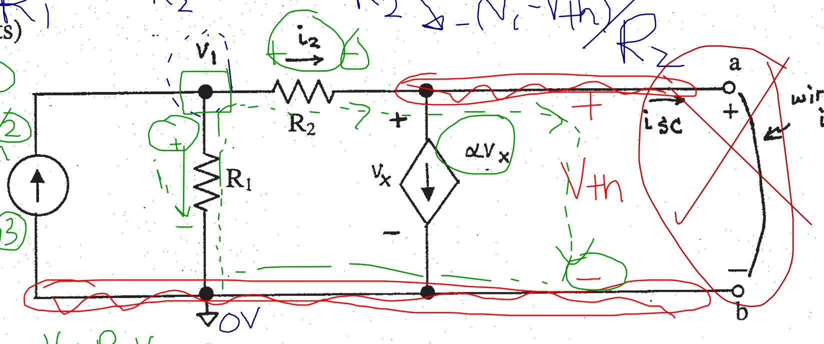
$$-i_s + \frac{V_1}{R_1} + \frac{(V_1-V_{th})}{R_2} = 0, \quad \frac{V_{th}-V_1}{R_2} + \alpha V_{th} = 0$$

$$\frac{V_{th}-V_1}{R_2} \rightarrow -(V_1-V_{th})/R_2$$

88

2. (25 points)



$-i_s + i_1 + i_2 = 0$ ①

$i_2 = \alpha V_x = \alpha V_{th}$ ②

$i_1 R_1 - i_2 R_2 - V_{th} = 0$ ③

① $i_1 = i_s - \alpha V_{th}$

③ $i_s R_1 - \alpha V_{th} R_1 - \alpha V_{th} R_2 - V_{th} = 0$

Find the Thevenin's equivalent circuit at terminals a-b. v_x must not appear in your solution. **Hint:** Use node voltage method to find v above R_1. **Note:** $\alpha > 0$.

$V_{th}(\alpha(R_1+R_2)+1) = i_s R_1$

sol'n: $V_{Th} = v_{ab}$ with no load connected to a, b.

$R_{Th} = \frac{V_{Th}}{i_{sc}}$ where i_{sc} = i out of 'a' terminal with wire from a to b

Find v_1: Use Node V method

write v_x in terms of v_1: $v_x = v_1 - \alpha v_x R_2$

$v_x = v_1/(1+\alpha R_2)$

Node v_1 eqn: $-i_s + \frac{v_1}{R_1} + \frac{v_1 - v_x}{R_2} = 0A$

$$v_1\left[\frac{\left(1-\frac{1}{1+\alpha R_2}\right)}{R_2}\right] = \frac{\frac{\cancel{1+}\alpha \cancel{R_2}\cancel{-1}}{1+\alpha R_2}}{\cancel{R_2}}$$

$$= v_1 \frac{\alpha}{1+\alpha R_2}$$

$$= V_1\left(1-\alpha \frac{R_2}{R_1}\right)$$

$$v_1\left(\frac{1}{R_1} + \frac{\alpha}{1+\alpha R_2}\right) = i_s$$

$$v_1 = i_s \frac{1}{\frac{1}{R_1} + \frac{\alpha}{1+\alpha R_2}}$$

$$V_{Th} = v_x = \frac{v_1}{1+\alpha R_2} = i_s \frac{1}{\frac{1+\alpha R_2}{R_1} + \alpha}$$

$$\boxed{V_{Th} = \frac{i_s}{\frac{1+\alpha R_2}{R_1} + \alpha} = i_s\left[R_1 \| (R_2 + 1/\alpha)\right] \cdot \frac{1/\alpha}{R_2 + 1/\alpha}}$$

Latter formula from using $1/\alpha$ for R_{eq} of dep. s

$R_{Th} = \frac{V_{Th}}{i_{sc}}$

For i_{sc} we have $v_x = 0V$.

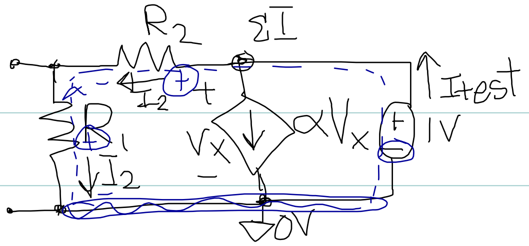
Then $v_1 = i_s \cdot R_1 \| R_2$ (no dep src)

and $i_{sc} = \frac{i_s R_1}{R_1 + R_2}$

$$R_{Th} = \frac{\cancel{i_s}}{\frac{1+\alpha R_2}{R_1} + \alpha} \cdot \frac{R_1 + R_2}{\cancel{i_s} R_1}$$

$$\boxed{R_{Th} = \frac{R_1 + R_2}{1+\alpha(R_1+R_2)} = R_1 \| \left(R_2 + \frac{1}{\alpha}\right)}$$

Latter formula from using $1/\alpha$ for dep src and looking in from a, b with i_s src off.



