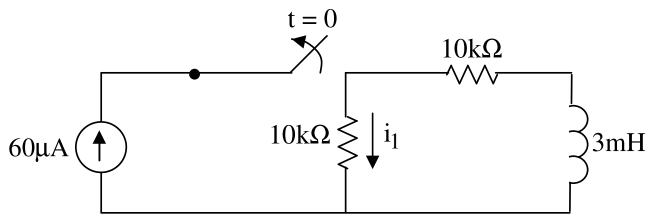
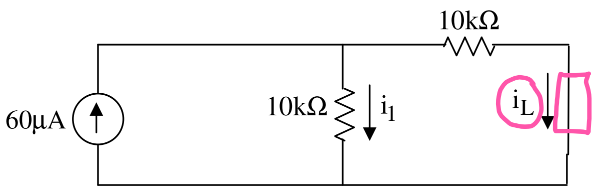
UNIVERSITY OF UTAH
ELECTRICAL & COMPUTER ENGINEERING DEPARTMENT

ECE 1270 **HOMEWORK #6 Solution** Summer 2009

1. After being closed a long time, the switch opens at $t = 0$. Find $i_1(t)$ for $t > 0$.



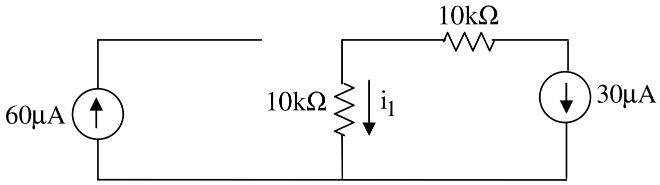
Step 1: (Redraw circuit at **t=0⁻** and solve for i_L. Inductor acts as a wire since it has sat for a long time)



This circuit is a current divider:

$$i_L = \frac{60\mu \cdot 10k}{10k + 10k} = 30\mu A$$

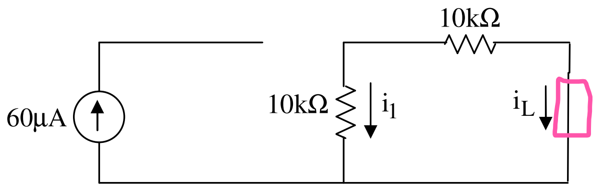
Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Inductor acts as a current source since the current in the inductor has to remain the same.)



Only one current in the branch:

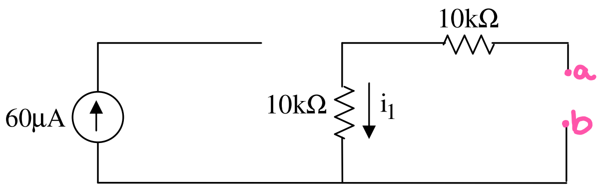
$$\boxed{i_1 = -30\mu}$$

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Inductor acts as a wire since it has sat for a long time in this position.)



There are no sources connected to the final circuit:

$\boxed{i_1 = 0A}$

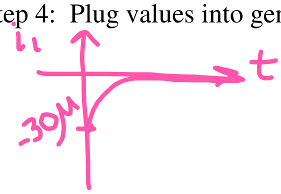


To find R_{eq} the inductor is removed from the final circuit to find path from top of inductor to bottom of inductor:

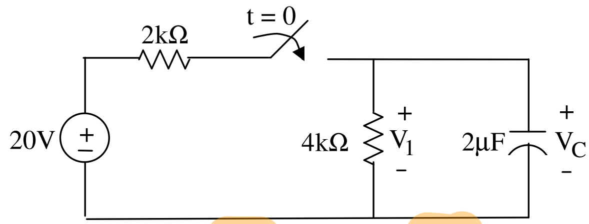
$$\tau = \frac{L}{R_{eq}} = \frac{3m}{10k + 10k} = \boxed{150n\sec}$$

Step 4: Plug values into general equation:

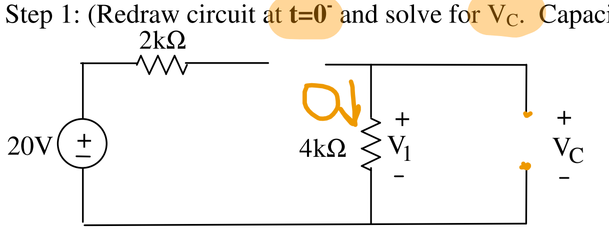
$$i_1(t) = 0 + [-30\mu - 0]e^{-t/150n\sec}A = -30\mu e^{-t/150n\sec}A$$



2. After being open for a long time, the switch closes at t = 0. Find $V_1(t)$ for $t > 0$.

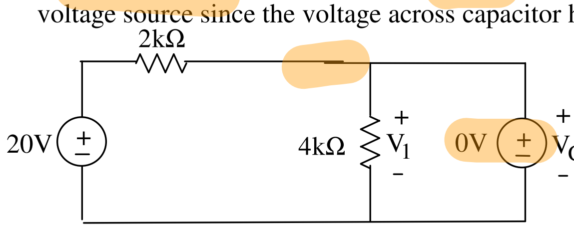


Step 1: (Redraw circuit at **t=0⁻** and solve for V_C. Capacitor acts as an open since it has been a long time)



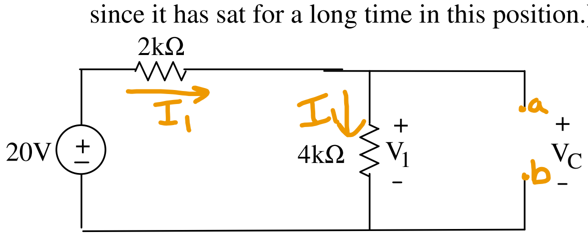
There is no source connected between V_c so $\boxed{V_1 = V_C = 0}$

Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Capacitor acts as a voltage source since the voltage across capacitor has to remain the same.)



$$\boxed{V_1 = 0V}$$

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Capacitor acts as an open since it has sat for a long time in this position.)



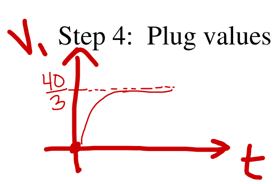
V1 is found by a voltage divider:

$$V_1 = \frac{20 \cdot 4k}{2k + 4k} = \frac{80k}{6k} = \boxed{\frac{40}{3}}V$$

To find R_{eq} the capacitor is removed from the final circuit(same circuit) to find path from top to bottom of capacitor. Independent sources are removed and the equivalent resistance is found:

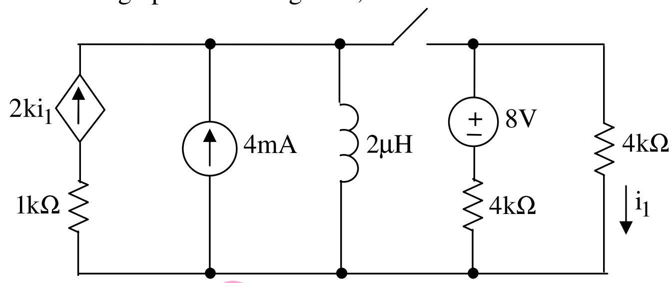
$$\tau = R_{eq} \cdot C = (4k \| 2k) \cdot 2\mu = \left(\frac{1}{\frac{1}{4k} + \frac{1}{2k}} \right) \cdot 2\mu = \boxed{\frac{16m}{6}\sec}$$

Step 4: Plug values into general equation:

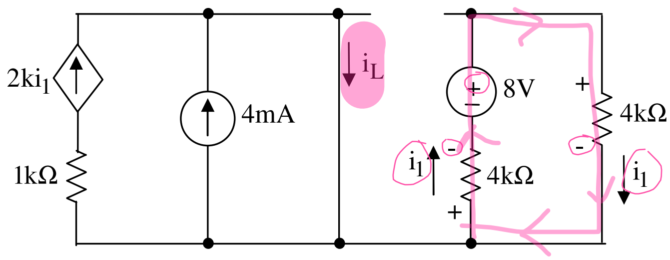


$$\boxed{V_1(t) = \frac{40}{3} + \left[0 - \frac{40}{3} \right] e^{-6t/16m\sec} V} = \frac{40}{3}\left(1 - e^{-6t/16m\sec}\right)V$$

3. After being open for a long time, the switch closes at t = 0. Find $i_1(t)$ for t > 0.



Step 1: (Redraw circuit at **t=0⁻** and solve for i_L. Inductor acts as a wire since it has sat for a long time)



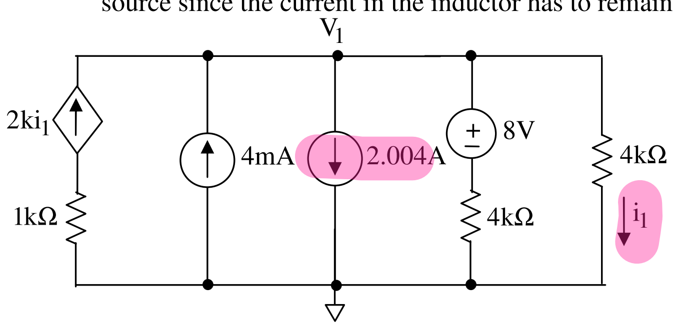
When the circuit contains a dependent source, an extra step is needed to determine the value of the dependent variable:

Note: $-i_1 \cdot 4k + 8 - i_1 \cdot 4k = 0 \Rightarrow i_1 = \dfrac{8}{8k} = 1mA$

Taking a current summation at the top node can be used to find i_L gives:

$$i_L = 2k \cdot i_1 + 4m \Rightarrow i_L = 2k \cdot 1m + 4m = 2.004A$$

Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Inductor acts as a current source since the current in the inductor has to remain the same.)



