Homework 3 due tonight by 5pm

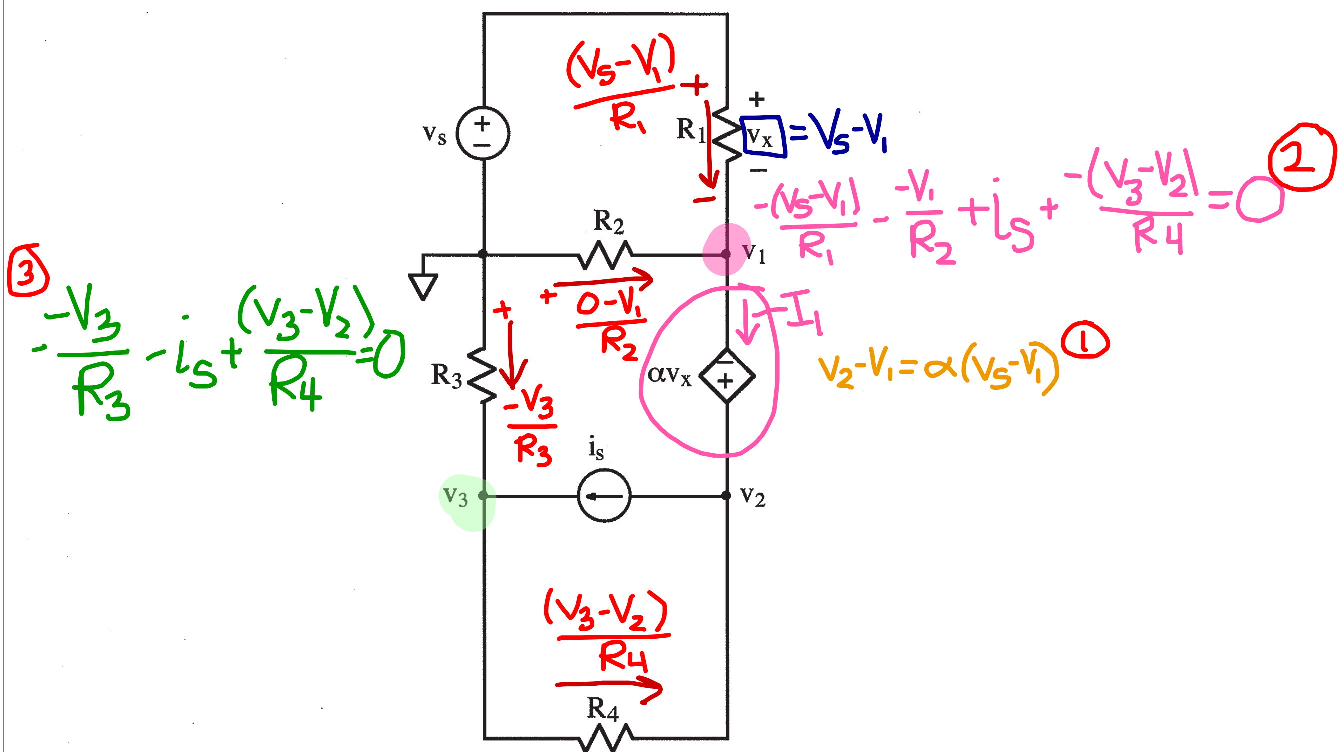
Homework 4 due Saturday by noon

Exam next Tuesday(open notes/book)

1. node Voltage (eq only)
2. Mesh current (eq only)
3. thevenin
4. Power in a source(dep or ind)

Solution Prob 1.a

Ex:



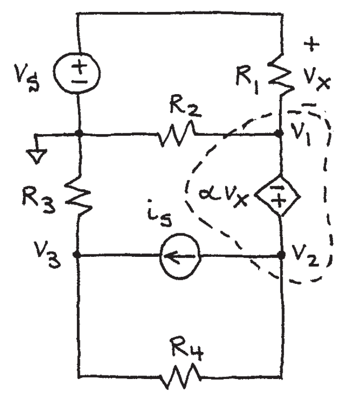
For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

soln: First, we define v_x in terms of node-v's:

$$v_x = v_s - v_1$$

Second, we see that v_1 and v_2 are connected by a v-src. Thus, we have a supernode for v_1 and v_2. We draw a bubble enclosing v_1 and v_2 along with the αv_x source.