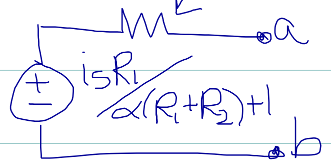
$$+I_2+\alpha \underset{1}{\underset{\shortparallel}{V_x}}-I_{test}=0$$

$$+I_2(R_1)+I_2R_2-1=0$$

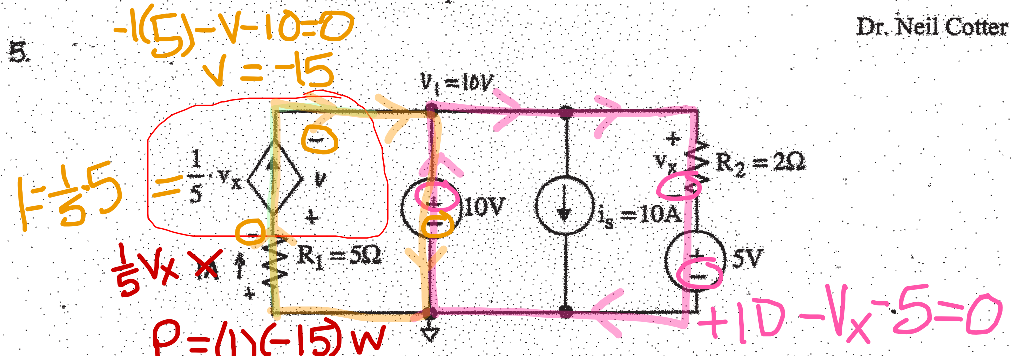
$$I_2=\frac{1}{R_1+R_2}$$

$$I_{test}=\alpha(1)+\frac{1}{R_1+R_2}=\frac{\alpha(R_1+R_2)+1}{(R_1+R_2)}$$

$$R_{th}=\frac{1}{I_{test}}=\boxed{\frac{R_1+R_2}{\alpha(R_1+R_2)+1}}$$



5.



Calculate the power consumed by the dependent current source, (labeled $\frac{1}{5}v_x$). **Note:** If a source supplies power, the power it consumes is negative.

Sol'n: V on top rail = 10V from 10V src (if ref on bottom).

Then $V_x = \boxed{5V} = 10V - 5V$

$\therefore \frac{1}{5\Omega} V_x = \frac{1}{5\Omega} 5V = 1A$ for dependent src

Using v-loop on left, $-1A \cdot R_1 - v - 10V = 0V$

or $v = -10V - 1A \cdot \underset{5\Omega}{\underset{\shortparallel}{R_1}} = -15V$

pwr ≡ $p = \frac{1}{5} V_x \cdot v = 1A\,(-15V) = -15\ W$

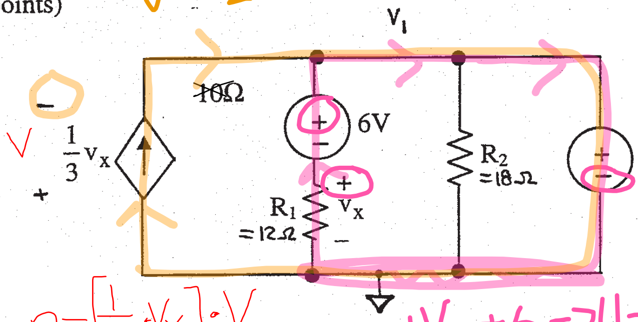
$p = -15\ W$

90

$p = \frac{1}{3} \cdot (18)(-24) =$

$V = -24$

3. (25 points)



$p = \left[\underbrace{\frac{1}{3} V_x}_{I}\right] \cdot V$

$+V_x + 6 - 24 = 0 \rightarrow V_x = 18$

Calculate the power consumed by the dependent current source, (labeled $\frac{1}{3} v_x$). **Note:** If a source supplies power, the power it consumes is negative.

soln: Find v_1. Then v across dep src is $-v_1$.