Solve the circuit for i_1. Mesh currents or node-voltage can be used. Node-voltage method:

- With dependent sources: solve for dependent variable in terms of either the mesh current or the node-voltage.

$$i_1 = \left(\frac{V_1}{4k}\right)$$

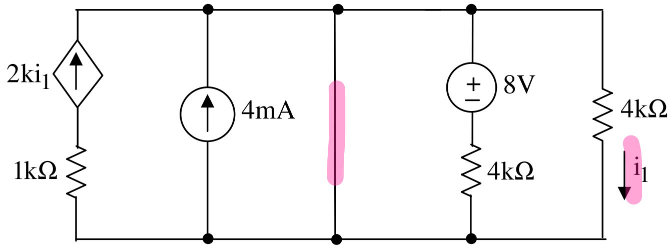
$$-2k \cdot i_1 - 4m + 2.004 + \left(\frac{V_1 - 8}{4k}\right) + i_1 = 0$$

$$-2k \cdot \left(\frac{V_1}{4k}\right) - 4m + 2.004 + \left(\frac{V_1 - 8}{4k}\right) + \left(\frac{V_1}{4k}\right) = 0$$

$$V_1\left(\frac{-2k}{4k} + \frac{1}{4k} + \frac{1}{4k}\right) = +4m - 2.004 + \frac{8}{4k} \quad \Rightarrow \quad V_1 = -1.998 \cdot \left(\frac{4k}{-2.002k}\right) \approx 4V$$

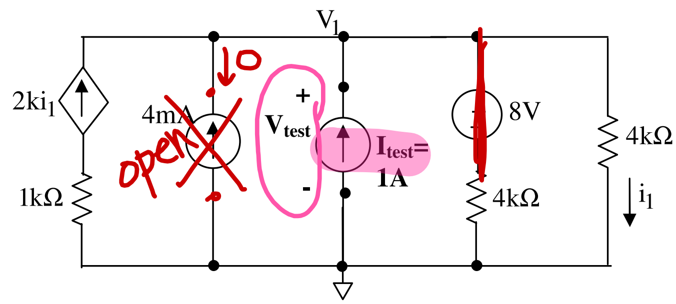
The desired variable is i_1: $\quad i_1 = \left(\dfrac{V_1}{4k}\right) = \dfrac{4}{4k} = \boxed{1mA}$

Step 3: **Final Value**(Redraw circuit at t=∞ and solve for unknown variable. Inductor acts as a wire since it has sat for a long time in this position.)



This wire puts 0V across 4k ohm resistor so:

$\boxed{i_1=0}$



To find R_{eq} the inductor is removed from the final circuit to find path from top of inductor to bottom of inductor(Thevenin Resistance):

Using a test source: $R_{th} = \dfrac{V_{test}}{I_{test}}$

Setting I_{test}=1A means that only V_{test} needs to be found.

$V_{test} = V_1$

$$-2K\left[\frac{V_1}{4K}\right]-1+\frac{V_1}{4K}+\frac{V_1}{4K}=0$$

$$V_1\left[\frac{-2K+2}{4K}\right]=1 \rightarrow V_1 \approx 1\left(\frac{4K}{2{,}002}\right)$$

$$V_1 \approx -2V$$

$$i_1 = \left(\frac{V_1}{4k}\right)$$

$$-2k\cdot i_1 - 4m - 1 + \left(\frac{V_1-8}{4k}\right) + i_1 = 0$$

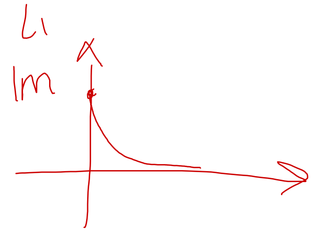
$$-2k\cdot\left(\frac{V_1}{4k}\right) - 4m - 1 + \left(\frac{V_1-8}{4k}\right) + \left(\frac{V_1}{4k}\right) = 0$$

$$V_1\left(\frac{-2k}{4k}+\frac{1}{4k}+\frac{1}{4k}\right) = 4m+1+\frac{8}{4k} \quad\Rightarrow\quad V_1 = 1.006\cdot\left(\frac{4k}{-2.002k}\right)\approx -2V$$

$$V_{test} = -2V$$

$$R_{th} = \frac{V_{test}}{I_{test}} = \left|\frac{-2V}{1A}\right| = 2\Omega$$

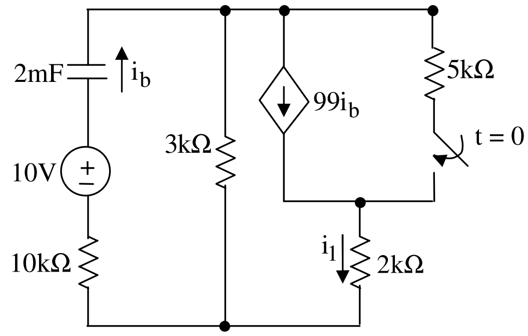
$$\tau = \frac{L}{R_{eq}} = \frac{2\mu}{2} = \boxed{1\mu\sec}$$



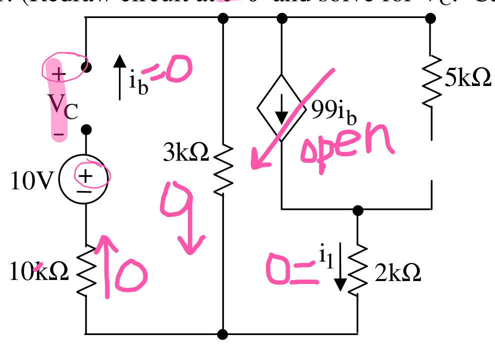
Step 4: Plug values into general equation:

$$i_1(t) = 0 + [1m - 0]e^{-t/1\mu\sec}A = 1me^{-t/1\mu\sec}A$$

4. After being open for a long time, the switch closes at t = 0. Find $i_1(t)$ for t > 0.



Step 1: (Redraw circuit at **t=0**⁻ and solve for V_C. Capacitor acts as an open since it has been a long time)



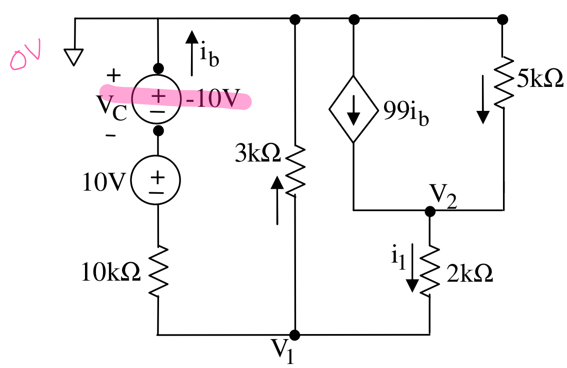
Solving for the dependent variable:

ib=0 which opens the dependent source => 99ib=0

Taking a V-loop to get V_c value(Be Careful- It is **not** 0 when there is a path with a V src.)

$$+0+10+V_C-0=0 \quad \Rightarrow \quad V_C=-10V$$

Step 2: **Initial Value** (Redraw circuit at **t=0⁺** and solve for unknown variable. Capacitor acts as a voltage source since the voltage across capacitor has to remain the same.)



Using node-voltage to solve this circuit to find i_1:
First find dependent variable in terms of node-voltage variable, V_1.

$$i_b=\frac{V_1-(-10)-(-(-10))}{10k}=\frac{V_1}{10k}$$

Next, take current summation equation at V_1 node:

$$\frac{V_1}{10k}+\frac{V_1}{3k}-\frac{(V_1-V_2)}{2k}=0$$

Current summation equation at V_2 node:

$$\frac{(V_2-V_1)}{2k}-99\cdot\frac{V_1}{10k}-\frac{(0-V_2)}{5k}=0$$

Using the first equation and solving for V_1:

$$\frac{V_1}{10k}+\frac{V_1}{3k}-\frac{V_1}{2k}=\frac{V_2}{2k}$$

$$V_1\left(\frac{6}{60k}+\frac{20}{60k}-\frac{30}{60k}\right)=\frac{V_2}{2k}$$

$$V_1=\frac{V_2}{2k}\left(\frac{60k}{-4}\right)=\frac{-15V_2}{2}$$

Plugging into the second equation:

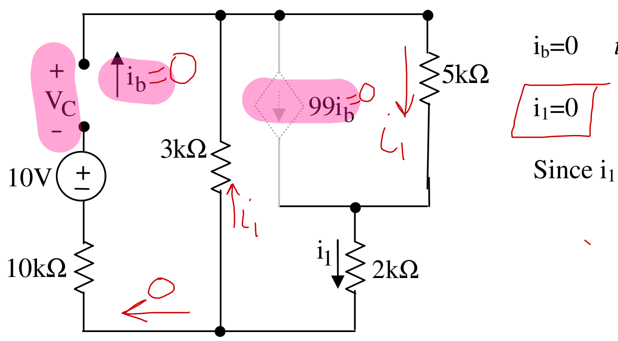
$$\frac{V_2}{2k}-\left(\frac{-15V_2}{4}\right)\left(\frac{1}{2k}\right)-99\cdot\frac{1}{10k}\left(\frac{-15V_2}{4}\right)+\frac{V_2}{5k}=0$$

$$V_2\left(\frac{1}{2k}+\frac{15}{8k}+\frac{99(15)}{40k}+\frac{1}{5k}\right)=0 \quad \Rightarrow \quad V_2=0$$

$$V_1=0$$

$$\boxed{i_1=0}$$

Step 3: **Final Value**(Redraw circuit at **t=∞** and solve for unknown variable. Capacitor acts as an open since it has sat for a long time in this position.)

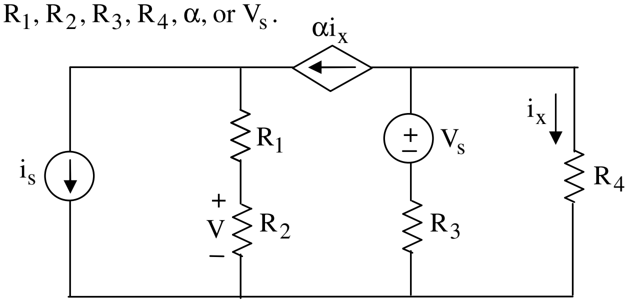


$i_b=0$ *this means that 99ib source becomes open.*

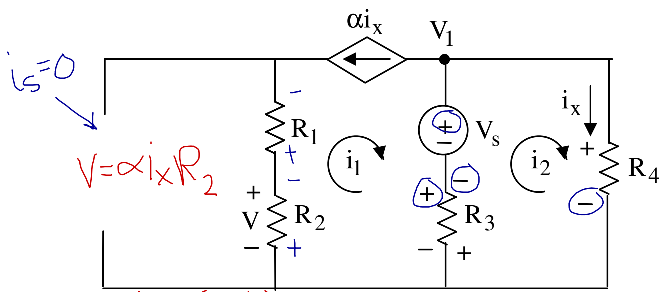
$\boxed{i_1=0}$

Since i_1 is always the same => $\boxed{i_1(t)=0.}$

5. Using superposition, derive an expression for V that contains no circuit quantities other than $i_s, R_1, R_2, R_3, R_4, \alpha$, or V_s.