Solution Prob 1.a (cont.)



We sum the currents flowing out of the bubble:

(1) $\frac{v_1 - v_s}{R_1} + \frac{v_1}{R_2} + i_s + \frac{v_2 - v_3}{R_4} = 0A$

Third, we write a voltage eq'n for v_1 and v_2:

(2) $v_1 + \alpha(\underbrace{v_s - v_1}_{v_x}) = v_2$

Fourth, we write a current-sum eq'n for node v_3:

(3) $\frac{v_3}{R_3} - i_s + \frac{v_3 - v_2}{R_4} = 0A$

We now have three independent eq'ns, (1), (2), and (3) that we could solve to find v_1, v_2, and v_3.

ECE 1270

Solution Prob 2

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Ex:

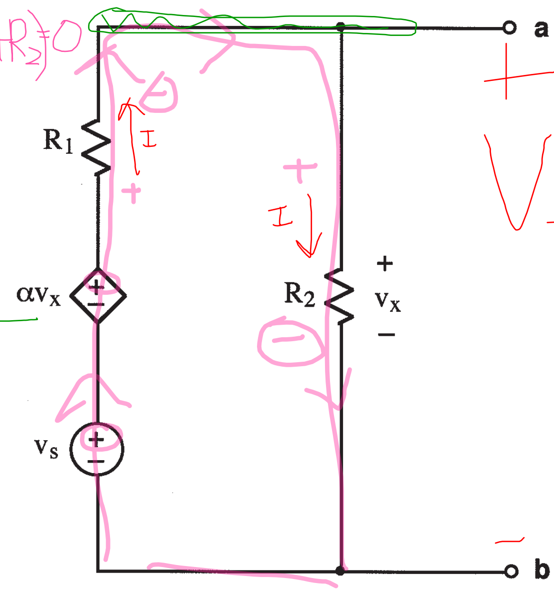
$+V_S + \alpha(IR_2) - I(R_1 + R_2) = 0$

$I = \dfrac{V_S}{(R_1 + R_2(1-\alpha))}$

$V_{th} = IR_2$

OR

$\dfrac{V_{th} - \alpha(V_{th}) - V_S}{R_1} + \dfrac{V_{th}}{R_2} = 0$



$+$

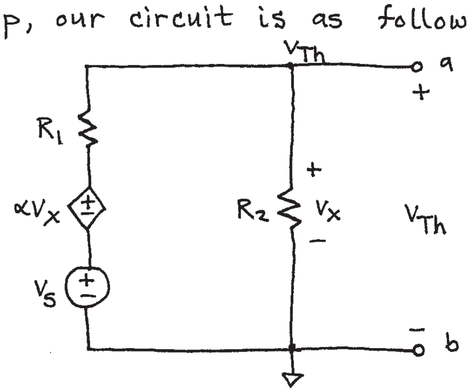
$V_{th} = V_x$

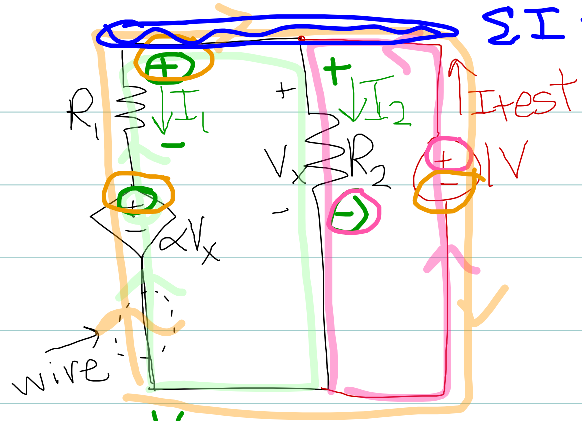
$-$

Find the Thevenin equivalent circuit at terminals a-b. v_x must not appear in your solution. **Hint:** Use the node-voltage method. **Note:** $0 < \alpha < 1$.

soln: $V_{Th} = v_{a,b}$ with nothing connected across a, b

Using the node-voltage method, with the reference at the bottom and v_{Th} at the top, our circuit is as follows:





ΣI:

$I_{test} = I_1 + I_2$

$= \frac{1+\alpha}{R_1} + \frac{1}{R_2}$

$= \frac{R_2(1+\alpha)+R_1}{R_1 R_2}$

$R_{th} = \frac{1V}{I_{test}} = \boxed{\frac{R_1 R_2}{R_2(1+\alpha)+R_2}}$

$V_x = I_2 R_2$

$+\alpha(I_2 R_2) + I_1 R_1 - I_2 R_2 = 0$

$+1 - I_2 R_2 = 0 \rightarrow I_2 = \frac{1}{R_2}$

$+\alpha\underbrace{(I_2 R_2)}_{1V} + I_1 R_1 - 1 = 0$

$I_1 = \frac{1+\alpha}{R_1}$

Solution Prob 2 (cont.)