Write v_x in terms of v_1: $v_x = v_1 - 6V$

Node V for v_1 node: $-\frac{1}{3}(v_1 - 6V) + \frac{v_1 - 6V}{R_1} + \frac{v_1}{R_2} + ?!$

We don't need eq'n! $v_1 = 24V$ from src on right.

$$\therefore \; p = \frac{1}{3} v_x \cdot (-v_1) = \frac{1}{3}(v_1 - 6V)(-v_1)$$

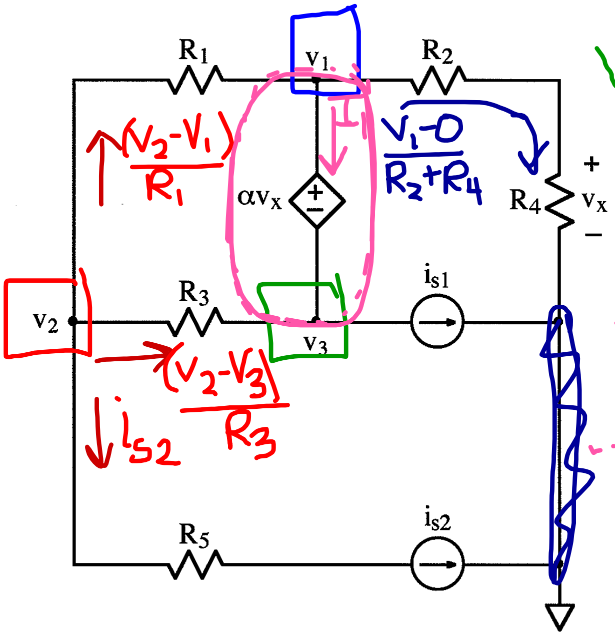
$$= \frac{1}{3}(24V - 6V)(-24V)$$

$$= \frac{1}{3}(18V)(-24V)$$

$$\boxed{p = -144\ W}$$

HOMEWORK #4 Solution Prob 1

Ex:



$V_1 - V_3 = \alpha V_x$

$V_x = \left[\frac{V_1 R_4}{R_2+R_4}\right]$

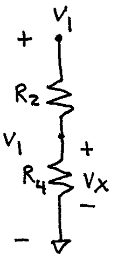
$-\frac{(V_2-V_1)}{R_1} + \frac{V_1}{R_2+R_4} + \cdots$

$\cdots -\frac{(V_2-V_3)}{R_3} + i_{s1} = 0$

For the circuit shown, write three independent equations for the node-voltages, $v_1, v_2,$ and v_3. The quantity v_x must not appear in the equations.

soln: We first write v_x in terms of node-v's.

We use a v-divider since we have v_1 across R_2 in series with R_4:



$$v_x = v_1 \cdot \frac{R_4}{R_2 + R_4}$$

We have a v-src connecting v_1 to v_3. So v_1, v_3 form a supernode.

We write a current summation eq'n for v_1, v_3. We find sum of i's flowing out of bubble containing v_1, v_3, and the dependent v-src.

(1) v_1, v_3 node: $\boxed{\frac{v_1 - v_2}{R_1} + \frac{v_1 - 0V}{R_2 + R_4} + \frac{v_3 - v_2}{R_3} + i_{s1} = 0A}$

we also write a voltage eq'n for v_1 and v_3. Note that we substitute for v_x to obtain an eq'n containing only node voltages.

(2) $\boxed{v_1 = v_3 + \alpha \left(\underbrace{v_1 \frac{R_4}{R_2 + R_4}}_{v_x} \right)}$

For v_2, we just sum currents out of node.

(3) v_2 node: $\boxed{\frac{v_2 - v_1}{R_1} + \frac{v_2 - v_3}{R_3} + i_{s2} = 0A}$

We now have our 3 eq'ns for v_1, v_2, and v_3.

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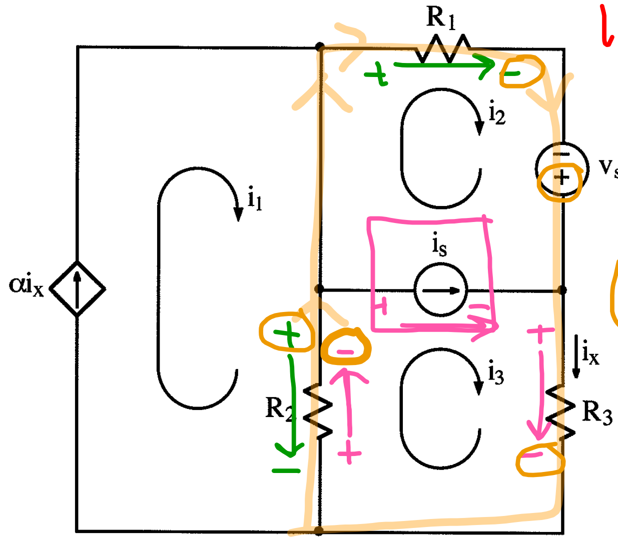
HOMEWORK #4 Solution Prob 3