Step 1: is=0(off), Vs=on



$$+\alpha i_x + \frac{(V_1-V_S)}{R_3} + i_x = 0$$

$$i_x = \frac{V_1}{R_4}$$

Using mesh currents:

$$\underline{i_1 = -\alpha i_x}$$

$$\underline{i_2 = i_x}$$

Using a voltage loop and then substituting above eq:

$$(+i_1 - i_2)R_3 + V_S - i_2 R_4 = 0$$

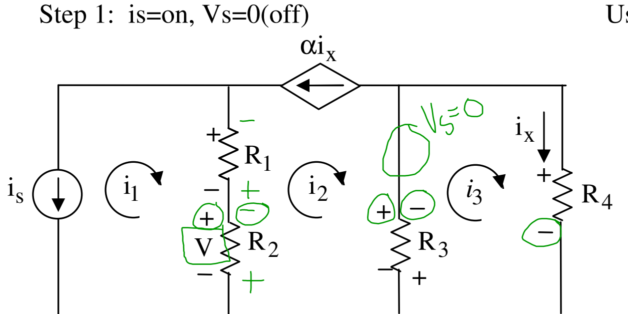
$$(-\alpha i_x - i_x)R_3 + V_S - i_x R_4 = 0$$

$$i_x(+\alpha R_3 + R_3 + R_4) = V_S$$

$$i_x = \frac{V_S}{(+\alpha R_3 + R_3 + R_4)}$$

$$V = \alpha i_x R_2 = \boxed{\frac{\alpha V_S R_2}{(+\alpha R_3 + R_3 + R_4)}}$$

Step 1: is=on, Vs=0(off)



$$V=(+i_1-i_2)R_2$$

Using mesh currents:

$$\cancel{i_1 = i_s} \quad i_1 = -i_s$$

$$i_2 = -\alpha i_x$$

$$i_3 = i_x$$

Using a voltage loop and then substituting above eq:

$$(+i_2 - i_3)R_3 - i_3 R_4 = 0$$

$$(-\alpha i_x - i_x)R_3 - i_x R_4 = 0$$

$$i_x(+\alpha R_3 + R_3 + R_4) = 0$$

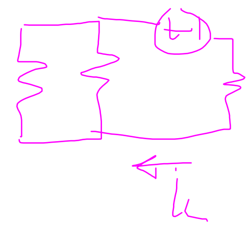
$$i_x = 0$$

$$i_2 = 0$$

$$V = (+i_1 - i_2)R_2 = \boxed{-i_s R_2}\ \cancel{i_s R_2}$$

The total V is the sum of both solutions:

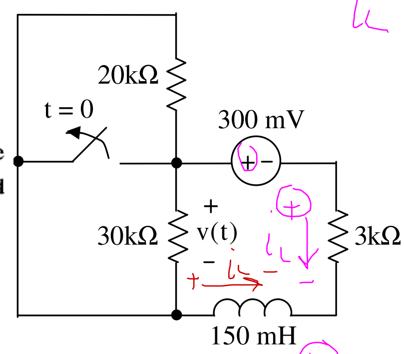
$$\boxed{V = i_s R_2 + \frac{\alpha V_S R_2}{(+\alpha R_3 + R_3 + R_4)}}$$



$W_L = \frac{1}{2} L i_L^2$

$i_L(t \to \infty)$

6. After being closed for a long time, the switch opens at t=0.
   a) Calculate the energy stored on the inductor as t -> ∞.
   b) Write a numerical expression for *v(t)* for t> 0.



SOL'N: a) As t–>∞, the switch is open and the *L* acts like a wire. The 20 kΩ and the 30 kΩ are in parallel, (which is 12 kΩ), and we use Ohm's law to find i_L(t–>∞):

$$i_L(t \to \infty) = \frac{300\text{ mV}}{12\text{ k}\Omega + 3\text{ k}\Omega} = \frac{300\text{ mV}}{15\text{ k}\Omega} = \boxed{20\ \mu\text{A}}$$

The stored energy is a function of the square of the current in the inductor:

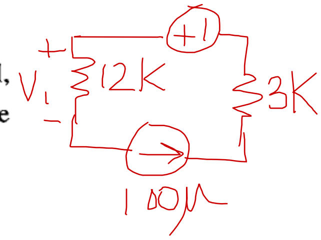
$$w_L(t \to \infty) = \frac{1}{2} L i_L^2(t \to \infty) = \boxed{\frac{1}{2} 150\text{m} \cdot (20\mu)^2}\text{ J} = \boxed{30\text{ pJ}}$$

$+i_L(3K) + 300mV = 0$

$i_L = -100\mu A$

b) We first find the current in the inductor for $t = 0^-$. The switch is closed, shorting out both the 20 kΩ and 30 kΩ resistors. The inductor looks like a wire. This leaves only the 300 mV source and 3 kΩ resistor:

$$i_L(0^-) = \frac{300\text{ mV}}{3\text{ k}\Omega} = \boxed{100\ \mu\text{A}}$$



At $t = 0^+$, the switch is open, the 20 kΩ and the 30 kΩ are in parallel, (which is 12 kΩ), and the inductor acts like a current source with the same current as the inductor had at $t = 0^-$. The voltage, $v(t = 0^+)$, is given by the inductor current times the parallel resistance of 12 kΩ.

$$v(t = 0^+) = 100\ \mu\text{A} \cdot 12\text{ k}\Omega = \boxed{1.2\text{ V}}$$

From earlier, we have that the inductor current as *t* approaches infinity is 10 μA. The voltage, *v*(*t*–>∞), is given by this inductor current times the parallel resistance of 12 kΩ.

$$v(t \to \infty) = 20\ \mu\text{A} \cdot 12\text{ k}\Omega = \boxed{0.24\text{ V}}$$

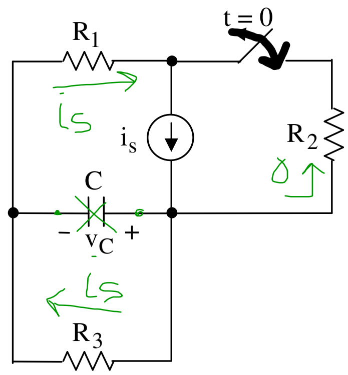
Now we use the general form of solution for *RL* problems:

$$v(t > 0) = v(t \to \infty) + [v(0^+) - v(t \to \infty)]e^{-t/(L/R_{\text{Th}})}$$

or, with L/R_{Th} = 150 mH/(12 kΩ + 3 kΩ) = $\boxed{10\ \mu\text{s}}$:

$$v(t > 0) = 0.24 + [1.2 - 0.24]e^{-t/10\ \mu\text{s}}\text{ V} = \boxed{0.24 + 0.96e^{-t/10\ \mu\text{s}}\text{ V}}$$

7. After being open for a long time, the switch closes at $t=0$.
   a) Write an expression for $v_c(t=0^+)$.
   b) Write an expression for $v_c(t>0)$ in terms of i_s, R_1, R_2, R_3, and C.

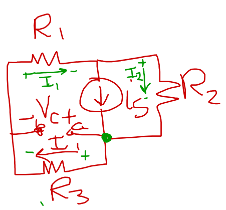


SOL'N: a) We first find the voltage across the capacitor for $t = 0^-$. The switch is open, eliminating R_2 from consideration. The capacitor looks like an open. This leaves only the i_s source driving R_1 and R_3 in series. The capacitor is directly across R_3 and so has the same voltage as R_3:

$$\boxed{v_C(0^-) = i_s R_3}$$

At $t = 0^+$, the capacitor has the same voltage as at $t = 0^-$.

$$\boxed{v_C(0^+) = i_s R_3}$$



b) As $t \to \infty$, the switch is closed, the C acts like an open, and we have a current divider with R_2 on one side and $R_1 + R_3$ on the other side. The current through R_3 is

$$I_1 = i = i_s \frac{R_2}{R_1 + R_2 + R_3}.$$

$-i_s + I_1 - I_2 = 0$

$-I_1(R_3+R_1) - I_2R_2 = 0$

$I_2 = I_1 - i_s$

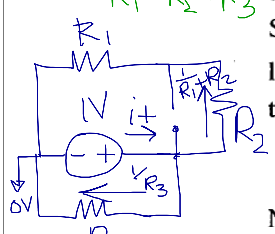
$-I_1(R_1+R_3) - R_2(I_1 - i_s) = 0$

$I_1 = +i_s \dfrac{R_2}{R_1+R_2+R_3}$

The voltage, v_C, as $t \to \infty$ is the same as the voltage across R_3, and is given by i times R_3:

$$v_C(t \to \infty) = i_s \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

For the time constant of the circuit, we take the Thevenin resistance from the terminals where the C is attached with the switch closed for $t > 0$. Since we have only an independent source, we turn off the source, i_s, and look into the circuit from the terminals where C is attached (but without the C). We see R_3 in parallel with $R_1 + R_2$.



$$R_{Th} = R_3 \| (R_1 + R_2)$$

$-i_t + \dfrac{1}{R_3} + \dfrac{1}{R_1+R_2} = 0$

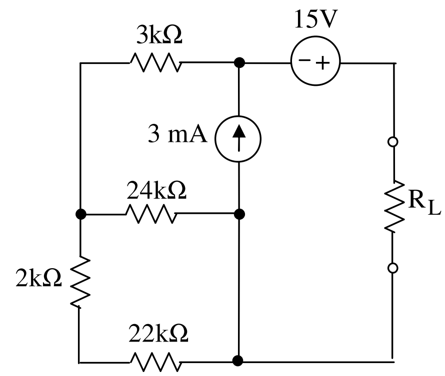
Now we use the general formula for RC circuit solutions:

$$v(t > 0) = v(t \to \infty) + [v(0^+) - v(t \to \infty)]e^{-t/R_{Th}C}$$

or

$$v(t > 0) = i_s \frac{R_2 R_3}{R_1 + R_2 + R_3} + [i_s R_3 - i_s \frac{R_2 R_3}{R_1 + R_2 + R_3}]e^{-t/R_3 \| (R_1 + R_2)C}$$

Use the circuit below for both problem 8 and 9.