We write v_x in terms of v_{Th}:

$$v_x = v_{Th}$$

The current summation for the v_1 node has only two terms:

$$\frac{v_{Th} - (v_s + \alpha v_{Th})}{R_1} + \frac{v_{Th}}{R_2} = 0 \text{ mA}$$

or

$$v_{Th}\left(\frac{1}{R_1} - \frac{\alpha}{R_1} + \frac{1}{R_2}\right) = \frac{v_s}{R_1}$$

Multiplying both sides by R_1 yields

$$v_{Th}\left(1 - \alpha + \frac{R_1}{R_2}\right) = v_s$$

or

$$v_{Th} = \frac{v_s}{1 - \alpha + \frac{R_1}{R_2}} = v_s \frac{R_2}{R_2(1-\alpha) + R_1}$$

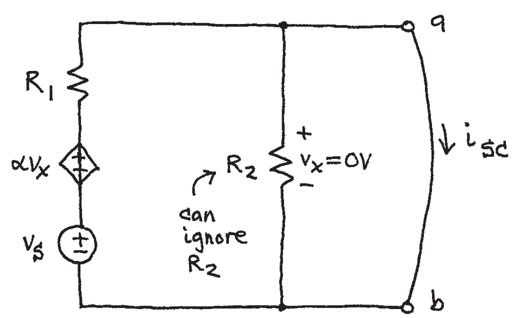
To find R_{Th}, we use $R_{Th} = \frac{v_{Th}}{i_{sc}}$

where $i_{sc} \equiv$ short circuit current flowing in a wire from a to b.

With a wire connecting a and b, $v_x = 0\text{V}$ and the dependent source becomes a wire.

Also, no current will flow thru R_2, and we may ignore R_2.

Solution Prob 2 (cont.)



If we ignore R_2, we calculate i_{sc} from the outer loop:

$$i_{sc} = \frac{v_s}{R_1}$$

Thus, we have

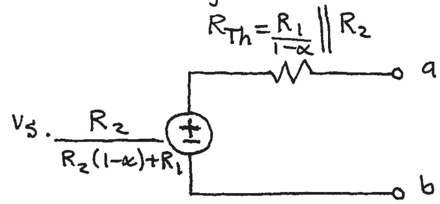
$$R_{Th} = \frac{v_{Th}}{i_{sc}} = \frac{\dfrac{v_s}{1-\alpha+\dfrac{R_1}{R_2}}}{\dfrac{v_s}{R_1}} = \frac{R_1}{1-\alpha+\dfrac{R_1}{R_2}}$$

Other equivalent forms for R_{Th}:

$$R_{Th} = \frac{R_1 R_2}{R_2(1-\alpha)+R_1}$$

$$R_{Th} = \frac{1}{\dfrac{1-\alpha}{R_1}+\dfrac{1}{R_2}} = \frac{R_1}{1-\alpha} \,\Big\|\, R_2$$

The circuit diagram for the Thevenin equivalent:



Note: For the first part of this problem, when we calculate v_{Th}, we may use an equivalent resistance in place of the dependent source.

With a,b open circuit, the same current, i, flows in R_2 and the dependent source.

$$\therefore \quad v_x = i R_2 \quad \text{or} \quad i = \frac{v_x}{R_2}$$

The equivalent R for the dependent source is

$$R_{eq} = \frac{\alpha v_x}{-i} = \frac{\alpha v_x}{\frac{v_x}{R_2}} = -\alpha R_2$$