Ex:



$i_1 = \alpha i_x$

$i_s = i_3 - i_2$

$i_x = i_3$

$(-i_3 + i_1)R_2 - i_2 R_1 + \cdots$

$\cdots V_s - i_3 R_3 = 0$

For the circuit shown, write three independent equations for the three mesh currents, i_1, i_2, and i_3. The quantity i_x must not appear in the equations.

sol'n: We first write i_x in terms of mesh currents.

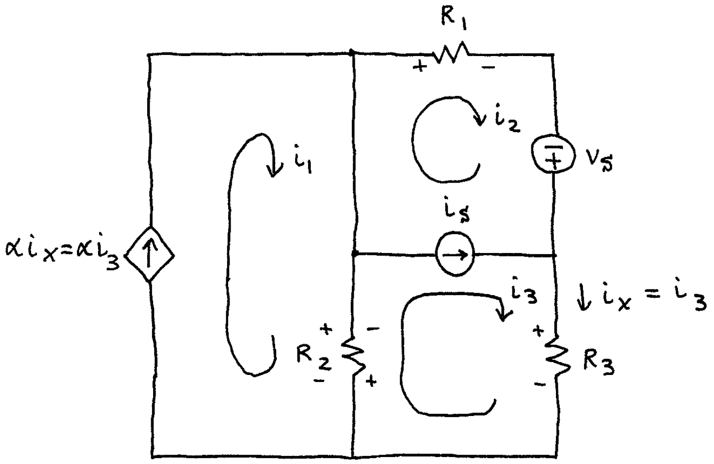
Since i_x is a current on the outside edge of the circuit, (flowing thru R_3), it is equal to the mesh (also called "loop") current.

$$i_x = i_3$$

Next we look for super meshes where a current src is between two loops. We have a supermesh for i_2, i_3 with i_s in between.

We draw the circuit model before writing our eq'ns.

HOMEWORK #4 Solution Prob 3 (cont.)



i_2, i_3 loop: $\boxed{-i_3 R_2 + i_1 R_2 - i_2 R_1 - i_3 R_3 = 0V}$
(supermesh uses loop around right half of circuit)

We add a current eq'n for i_s src between loops.

$\boxed{i_s = i_3 - i_2}$ (i_2 has − sign because it is measured in direction opposite to direction of i_s)

$i_2 = i_3 - i_s$

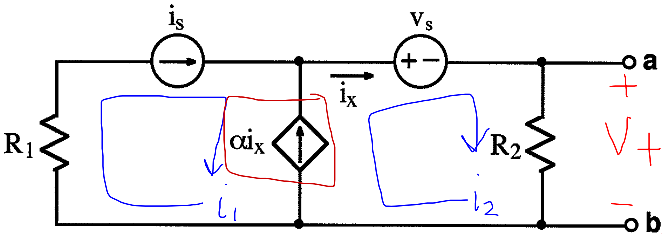
Finally, for the i_1 loop we encounter a curious situation. Since we have a current source on the outside edge of the circuit, we must have that i_1 = current for src.

Thus, $\boxed{i_1 = \alpha i_3.}$ This is the eq'n for i_1.

We now have 3 eq'ns in i_1, i_2, i_3 which we could solve to find i_1, i_2, and i_3.

HOMEWORK #4 Solution Prob 4

Ex:

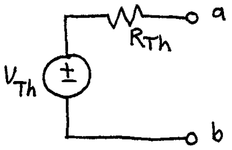


$V_{th} = i_2 R_2 = \frac{-i_s R_2}{(\alpha - 1)}$

$i_1 = i_s, \; i_2 = i_x, \; \alpha i_x = i_2 - i_1 \rightarrow i_x(\alpha - 1) = -i_s$

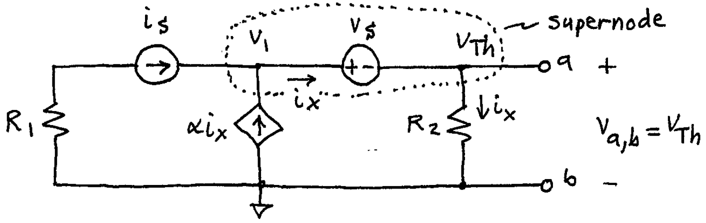
Find the Thevenin's equivalent circuit at terminals a-b. i_x must not appear in your solution. **Note:** $\alpha \neq 1$.

sol'n: We must find V_{Th} and R_{Th} for the Thevenin equivalent circuit that has the same behavior as the above circuit when viewed from a, b terminals.