8. Calculate the value of R_L that would absorb maximum power.

Use $R_L = R_{Th}$ for maximum power transfer. To find R_{Th}, we turn off the independent sources and look in from the terminals where R_L is attached (with R_L removed). The voltage source becomes a wire, and the current source becomes an open circuit.

$$R_{Th} = (2\text{ k}\Omega + 22\text{ k}\Omega) \| 24\text{ k}\Omega + 3\text{ k}\Omega = 12\text{ k}\Omega + 3\text{ k}\Omega = 15\text{ k}\Omega$$

We use this Thevenin resistance value for R_L:

$$R_L = 15\text{ k}\Omega$$

9. Calculate that value of maximum power R_L could absorb.

The maximum power transferred is

$$p_{max} = \frac{v_{Th}^2}{4R_{Th}}$$

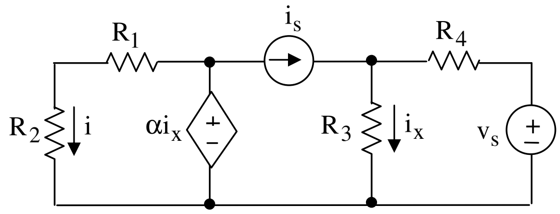
The Thevenin equivalent voltage is the voltage across R_L without R_L. Since there is no R_L, the 3 mA current must all flow through the 3 kΩ resistance and then divide as it flows through the other resistors. It turns out that the 3 mA flows through R_{Th}, and the voltage arising from the 3 mA is found using Ohm's law. To this voltage, we add the 15 V from the voltage source to get v_{Th}.

$$v_{Th} = 3\text{ mA} \cdot 15\text{ k}\Omega + 15\text{ V} = 45 + 15\text{ V} = 60\text{ V}$$

Now we use the formula for maximum power transferred:

$$p_{max} = \frac{v_{Th}^2}{4R_{Th}} = \frac{60^2}{4 \cdot 15k}\text{ W} = 60\text{ mW}$$

10. Using superposition, derive an expression for i that contains no circuit quantities other than $i_s, R_1, R_2, R_3, R_4, \alpha$, or V_s.



SOL'N: First, we turn off the independent source, i_s. This means the current source turns into an open circuit, separating the circuit into two pieces. On the right, we have v_s in series with R_3 and R_4.

$$i_{x1} = \frac{v_s}{R_3 + R_4}$$

On the left side, we have the dependent voltage source (whose voltage is now known) and R_1 in series with R_2.

$$i_1 = \frac{\alpha i_{x1}}{R_1 + R_2} = \frac{\alpha v_s}{(R_1 + R_2)(R_3 + R_4)}$$

Second, we turn off the independent source, v_s. This turns the voltage source into a wire, and we have a current divider for i_s flowing through R_3 parallel R_4.

$$i_{x2} = \frac{i_s R_4}{R_3 + R_4}$$

On the left side, the dependent source fixes the voltage across R_1 and R_2. Thus, we have the dependent voltage source (whose voltage is now known) and R_1 in series with R_2, just as we did before.

$$i_2 = \frac{\alpha i_{x2}}{R_1 + R_2} = \frac{\alpha i_s R_4}{(R_1 + R_2)(R_3 + R_4)}$$

We sum the two currents to find the total value of i:

$$i = i_1 + i_2 = \frac{\alpha(v_s + i_s R_4)}{(R_1 + R_2)(R_3 + R_4)}$$

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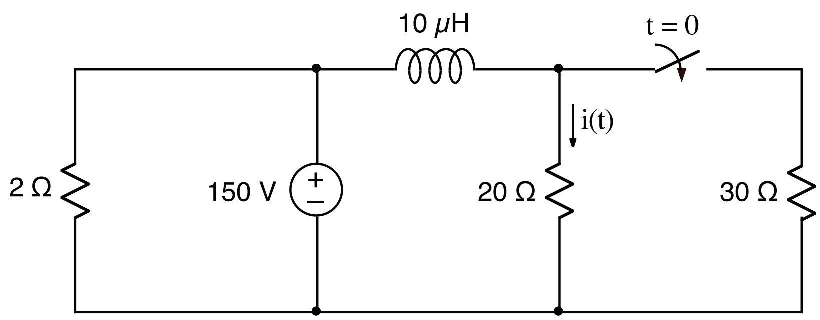
HOMEWORK #6

THE UNIVERSITY OF UTAH

1.

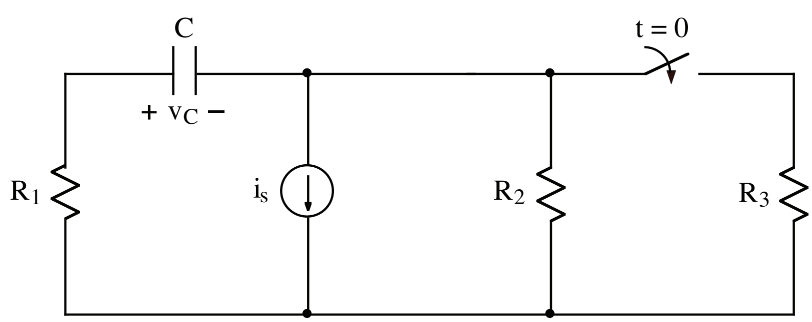


After being ~~closed~~ open for a long time, the switch closes at $t = 0$.

Calculate the energy stored on the inductor as $t \to \infty$.

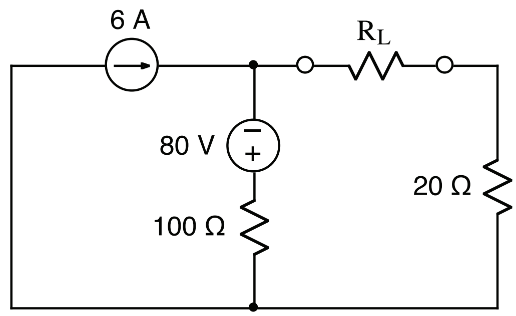
2. For the circuit in problem 1, find $i(t)$ for $t > 0$.

3.



After being open for a long time, the switch closes at $t = 0$. Write an expression for $v_c(t \geq 0)$ in terms of R_1, R_2, R_3, i_s, and C.

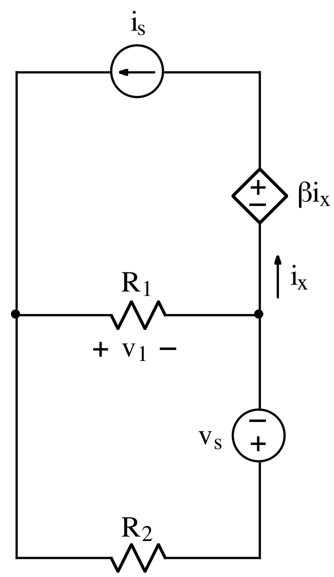
4.



a) Calculate the value of R_L that would absorb maximum power.

b) Calculate that value of maximum power R_L could absorb.

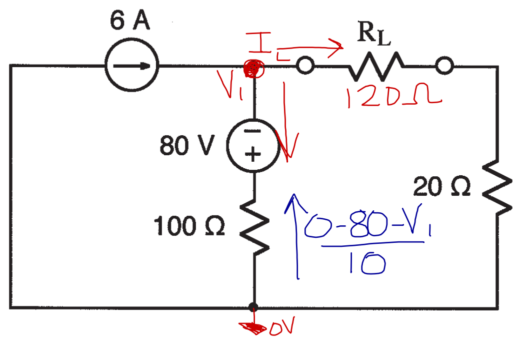
5.



Using superposition, derive an expression for v_1 that contains no circuit quantities other than i_s, v_S, R_1, R_2, and β, where $\beta > 0$.

HOMEWORK #6 Solution Prob 4

Ex:



$P = I_L^2(120) = 573.3\,W$

$-6 + \frac{V_1+80}{100} + \frac{V_1}{140} = 0$

$i_L = \frac{V_1}{140}$

$V_1\left(\frac{1}{100} + \frac{1}{140}\right) = +6 - \frac{8}{10}$

$V_1 = \frac{910}{3}$

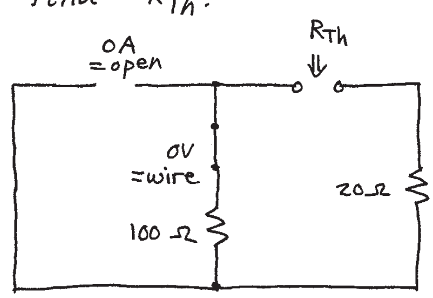
$i_L = \frac{910}{3(140)}$

a) Calculate the value of R_L that would absorb maximum power.

b) Calculate that value of maximum power R_L could absorb.

Sol'n: a) $R_L = R_{Thev}$ for max pwr

Turn off sources, remove R_L, and look into circuit from R_L terminals to find R_{Th}.



$R_{Th} = 100\,\Omega + 20\,\Omega = 120\,\Omega$

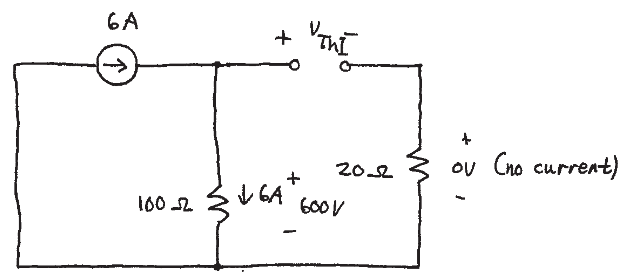
$\therefore\ R_L = 120\,\Omega$

b) max pwr $= \dfrac{\left(\frac{V_{Th}}{2}\right)^2}{R_{Th}}$ from Thev equiv with $R_L = R_{Th}$. (R_L sees $V = \frac{V_{Th}}{2}$.)

v_{Th} = V across R_L terminal with R_L removed.

Use superposition to find v_{Th}.