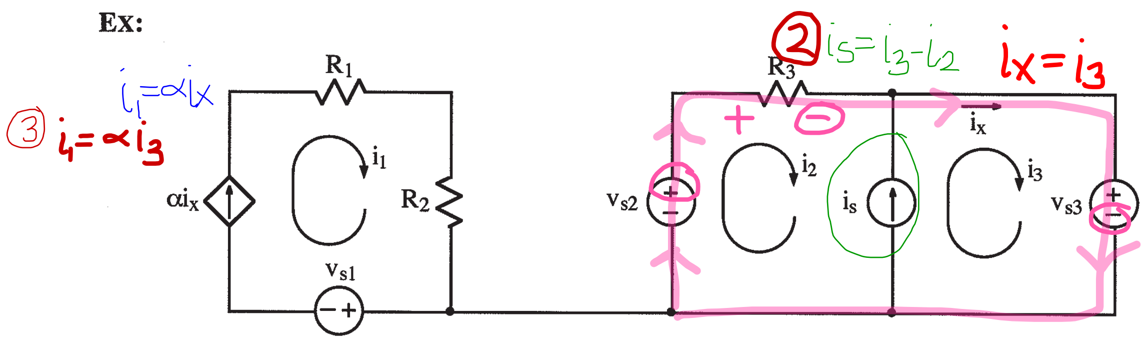
Using R_{eq} in place of αv_x allows us to obtain v_{Th} from a v-divider formula:

$$v_{Th} = v_s \frac{R_2}{R_2 - \alpha R_2 + R_1} = v_s \frac{R_2}{R_2(1-\alpha)+R_1}$$

This agrees with our node-voltage result.

Solution Prob 1.c

Ex:



For the circuit shown, write three independent equations for the three mesh currents i_1, i_2, and i_3. The quantity i_x must not appear in the equations.

① $+V_{S2} - i_2 R_3 - V_{S3} = 0$

sol'n: First, we express i_x in terms of mesh currents. Since i_x is on the outer edge of the circuit, it is equal to the mesh current i_3:

$$i_x = i_3$$

Second, we examine the i_1 loop and find that the dependent source is on the outside edge. Thus, $\alpha i_x = i_1$. Substituting for i_x gives an equation for i_1:

(1) $i_1 = i_3$

Third, we examine the i_2 loop and find that the current source, i_S, between the i_2 and i_3 loops means we have an i_2, i_3 super mesh.

The v-loop for the i_2, i_3 super mesh is

(2) $V_{S2} - i_2 R_3 - V_{S3} = 0V$

Solution Prob 1.c (cont.)

Note: The problem asks only for circuit eq'ns, but we could easily solve for i_2:

$$i_2 = \frac{v_{s2} - v_{s3}}{R_3}$$

Fourth, we write an eq'n for the i_s source between the i_2 and i_3 loops.

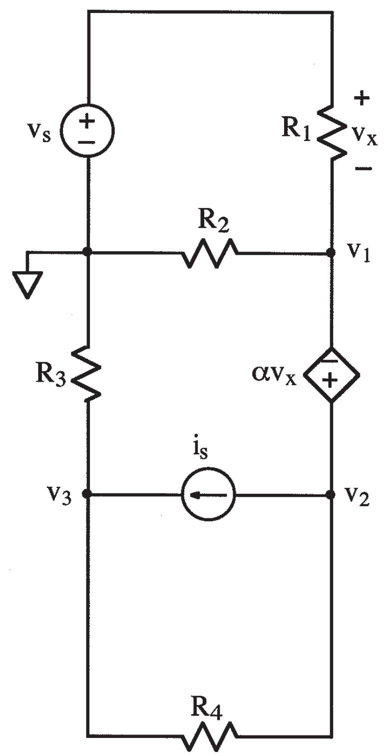
(3) $$i_s = i_3 - i_2$$

Note that i_3 flows in the same direction as i_s and i_2 flows opposite the direction of the arrow for i_s.

The eq'ns numbered (1), (2), and (3) are independent, (meaning we could solve them to find i_1, i_2, and i_3.