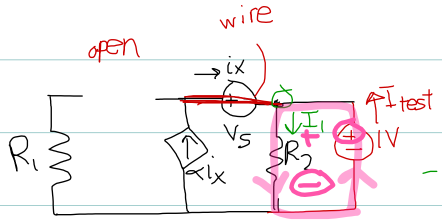


$V_{Th} = V_{a,b}$ for circuit with nothing connected across a, b.

We can use node-v method or another method of our choice to find $V_{a,b}$



We first define i_x in terms of node voltages. Here, i_x flows thru R_2. Thus $i_x = \frac{V_{Th}}{R_2}$.



$-i_x + I_1 - I_{test} = 0$

$I_1 = I_{test}$

$i_x = \alpha i_x$

$i_x - \alpha i_x = 0$

$i_x(1-\alpha) = 0$

$i_x = 0$

$+1 - I_{test} \cdot R_2 = 0$

$I_{test} = \frac{1}{R_2}$

$R_{th} = \frac{1}{test} = R_2$

$$\frac{-i_s R_2}{\alpha - 1}$$

We have a supernode for v_1 and v_{Th}.

So we sum currents out of a bubble around v_1, v_{Th}, and v_s.

v_1, v_{Th} node: $-i_s - \alpha \frac{v_{Th}}{R_2} + \frac{v_{Th}}{R_2} = 0A$

We could continue on to write a voltage eq'n for v_1 and v_{Th}: $v_1 = v_{Th} + v_s$

But our first eq'n has only v_{Th} in it; we can solve the first eq'n for v_{Th} and stop there.

Rearranging gives $v_{Th}\left(\frac{1}{R_2} - \frac{\alpha}{R_2}\right) = i_s$

or $v_{Th} = \frac{i_s R_2}{1-\alpha}$

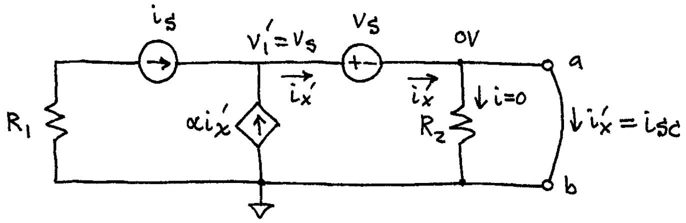
To find R_{Th}, we can use the method of shorting out a and b and measuring the current in the wire. This is i_{sc} for $i_{\underline{s}hort\ \underline{c}ircuit}$. If we look at a Thevenin

equivalent circuit with a wire from a to b, we have current $i_{sc} = \frac{v_{Th}}{R_{Th}}$.

Thus, $R_{Th} = \frac{v_{Th}}{i_{sc}}$.

We redraw our circuit with a wire from a to b.

HOMEWORK #4 Solution Prob 4 (cont.)



This is a different circuit than before. We have 0V at **a**, (instead of v_{Th}), and no current flows in R_2 since it is bypassed by a wire. Also, $i_{sc} = i_x'$.

We also have $v_1 = 0V + v_s = v_s$. Circuit is solved. Or is it? We still need to find $i_{sc} = i_x'$

Using a current summation at v_1, we have

$$-i_s - \alpha i_x' + i_x' = 0A$$

or $$i_x'(1-\alpha) = i_s$$

or $$i_x' = \frac{i_s}{1-\alpha} = i_{sc}$$

Using $R_{Th} = \frac{v_{Th}}{i_{sc}}$ gives $R_{Th} = \dfrac{\frac{i_s R_2}{1-\alpha}}{\frac{i_s}{1-\alpha}}$