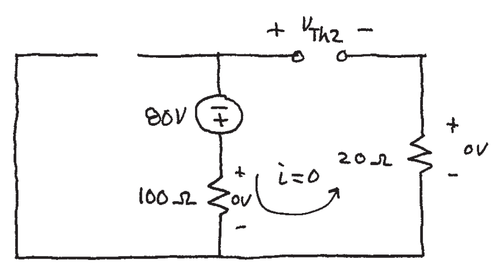
case I: 6A on, 80V off



$$v_{Th1} = V \text{ across } 100\,\Omega \text{ resistor} = 6A \cdot 100\,\Omega$$

$$\therefore\ v_{Th1} = 600V$$

case II: 6A off, 80V on



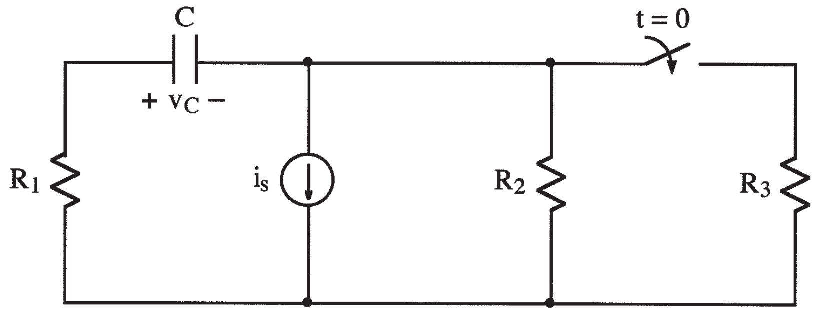
We have no current no V drop across R's.
The 80V appears as $-v_{Th2}$ across the terminals. $v_{Th2} = -80V$.

$$v_{Th} = v_{Th1} + v_{Th} = 600 - 80\ V = 520V$$

$$p_{max} = \left(\frac{520}{2}\right)^2 / R_{Th} = \left(\frac{520}{2}\right)^2 / 120\,\Omega = \frac{1690}{3}\ W = 563.3W$$

HOMEWORK #6 Solution Prob 3

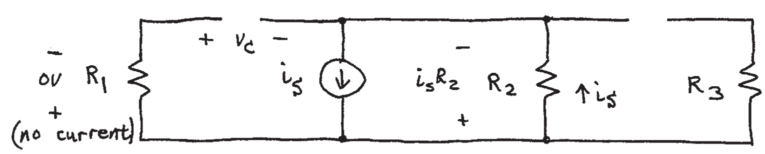
Ex:



After being open for a long time, the switch closes at t = 0. Write an expression for $v_c(t \geq 0)$ in terms of R_1, R_2, R_3, i_s, and C.

Sol'n: At $t = 0^-$ the switch is open and C = open.

$t = 0^-$:



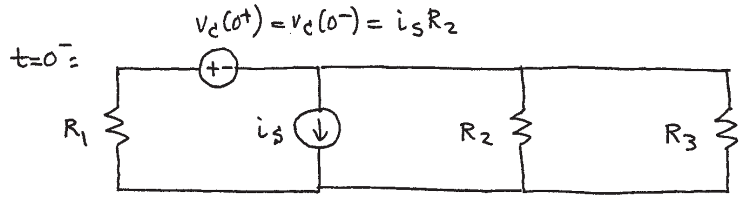
i_s flows thru R_2 producing v-drop $i_s R_2$.

Since there is no current in R_1, this voltage appears across C.

$$v_c(0^-) = i_s R_2$$

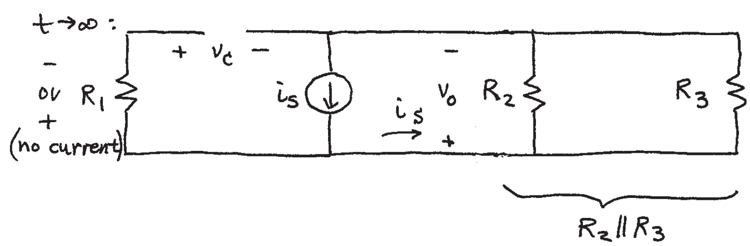
Note that + sign of $i_s R_2$ v-drop connects to + sign of v_c thru R_1 (0V drop ≈ wire) and − sign of $i_s R_2$ v-drop connects to − sign of v_c thru wire.

At $t=0^+$, we treat C as v-source with value $v_C(0^+) = v_C(0^-)$. Switch is closed.



Since the value we need is $v_C(0^+)$, there is nothing further to solve.

For $t \to \infty$, we treat C as open, switch closed.



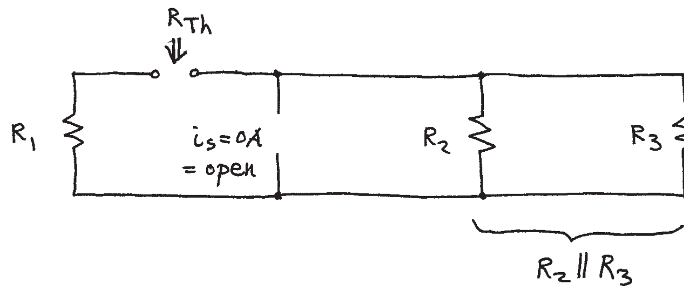
Now we have $v_C = i_S \cdot R_2 \| R_3$. This is the same as $t=0^-$ except that we have $R_2 \| R_3$ instead of R_2.

$$v_C(t \to \infty) = i_S\, R_2 \| R_3$$

The time constant is $R_{Th} C$.

We remove C and look into the circuit from terminals where C attaches. We also turn off i_S. What we see is R_{Th}.

HOMEWORK #6 Solution Prob 3 (cont.)



We have $R_{Th} = R_1 + R_2 \| R_3$

Now plug terms into general sol'n:

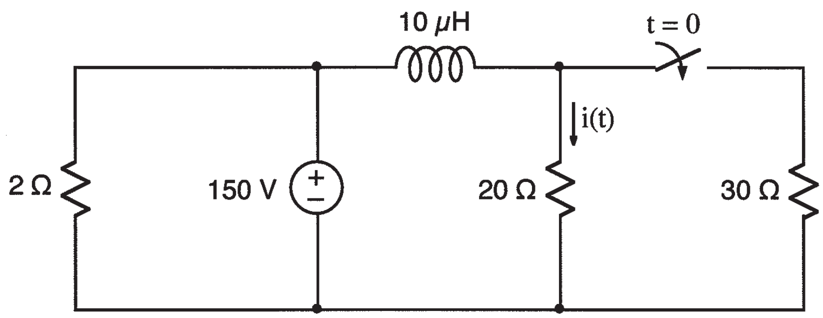
$$v_c(t>0) = v_c(t\to\infty) + \left[v_c(0^+) - v_c(t\to\infty)\right] e^{-t/R_{Th}C}$$

Here, we have:

$$v_c(t>0) = i_s \cdot R_2 \| R_3 + \left(i_s R_2 - i_s R_2 \| R_3\right) e^{\frac{-t}{(R_1 + R_2 \| R_3)C}}$$

HOMEWORK #6 Solution Prob 2

Ex:

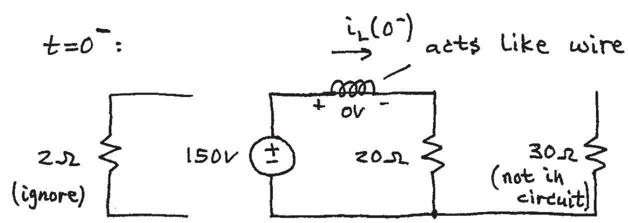


After being closed for a long time, the switch closes at t = 0.

Find i(t) for t > 0.

Sol'n: At $t = 0^-$, switch is open and L = wire. We note that the 2Ω resistor is a 2nd circuit across the 150V source and may be ignored.

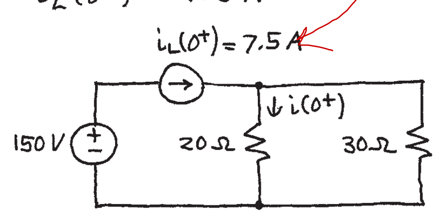
$t = 0^-$:



$$i_L(0^-) = \frac{150\,V}{20\,\Omega} = 7.5\ A$$

At $t = 0^+$, switch is closed and we model L as current source with $i_L(0^+) = i_L(0^-) = 7.5\ A$.

$t = 0^+$:



HOMEWORK #6 Solution Prob 2 (cont.)

We have a current divider.

$$i(0^+) = i_L(0^+) \cdot \frac{30\,\Omega}{20\,\Omega + 30\,\Omega} = 7.5A \cdot \frac{3}{5} = \boxed{4.5\,A}$$

$$i_L = \frac{150}{20} + \frac{150}{30} = \frac{150(30+20)}{30(20)}$$

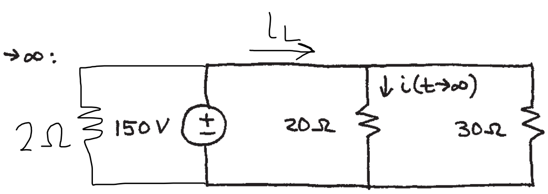
$$W_L(t\to\infty) = \frac{1}{2} L i_L^2$$

$L = 10\,\mu H$

$$W_L(t\to\infty) = 78\,\mu J$$

As $t\to\infty$, switch is closed and L = wire.

$t\to\infty$:

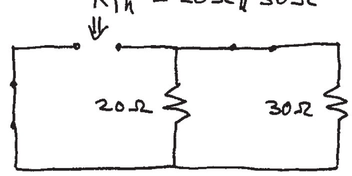


We have 150V across the 20Ω resistor.

$$i(t\to\infty) = \frac{150V}{20\Omega} = \boxed{7.5\,A}$$

The time constant is $\frac{L}{R_{Th}}$ where we look into the circuit from the terminals where L is attached. We can find R_{Th} by turning off the 150V source and seeing what R value we have looking into the circuit from the terminals where L is attached.

$R_{Th} = 20\Omega \| 30\Omega$



Note: switched is closed since $t>0$.

$$R_{Th} = 20\Omega \| 30\Omega = 12\,\Omega$$

HOMEWORK #6 Solution Prob 2 (cont.)

The time constant is $\frac{L}{R_{Th}} = \frac{10\,\mu H}{12\,\Omega} = \frac{5}{6}\,\mu s.$

We plug values into the general form of solution:

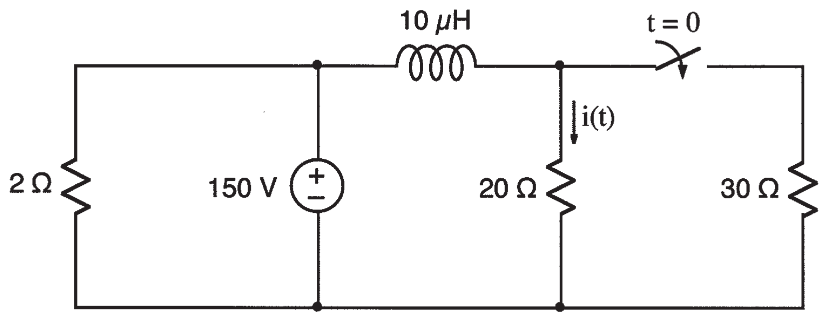
$$i(t>0) = i(t\to\infty) + [i(t=0^+) - i(t\to\infty)]\, e^{-t/\frac{L}{R_{Th}}}$$

$$\boxed{i(t>0) = 7.5A + [4.5A - 7.5A]\, e^{-t/\frac{5}{6}\mu s}}$$

or $i(t>0) = 7.5A - 3Ae^{-t/\frac{5}{6}\mu s}$

HOMEWORK #6 Solution Prob 1

Ex:



After being closed for a long time, the switch closes at t = 0.