Solution Prob 1.b

Ex:



(1) $$\frac{v_1 - v_s}{R_1} + \frac{v_1}{R_2} + i_s + \frac{v_2 - v_3}{R_4} = 0A$$

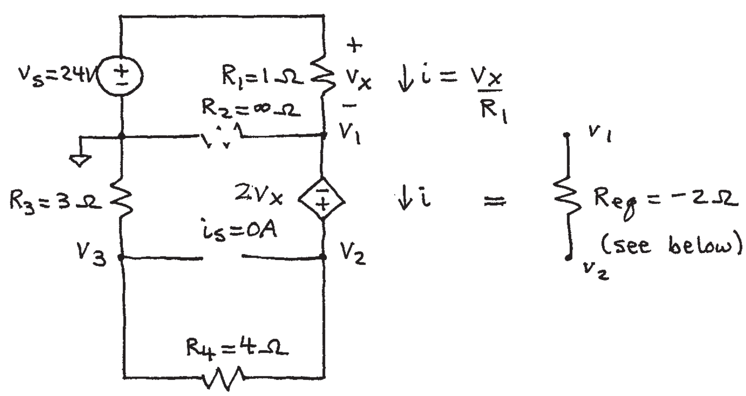
(2) $$v_1 + \alpha(v_s - v_1) = v_2$$

(3) $$\frac{v_3}{R_3} - i_s + \frac{v_3 - v_2}{R_4} = 0A$$

Make a consistency check on the above node-voltage equations by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and for your consistency check, and show that the equations are satisfied for these values. (In other words, plug the values into the equations and show that the left side and the right side of each equation are equal.)

sol'n: Many consistency checks are possible. Here, we consider a check in which the dependent source remains on.

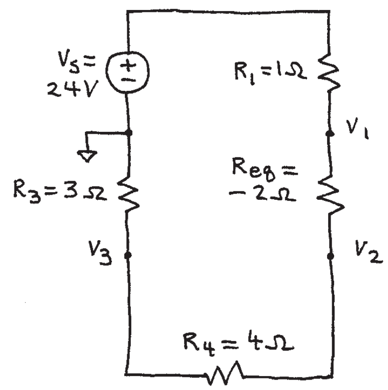
Let $v_s = 24V$
$R_1 = 1\Omega$
$R_2 = \infty\ \Omega$ (open circuit)
$R_3 = 3\Omega$
$R_4 = 4\Omega$
$\alpha = 2$
$i_s = 0A$ (off = open circuit)



We observe that current $i = v_x/R_1$ flows thru R_1, creating v_x, and thru the dependent source, creating $-\alpha v_x$. Thus, we may replace the dependent source with an equivalent R:

$$R_{eq} = \frac{v}{i} = \frac{-\alpha v_x}{i} = \frac{-2\,v_x}{\frac{v_x}{R_1 = 1\Omega}} = -2\Omega$$

Using R_{eq} in place of αv_x gives the follow circuit:



Now we can use v-divider eq'ns to find v_1, v_2, and v_3:

$$v_3 = V_S \cdot \frac{R_3}{R_1 + R_{eq} + R_4 + R_3} = 24V \cdot \frac{3\Omega}{1-2+4+3\ \Omega}$$

$$v_3 = 24V \cdot \frac{3\Omega}{6\Omega} = 12V$$

$$v_2 = V_S \frac{R_3 + R_4}{R_1 + R_{eq} + R_4 + R_3} = 24V \cdot \frac{3+4\ \Omega}{6\Omega}$$

$$v_2 = 28V$$

$$v_1 = V_S \frac{R_{eq} + R_3 + R_4}{R_1 + R_{eq} + R_4 + R_3} = 24V\left(\frac{-2+3+4}{6\Omega}\right)\Omega$$

$$v_1 = 20V$$

Solution Prob 1.b (cont.)

We plug all our numbers into the eq'ns given in the problem to verify that equality holds.

(1) $$\frac{20V - 24V}{1\Omega} + \frac{20V}{\infty\Omega} + 0A + \frac{28V - 12V}{4\Omega}$$

$$= \frac{-4V}{1\Omega} + 0A + 0A + \frac{16V}{4\Omega} = -4A + 4A = 0A \checkmark$$

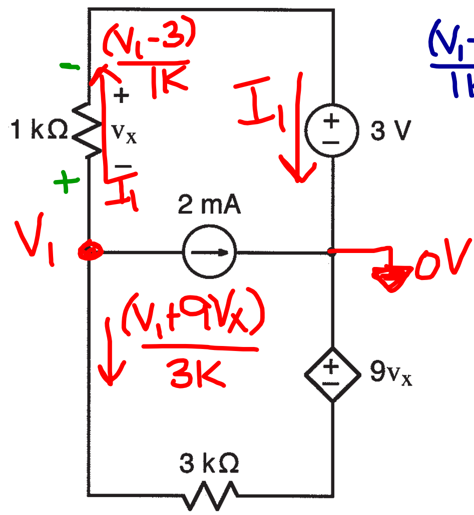
(2) $$20V + 2(24V - 20V) = 20V + 2(4V) = 28V$$