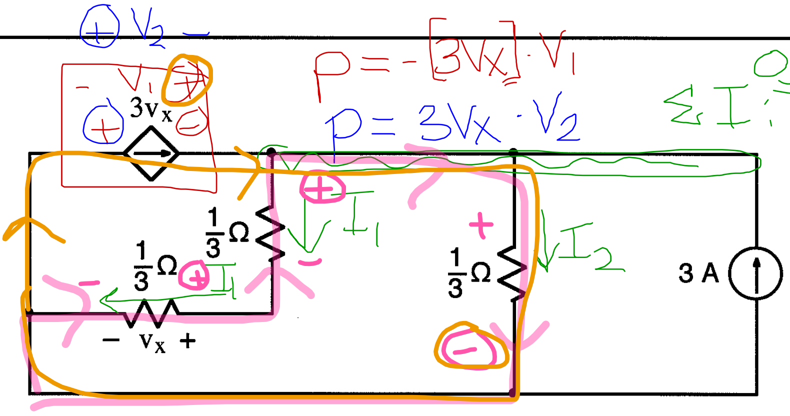
or $R_{Th} = R_2$ (Nothing else plays a role in R_{Th}.)

Consistency check: Set $\alpha = 0 \Rightarrow$ dependent src = open. Then R_1, v_s in series with current src i_s irrelevant. We have Norton equiv, i_s and R_2: $v_{Th} = i_s R_2$, $R_{Th} = R_2$ ✓

$+V_1 - I_2(\frac{1}{3}) = 0 \rightarrow V_1 = 1$

HOMEWORK #4 Solution Prob 5

Ex:



$P = -[3V_x]\cdot V_1$

$P = 3V_x \cdot V_2$

$\sum I_i \overset{0}{=} -3V_x + I_1 + I_2 - 3$

$V_x = I_1(\frac{1}{3})$

$\underbrace{\cancel{-3(\tfrac{1}{3})I_1 + I_1}}_{0}$

$I_2 = 3$

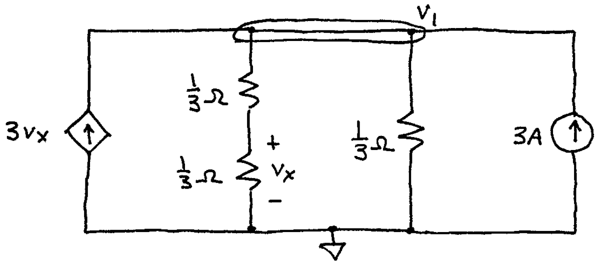
Calculate the power ~~dissipated~~ in the dependent current source, (labeled 3v$_x$).

$+\frac{2}{3}I_1 = \frac{1}{3}(3) \rightarrow I_1 = 3/2 \rightarrow V_x = \frac{1}{2}$

sol'n: Any method of solution is allowed, (provided it is a valid approach).

$P = -3\cdot\frac{1}{2}(1) = \boxed{-3/2\,W}$

We'll use node-voltage method with reference on bottom and v_1 on top. It also helps to redraw circuit.



We write v_x in terms of v_1 by using a voltage divider:

$$v_x = v_1 \cdot \frac{1/3\ \Omega}{\frac{1}{3}\Omega + \frac{1}{3}\Omega} = \frac{v_1}{2}$$

Now we write the current summation eq'n for node v_1.

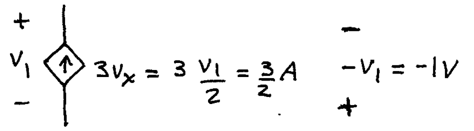
HOMEWORK #4 Solution Prob 5 (cont.)

$$-3\underbrace{\left(\frac{v_1}{2}\right)}_{v_x} + \frac{v_1}{\frac{1}{3}\Omega+\frac{1}{3}\Omega} + \frac{v_1}{\frac{1}{3}\Omega} - 3A = 0A$$

or $v_1\left(-\frac{\cancel{3}}{\cancel{2}\Omega} + \frac{\cancel{3}}{2\Omega} + \frac{3}{\Omega}\right) = 3A$

or $v_1 = 1A\cdot\Omega = 1V$

So we have

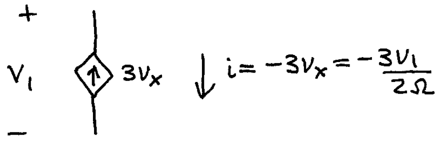


Power is $p = iv$ where i, v follow passive sign convention.

$$p = \frac{3}{2}A\cdot(-1V) = -\frac{3}{2}W$$

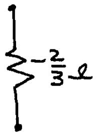
Note: In this problem we can replace the dependent src with a resistor, (even before we know the value of v_1).

