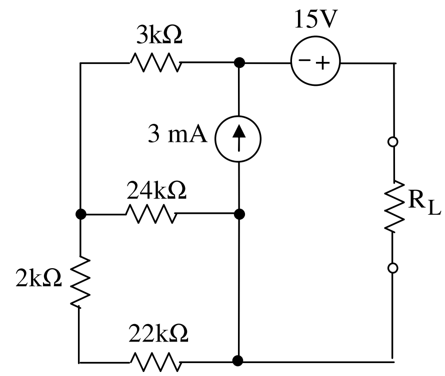
$$v(t > 0) = v(t \to \infty) + [v(0^+) - v(t \to \infty)]e^{-t/R_{Th}C}$$

or

$$v(t > 0) = i_s \frac{R_2 R_3}{R_1 + R_2 + R_3} + [i_s R_3 - i_s \frac{R_2 R_3}{R_1 + R_2 + R_3}]e^{-t/R_3 \parallel (R_1+R_2)C}$$

Use the circuit below for both problem 8 and 9.

8. Calculate the value of R_L that would absorb maximum power.



Use $R_L = R_{Th}$ for maximum power transfer. To find R_{Th}, we turn off the independent sources and look in from the terminals where R_L is attached (with R_L removed). The voltage source becomes a wire, and the current source becomes an open circuit.

$$R_{Th} = (2\ \text{k}\Omega + 22\ \text{k}\Omega) \| 24\ \text{k}\Omega + 3\ \text{k}\Omega = 12\ \text{k}\Omega + 3\ \text{k}\Omega = 15\ \text{k}\Omega$$

We use this Thevenin resistance value for R_L:

$$R_L = 15\ \text{k}\Omega$$

9. Calculate that value of maximum power R_L could absorb.

The maximum power transferred is

$$p_{max} = \frac{v_{Th}^2}{4R_{Th}}$$

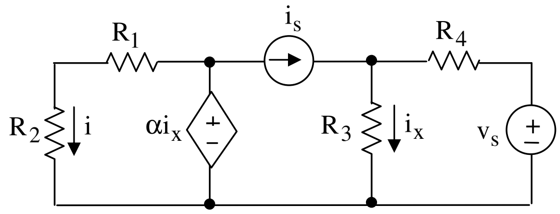
The Thevenin equivalent voltage is the voltage across R_L without R_L. Since there is no R_L, the 3 mA current must all flow through the 3 kΩ resistance and then divide as it flows through the other resistors. It turns out that the 3 mA flows through R_{Th}, and the voltage arising from the 3 mA is found using Ohm's law. To this voltage, we add the 15 V from the voltage source to get v_{Th}.

$$v_{Th} = 3\ \text{mA} \cdot 15\ \text{k}\Omega + 15\ \text{V} = 45 + 15\ \text{V} = 60\ \text{V}$$

Now we use the formula for maximum power transferred:

$$p_{max} = \frac{v_{Th}^2}{4R_{Th}} = \frac{60^2}{4 \cdot 15\text{k}}\ \text{W} = 60\ \text{mW}$$

10. Using superposition, derive an expression for i that contains no circuit quantities other than $i_s, R_1, R_2, R_3, R_4, \alpha$, or V_s.



SOL'N: First, we turn off the independent source, i_S. This means the current source turns into an open circuit, separating the circuit into two pieces. On the right, we have v_S in series with R_3 and R_4.

$$i_{x1} = \frac{v_S}{R_3 + R_4}$$

On the left side, we have the dependent voltage source (whose voltage is now known) and R_1 in series with R_2.

$$i_1 = \frac{\alpha i_{x1}}{R_1 + R_2} = \frac{\alpha v_S}{(R_1 + R_2)(R_3 + R_4)}$$

Second, we turn off the independent source, v_S. This turns the voltage source into a wire, and we have a current divider for i_S flowing through R_3 parallel R_4.

$$i_{x2} = \frac{i_S R_4}{R_3 + R_4}$$

On the left side, the dependent source fixes the voltage across R_1 and R_2. Thus, we have the dependent voltage source (whose voltage is now known) and R_1 in series with R_2, just as we did before.

$$i_2 = \frac{\alpha i_{x2}}{R_1 + R_2} = \frac{\alpha i_S R_4}{(R_1 + R_2)(R_3 + R_4)}$$

We sum the two currents to find the total value of i:

$$i = i_1 + i_2 = \frac{\alpha (v_S + i_s R_4)}{(R_1 + R_2)(R_3 + R_4)}$$