Calculate the energy stored on the inductor as $t \to \infty$.

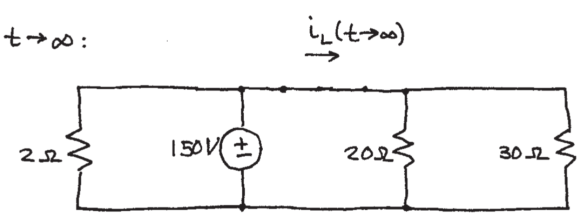
Sol'n: Energy $w = \frac{1}{2} L i_L^2$

For $t \to \infty$, currents and voltages reach constant values. Thus $v_L = L\frac{di}{dt} = L \cdot 0 = 0$

and the L acts like a wire. (It can carry current but $v_L(t \to \infty) = 0V$.)

The switch is closed as $t \to \infty$.

$t \to \infty$:



Note: It doesn't which direction we use to measure i_L since we will be using i_L^2 and the squared value will be positive.

HOMEWORK #6 Solution Prob 1 (cont.)

The 2Ω resistor is a 2nd circuit across the 150V source that we may ignore.

We have $i_L(t \to \infty) = \dfrac{150\,V}{20\Omega \| 30\Omega}$.

$$20\Omega \| 30\Omega = 10\Omega \cdot 2\|3 = 10\Omega \cdot \frac{2(3)}{2+3} = 12\Omega$$

Thus, $i_L(t \to \infty) = \dfrac{150\,V}{12\Omega} = \dfrac{25}{2}\,V = 12.5\,V.$

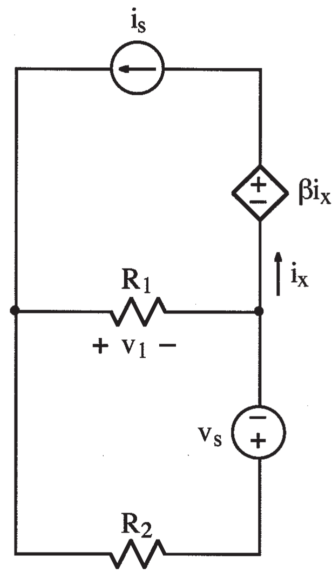
Energy $w_L(t \to \infty) = \dfrac{1}{2}\,10\mu H \cdot \left(\dfrac{25}{2}\,V\right)^2$

$$w_L(t \to \infty) = 781.25\,\mu J$$

Note: The units for energy are Joules.

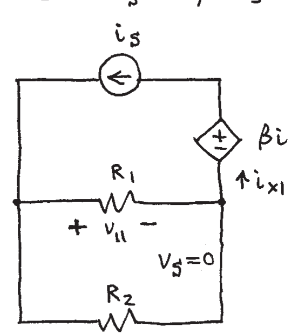
HOMEWORK #6 Solution Prob 5

Ex:



Using superposition, derive an expression for v_1 that contains no circuit quantities other than i_s, v_s, R_1, R_2, and β, where $\beta > 0$.

Sol'n: case I: i_s on, v_s off



$R_{eq} = -\frac{\beta i_{x1}}{i_{x1}} = -\beta$

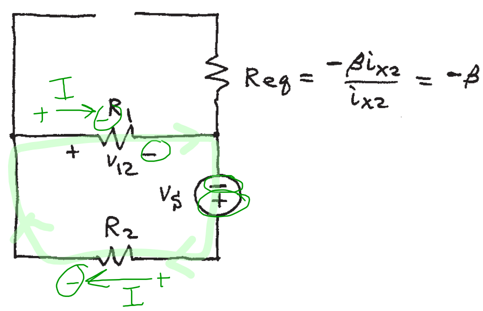
We have a current divider with R_1 & R_2.

$$i_{R1} = i_s \cdot \frac{R_2}{R_1 + R_2}$$

$$v_{11} = i_{R1} \cdot R_1 = i_s \cdot R_1 \| R_2$$

$$V_{11} = \frac{i_s R_2 R_1}{R_1 + R_2}$$

case II: i_s off, v_s on



$+V_S = I(R_1 + R_2)$

$V_{12} = IR_1$

$V_{12} = \frac{V_S R_1}{R_1 + R_2}$

We have voltage divider.

$$v_{12} = v_s \cdot \frac{R_1}{R_1 + R_2}$$

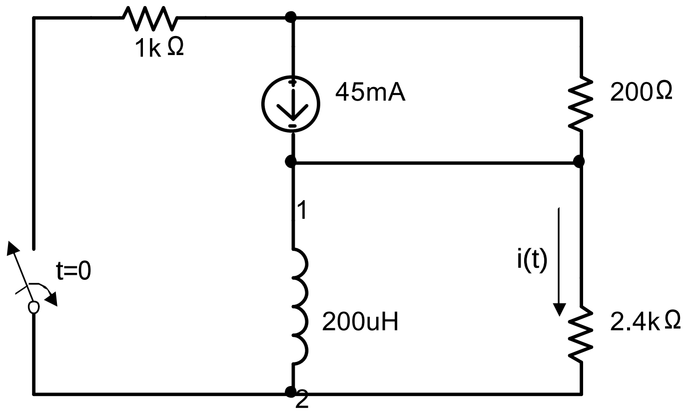
sum v_1's:

$$\boxed{v_1 = v_{11} + v_{12} = i_s \cdot R_1 \| R_2 + v_s \frac{R_1}{R_1 + R_2}}$$

UNIVERSITY OF UTAH
ELECTRICAL AND COMPUTER ENGINEERING DEPARTMENT

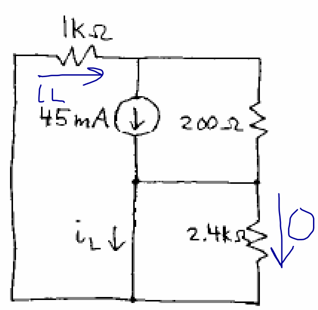
ECE 1270 HOMEWORK #6 Solution Summer 2007

1.

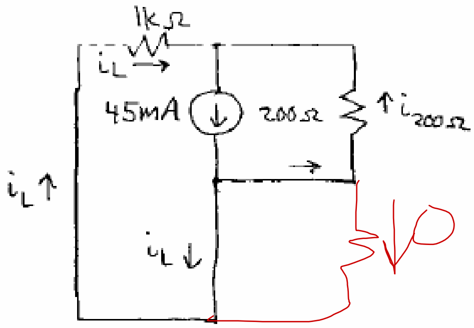


After being open for a long time, the switch closes at t=0.
a) Calculate the energy stored on the inductor as $t \rightarrow \infty$.
b) Write a numerical expression for i(t) for t>0.

sol'n: a) As $t \to \infty$, the L acts like a wire.



Since the 2.4kΩ is shorted out by wires, we may ignore it. This leaves a current divider formed by the 1kΩ and 200Ω resistors for current from the 45mA source.



$$i_L = 45mA \cdot \frac{200\Omega}{200\Omega + 1k\Omega} = 45mA \cdot \frac{1}{6}$$

$$i_L = 7.5 \text{ mA}$$

For an inductor, $w_L = \frac{1}{2} L i_L^2$:

$$w_L(t\to\infty) = \frac{1}{2} \cdot 200\,\mu H \cdot (7.5 mA)^2$$

$$= 100\,\mu H \cdot \left(\frac{3}{4}\right)^2 (10 \text{ mA})^2$$

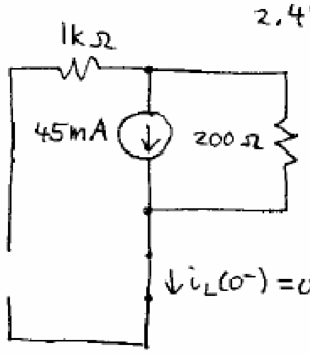
$$= \frac{9}{16} \cdot 100\,\mu \cdot 100\,\mu \text{ J}$$

$$= \frac{9}{16} \cdot 10k\,\mu\mu \text{ J}$$

$$= \frac{90}{16} \text{ nJ}$$

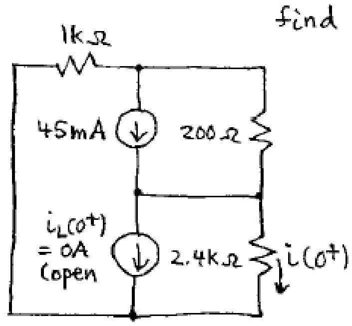
$$w_L(t\to\infty) = \boxed{5.625 \text{ nJ}}$$

b) $t=0^-$ model: L = wire, switch open
2.4kΩ shorted out (so ignore)
find $i_L(0^-)$

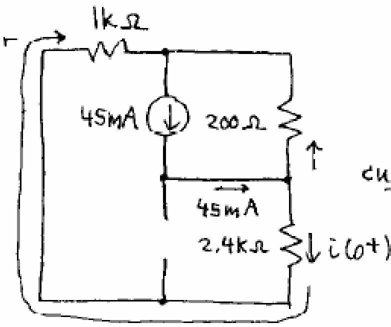


$t = 0^+$ model: L = current source: $i_L(0^+) = i_L(0^-)$

find $i(0+)$ (switch closed)



or



current divider

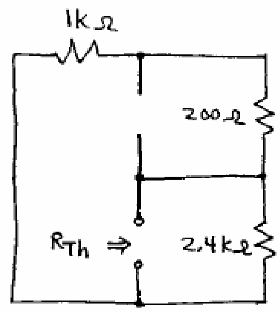
We have a current divider:

$$i(0^+) = 45mA \cdot \frac{200\Omega}{1k\Omega + 2.4k\Omega + 200\Omega} = \frac{45mA}{12}$$

or $i(0^+) = \boxed{\frac{15}{4} \ mA}$

From part (a) we know $i(t \to \infty) = 0$ since the 2.4kΩ is shorted by the L and the wire on the middle right side.

$\tau = \frac{L}{R_{Th}}$ where R_{Th} = Thevenin R for terminals where L is connected.