We have voltage v_1 across dependent src and current $-3v_x = -3\frac{v_1}{2}$

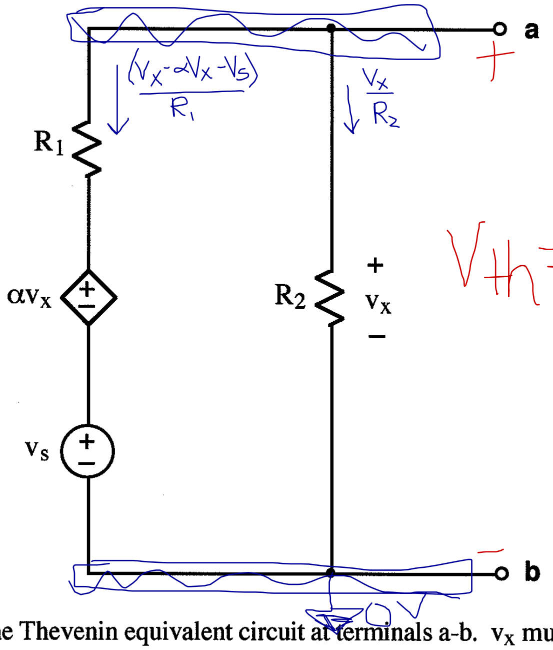


Then $R_{eq} = \frac{v_1}{-\frac{3}{2\Omega}v_1} = -\frac{2}{3}\Omega.$

Use this instead of dependent src:



Ex:



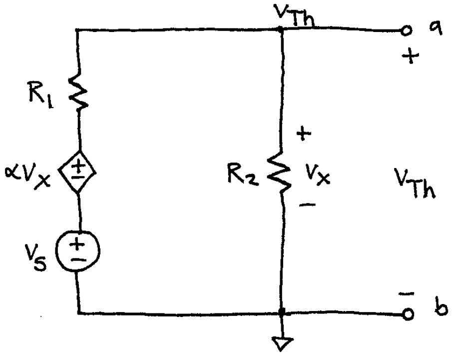
$$\frac{(V_X - \alpha V_X - V_S)}{R_1} + \frac{V_X}{R_2} = 0$$

$$V_{th} = V_X$$

Find the Thevenin equivalent circuit at terminals a-b. v_X must not appear in your solution. **Hint:** Use the node-voltage method. **Note:** $0 < \alpha < 1$.

soln: $V_{Th} = v_{a,b}$ with nothing connected across a, b

Using the node-voltage method, with the reference at the bottom and v_{Th} at the top, our circuit is as follows:



Exam #1 Solution Prob 2 (cont.)

We write v_x in terms of v_{Th}:

$$v_x = v_{Th}$$

The current summation for the v_1 node has only two terms:

$$\frac{v_{Th} - (v_s + \alpha v_{Th})}{R_1} + \frac{v_{Th}}{R_2} = 0 \text{ mA}$$

or

$$v_{Th}\left(\frac{1}{R_1} - \frac{\alpha}{R_1} + \frac{1}{R_2}\right) = \frac{v_s}{R_1}$$

Multiplying both sides by R_1 yields

$$v_{Th}\left(1 - \alpha + \frac{R_1}{R_2}\right) = v_s$$

or

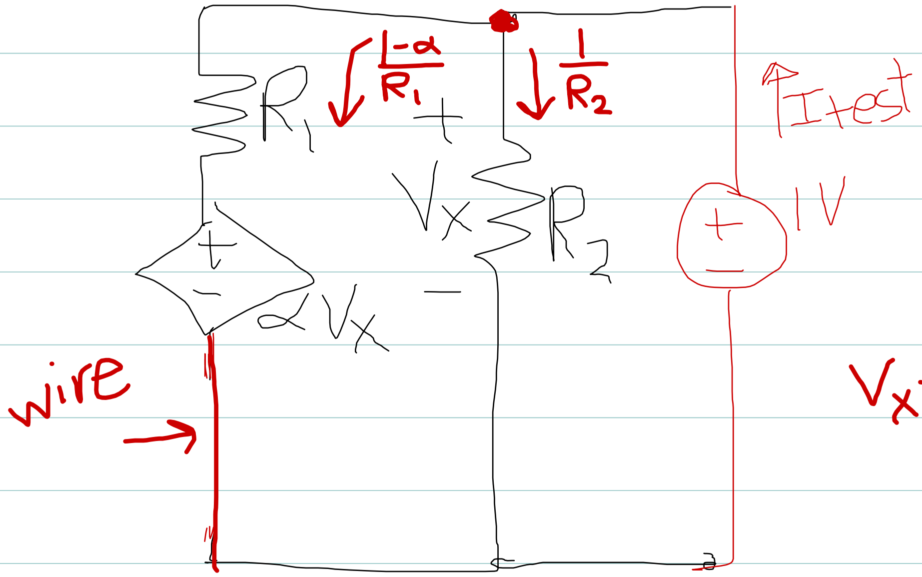
$$v_{Th} = \frac{v_s}{1 - \alpha + \frac{R_1}{R_2}} = \boxed{v_s \frac{R_2}{R_2(1-\alpha) + R_1}}$$

To find R_{Th}, we use $R_{Th} = \frac{v_{Th}}{i_{sc}}$

where $i_{sc} \equiv$ short circuit current flowing in a wire from a to b.