We turn off the independent source and look into the terminals where L is connected.

$$R_{Th} = 2.4k\Omega \,\|\, (1k\Omega + 200\Omega)$$

$$= 2.4k\Omega \,\|\, 1.2k\Omega$$

$$= 1.2k\Omega \cdot 2\|1 = 1.2k\Omega \cdot \frac{2}{3}$$

$R_{Th} = 800\Omega$ $\quad \tau = \frac{200\mu H}{800\Omega} = \frac{1}{4}\mu s$

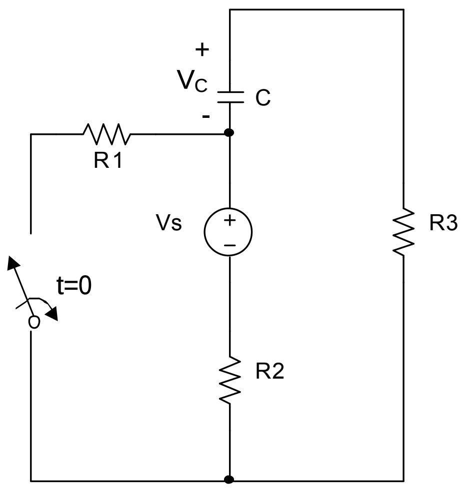
or $\tau = 250\ ns$

Use $i(t) = i(t\to\infty) + [i(0^+) - i(t\to\infty)]e^{-t/\tau}$ $\quad t>0$

$$\therefore\ i(t) = 0 + \left[\frac{15}{4}mA - 0\right]e^{-t/250ns} = \boxed{3.75mAe^{-t/250ns}}$$

for $t>0$

2.



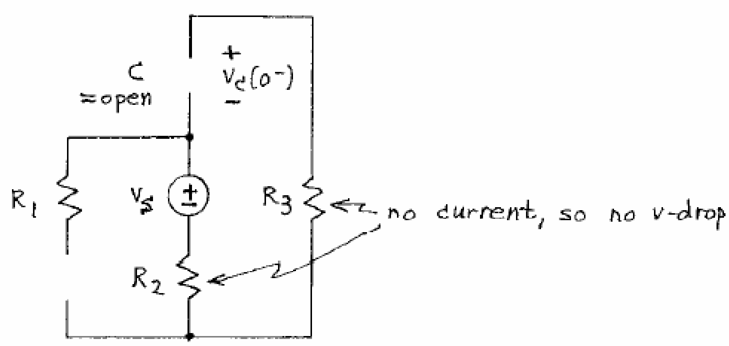
After being open for a long time, the switch becomes closed at t=0.

a) Write an expression for $V_c(t{=}0^+)$.

b) Write an expression for $V_c(t{>}0)$ in terms of R1, R2, R3, Vs, and C.

sol'n: a) We use $v_c(t=0^+) = v_c(0^-)$.

$t=0^-$ model: C acts like open circuit
switch is open



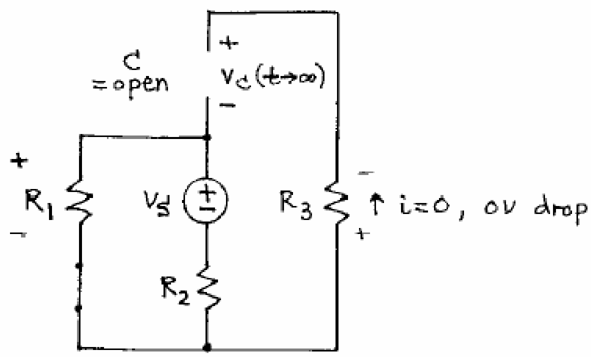
No current flows. $v_c(0^+) = v_c(0^-) = -v_s$

b) We use the general form of sol'n:

$$v_c(t>0) = v_c(t\to\infty) + [v_c(0^+) - v_c(t\to\infty)]\,e^{-t/\tau}$$

where $\tau = R_{Th}C$ (using Thevenin equiv where C connected)

$t\to\infty$ model: C = open, switch closed

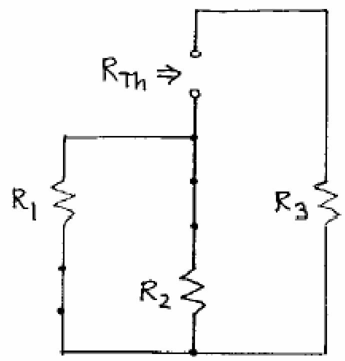


Since there is no v-drop across R_3, we have $v_c(t\to\infty)$ = − v-drop across R_1 from v-loop around outside of circuit.

v-drop across $R_1 = v_s \dfrac{R_1}{R_1+R_2}$ (v-divider)

$$\therefore\ v_c(t\to\infty) = -v_s\frac{R_1}{R_1+R_2}$$

$\tau = R_{Th}C$: We remove C and turn off v_s. Then we look into circuit from terminals where C connected.



We have $R_{Th} = R_1 \| R_2 + R_3$

Combining results, we have our solution:

$$v_C(t>0) = -v_s \frac{R_1}{R_1+R_2} + \left[-v_s + v_s \frac{R_1}{R_1+R_2}\right] e^{-t/(R_1\|R_2+R_3)C}$$

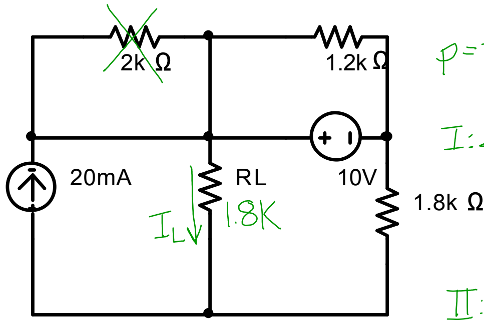
or

$$v_C(t>0) = -v_s \left\{ \frac{R_1}{R_1+R_2} + \frac{R_2}{R_1+R_2} e^{-t/(R_1\|R_2+R_3)C} \right\}$$

or

$$v_C(t>0) = -v_s + v_s \frac{R_2}{R_1+R_2} \left[1 - e^{-t/(R_1\|R_2+R_3)C}\right]$$

3.



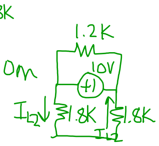
$P = I_L^2 R_{th} = 12.8m^2(1.8K) = 293.9mW$

superposition

I: 20A on, 10V off

$I_{L1} = \frac{20(1.8K)}{3.6K} = 10m$

II: 20A off, 10V on



a) Calculate the value of RL that would absorb maximum power.
b) Calculate that value of maximum power RL could absorb.

$I_{L2} = \frac{10}{3.6K} \rightarrow I_L = I_{L1} + I_{L2}$

soln: $R_L = R_{Th}$ for max power transfer where the Thevenin equivalent is with respect to terminals a and b (without R_L).

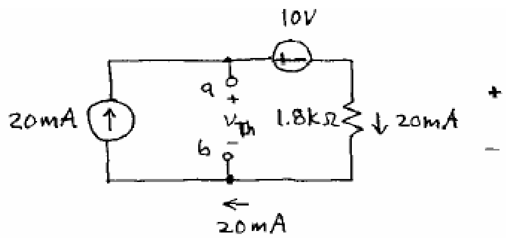
$I_L = 10m + \frac{10}{3.6K}$

$I_L = 12.8mA$

We observe that the 2kΩ resistor on top is shorted out by wires and may be ignored.

We also observe that the 1.2kΩ resistor on top is directly across the 10V source and may be treated as a separate circuit having no effect on the rest of the circuit other than to draw some current from the 10V source.

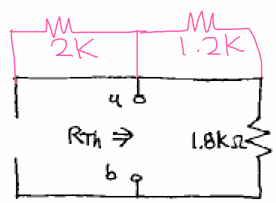
That leaves us with the following circuit:



V_{Th} = voltage across a, b terminals with no load from a to b.

We have $v_{ab} = 20mA \cdot 1.8k\Omega + 10V = 46V$

To find R_{Th}, we turn off the two independent sources and look into the circuit from the a and b terminals:



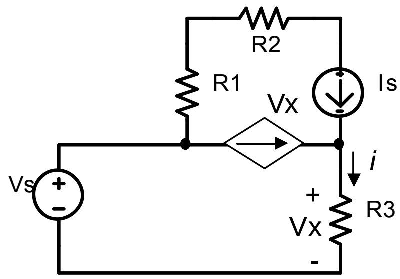
We see 1.8kΩ across a and b.

$R_{Th} = 1.8\ k\Omega$

$\therefore \boxed{R_L = 1.8k\Omega}$

b) $\text{max pwr} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(46V)^2}{4 \cdot 1.8k\Omega} \doteq 293.9\ mW$

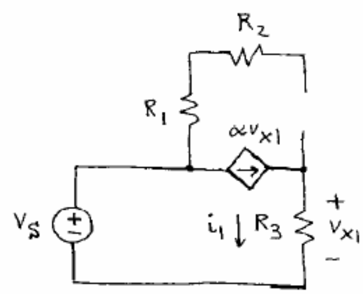
4.



Using superposition, derive an expression for *i* that contains no circuit quantities other than Is, Vs, R1, R2, R3, and , where >0.

sol'n: We turn on one independent source at a time, find i for each source, and sum the i's.