With a wire connecting a and b, $v_x = 0V$ and the dependent source becomes a wire.

Also, no current will flow thru R_2, and we may ignore R_2.

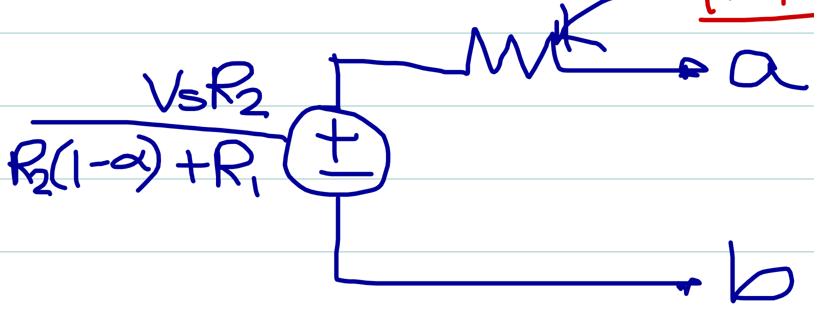


$V_x = 1V$

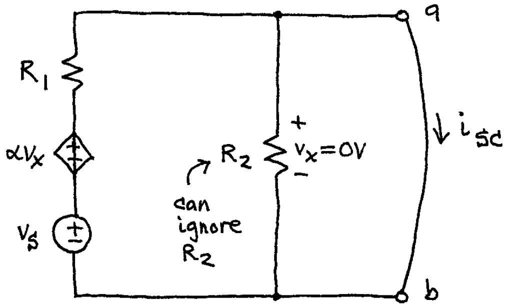
$$\Sigma I: -I_{test} + \frac{1}{R_2} + \frac{1-\alpha}{R_1} = 0$$

$$I_{test} = \frac{R_1 + R_2(1-\alpha)}{R_1 R_2}$$

$$R_{th} = \frac{1}{I_{test}} = \boxed{\frac{R_1 R_2}{R_1 + R_2(1-\alpha)}}$$



Exam #1 Solution Prob 2 (cont.)



If we ignore R_2, we calculate i_{sc} from the outer loop:

$$i_{sc} = \frac{v_s}{R_1}$$

Thus, we have

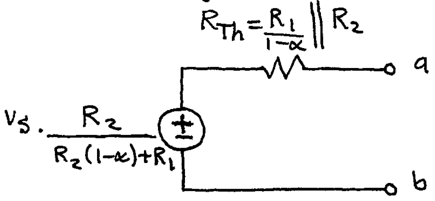
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{\dfrac{v_s}{1-\alpha+\dfrac{R_1}{R_2}}}{\dfrac{v_s}{R_1}} = \frac{R_1}{1-\alpha+\dfrac{R_1}{R_2}}$$

Other equivalent forms for R_{Th}:

$$R_{Th} = \frac{R_1 R_2}{R_2(1-\alpha)+R_1}$$

$$R_{Th} = \frac{1}{\dfrac{1-\alpha}{R_1}+\dfrac{1}{R_2}} = \frac{R_1}{1-\alpha} \Big\| R_2$$

The circuit diagram for the Thevenin equivalent:



Note: For the first part of this problem, when we calculate v_{Th}, we may use an equivalent resistance in place of the dependent source.

With a, b open circuit, the same current, i, flows in R_2 and the dependent source.

$$\therefore \; v_x = i R_2 \quad \text{or} \quad i = \frac{v_x}{R_2}$$

The equivalent R for the dependent source is

$$R_{eq} = \frac{\alpha v_x}{-i} = \frac{\alpha v_x}{\frac{v_x}{R_2}} = -\alpha R_2$$

Using R_{eq} in place of αv_x allows us to obtain v_{Th} from a v-divider formula:

$$v_{Th} = v_s \frac{R_2}{R_2 - \alpha R_2 + R_1} = v_s \frac{R_2}{R_2(1-\alpha)+R_1}$$

This agrees with our node-voltage result.