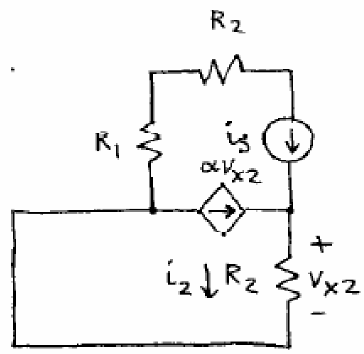
case I: v_s on and i_s off (= open)



We have $i_1 = \alpha v_{x1} = \dfrac{v_{x1}}{R_3}$

Unless $\alpha = \dfrac{1}{R_3}$ <u>exactly</u>, (which is impossible in practice), we must have $v_x = 0$, $i_1 = 0$.

case II: V_s off, i_s on
(=wire)



We have the following current summation at the node on the right side:

$$-i_s - \alpha i_2 R_2 + i_2 = 0$$

or

$$i_2(1-\alpha R_2) = i_s$$

or

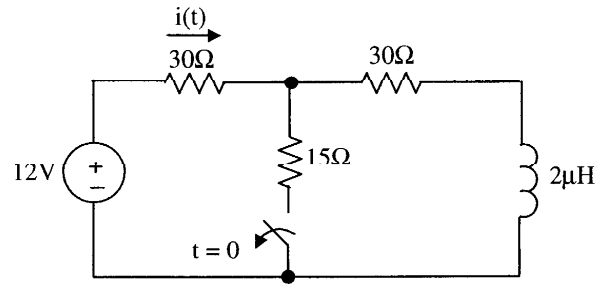
$$i_2 = \frac{i_s}{1-\alpha R_2}$$

Now we sum the i_1 and i_2:

$$i = i_1 + i_2 = 0 + \frac{i_s}{1-\alpha R_2}$$

$$i = \frac{i_s}{1-\alpha R_2}$$

1. (30 points)

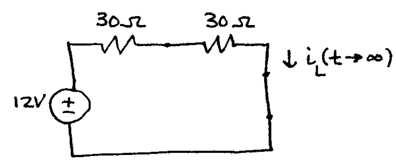


After being closed for a long time, the switch is <u>opened</u> at t = 0.

a. Calculate the energy stored on the inductor at $t \to \infty$.

b. Write a numerical expression for $i(t)$, $t > 0$.

sol'n: a) $t \to \infty$: model L as wire, switch open



$i_L(t\to\infty) = \frac{12V}{30+30\Omega} = \frac{12V}{60\Omega}$

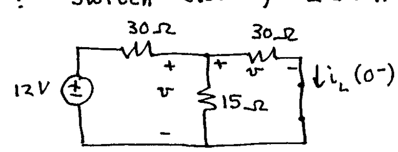
$\quad " \quad = \frac{1}{5}A$

Energy $w_L(t\to\infty) = \frac{1}{2} L i_L^2(t\to\infty) = \frac{1}{\cancel{2}}\cancel{2}\mu H \cdot \left(\frac{1}{5}A\right)^2$

$$\boxed{w_L(t\to\infty) = \frac{1}{25}\,\mu W \text{ or } 40\,nW}$$

Note: $i(t\to\infty) = i_L(t\to\infty) = \frac{1}{5}A = 200\,mA$

b) $t=0^-$: Switch closed, L = wire, find $i_L(0^-)$



$i_L(0^-) = \frac{v}{30\Omega}$

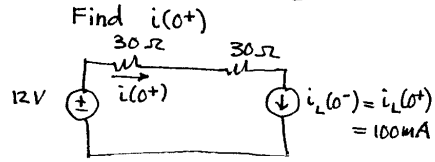
$v = \frac{15\|30\Omega}{15\|30 + 30\Omega} \cdot 12V$

$t=0^+$: switch open, L = current src $\;" = i_L(0^-)$

$15\|30\Omega = 15\Omega \cdot 1\|2 = 15\Omega \cdot \frac{2}{3} = 10\Omega$

$v = \frac{10}{10+30} \cdot 12V = 3V$

Find $i(0^+)$



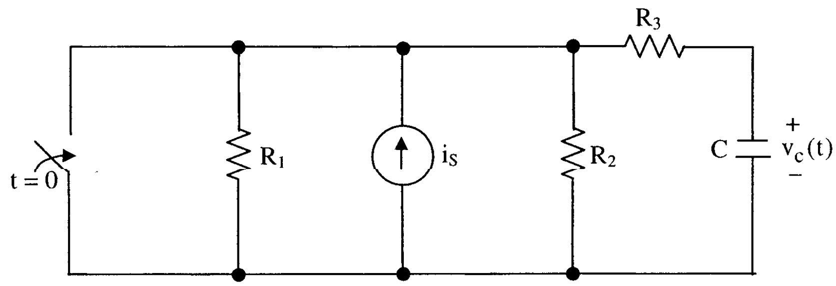
$i_L(0^-) = \frac{3V}{30\Omega} = 100\,mA$

$i(0^+) = i_L(0^+) = i_L(0^-) = 100\,mA$

$0<t<\infty$: R_{Thev} of circuit $= 30\Omega + 30\Omega = 60\Omega$ $\quad \therefore \tau = \frac{L}{R_{Th}} = \frac{2\mu H}{60\Omega} = \frac{1}{30}\mu s$

$$\boxed{i(t>0) = i(t\to\infty) + [i(0^+) - i(t\to\infty)]e^{-t/\tau} = 200 - 100e^{-t/\frac{1}{30}\mu s}\ mA}$$

2. (25 points)

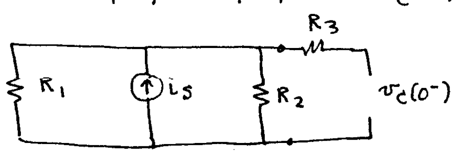


After being open for a long time, the switch is <u>closed</u> at t = 0.

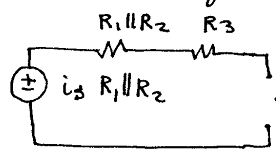
a. Write an expression for $v_c(t = 0^+)$.

b. Write an expression for $v_c(t)$, $t > 0$.

sol'n: a) $t=0^-$: switch open, C = open, find $v_C(0^-)$ since $v_C(0^+) = v_C(0^-)$



use Thevenin equiv of i_s, R_1, and R_2

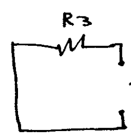


$v_C(0^-) = i_s \cdot R_1 \| R_2 = v_C(0^+)$

$$\boxed{v_C(0^+) = i_s \cdot R_1 \| R_2 \quad \text{or} \quad i_s \frac{R_1 R_2}{R_1 + R_2}}$$

b) $v_C(0^+) = i_s \cdot R_1 \| R_2$ so move on to $t \to \infty$ and R_{Thev}

$t \to \infty$: switch closed (so we can ignore R_1, i_s, and R_2), C = open, find $v_C(t \to \infty)$



$v_C(t \to \infty) = 0V$

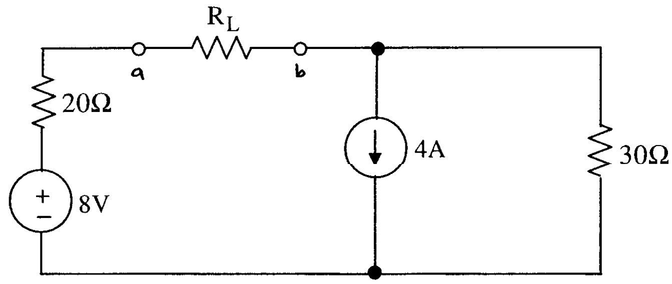
$0 < t < \infty$: from circuit for $t \to \infty$ we see that R_{Thev} of circuit where C is connected is just R_3

$\tau = R_{Th} C = R_3 C$

$$v_C(t \geq 0) = v_C(t \to \infty) + [v_C(0^+) - v_C(t \to \infty)]e^{-t/\tau}$$

$$\boxed{v_C(t \geq 0) = i_s \cdot R_1 \| R_2 \, e^{-t/R_3 C}}$$

3. (20 points)

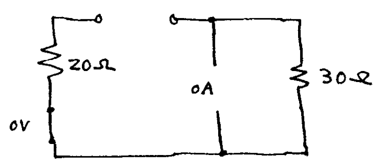


a. Calculate the value of R_L that would absorb maximum power.

b. Calculate that value of maximum power R_L could absorb.

soln: a) $R_L = R_{Thev}$ of circuit where R_L connected gives max pwr xfer

R_{Thev} = R looking into circuit at terminals a, b without R_L and src's = 0 (i.e. 8V = 0V = wire, 4A = 0A = open)

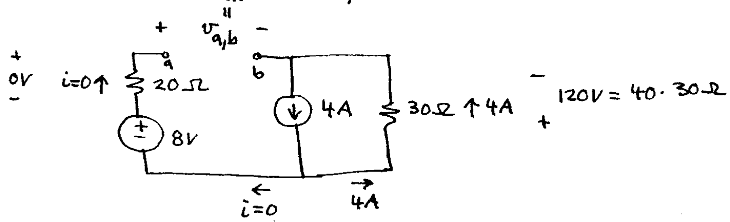


$R_{Th} = 20 + 30\ \Omega = 50\ \Omega$

$\therefore \boxed{R_L = 50\ \Omega}$

b) $\max p = \dfrac{v_{Th}^2}{4R_{Th}}$

Find $v_{Th} = v_{a,b}$ without R_L

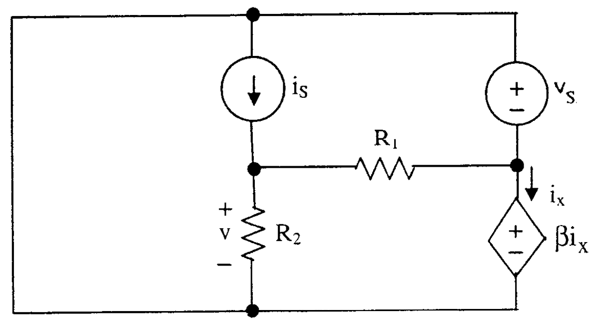


From outer v-loop, we have $v_{Th} = -8V - 120V = -128V$

$\max p = \dfrac{(-128V)^2}{4 \cdot 50\ \Omega} = 32 \cdot \dfrac{128}{50}\ W = \dfrac{32 \cdot 128 \cdot 20}{1k}\ W = \dfrac{81.92\ kW}{1k}$

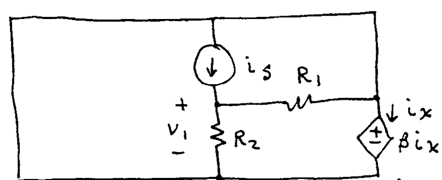
$\boxed{\max p = 81.92\ W}$

4. (25 points)



Using superposition, derive an expression for v that contains no circuit quantities other than i_S, v_S, R_1, R_2, and β, where $\beta > 0$.

soln: case I: i_s on, v_s off



$\Big\}\ R_{eq} = \dfrac{\beta i_x}{i_x} = \beta\ \Omega$

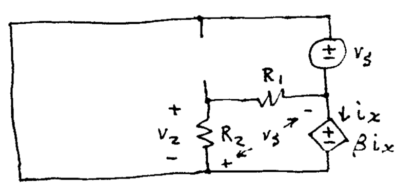
↰ wire around outside shorts $R_{eq} \Rightarrow \beta i_x = 0V \Rightarrow i_x = 0$

$\therefore$ ignore dependent src

i-divider with R_1 and R_2 in parallel

$v_1 = i_s \cdot R_1 \| R_2$

case II: i_s off, v_s on



$\Big\}\ R_{eq} = \dfrac{\beta i_x}{i_x} = \beta\Omega$ as before

v-divider with v_s across R_1 and R_2 in series

$$v_2 = -v_s \frac{R_2}{R_1 + R_2}$$

$$\boxed{v = v_1 + v_2 = i_s \cdot R_1 \| R_2 - v_s \frac{R_2}{R_1 + R_2}}$$