$$= v_2 \checkmark$$

(3) $$\frac{12V}{3\Omega} - 0A + \frac{12V - 28V}{4\Omega}$$

$$= 4A - 0A + \frac{-16V}{4\Omega} = 4A - 4A = 0A \checkmark$$

Ex:



$$\frac{(V_1-3)}{1K} + 2m + \frac{(V_1+9V_x)}{3K} = 0$$

$$V_x = -(V_1-3) = 3 - V_1$$

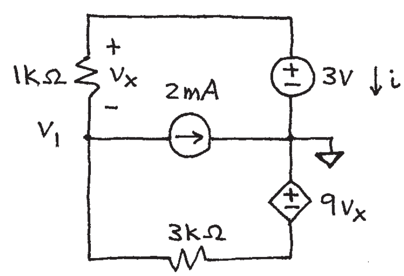
$$V_1\left(\frac{1}{1K} + \frac{1}{3K} - \frac{9}{3K}\right) = -2m - \frac{3(9)}{3K}$$

$$V_1 = 4.8V$$

$$I_1 = \frac{4.8-3}{1K} = +1.8mA$$

Calculate the power consumed by the 3V source. **Note:** If a source supplies power, the power it consumes is negative.

sol'n: Any method of solving the circuit is acceptable. Here, we use the node-voltage method. We place the reference at the node on the right side.



We first define v_x in terms of node voltage:

$$v_x = 3V - v_1$$

Solution Prob 3 (cont.)

Our node voltage eq'n at v_1:

$$\frac{v_1 - 3V}{1k\Omega} + 2mA + \frac{v_1 - (-9(3V - v_1))}{3k\Omega} = 0A$$

Rearranging the eq'n, we have

$$v_1\left(\frac{1}{1k\Omega} + \frac{1}{3k\Omega} - \frac{9}{3k\Omega}\right) = \frac{3V}{1k\Omega} - 2mA - \frac{9(3V)}{3k\Omega}.$$

Multiplying both sides by $3k\Omega$ gives

$$v_1(3 + 1 - 9) = 9V - 6V - 27V = -24V$$

or

$$v_1 = -\frac{24V}{-5} = \frac{24}{5}\,V.$$

The current, i, thru the 3V source is

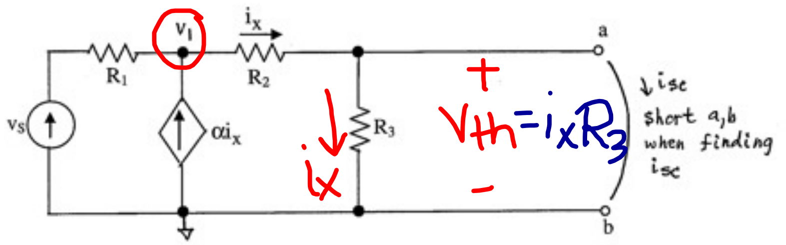
$$i = \frac{v_1 - 3V}{1k\Omega} = \frac{\frac{24}{5}V - \frac{15V}{5}}{1k\Omega} = \frac{9V}{5k\Omega} = \frac{9}{5}mA.$$

The power for the 3V source is $p = vi = 3V \cdot i$:

$$p = \boxed{3V \cdot \frac{9}{5}\,mA} = \boxed{\frac{27}{5}\,mW} = 5.4\,mW$$

$$-V_S - \alpha i_x + i_x = 0 \rightarrow i_x = \frac{V_S}{1-\alpha}$$

4.



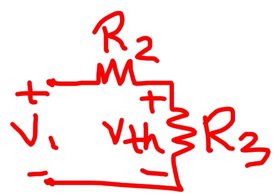
Find the Thevenin's equivalent circuit at terminals a-b. i_x must not appear in your solution. **Hint:** Use the node voltage method. **Note:** $\alpha > 0$.

sol'n: Assuming v_s is <u>current src</u>:

$v_{Thev} = v_{a,b}$ open circuit

Use node-voltage method to find v_1:

$$-v_s - \alpha \underbrace{\boxed{\frac{v_1}{R_2+R_3}}}_{=i_x} + \frac{v_1}{R_2+R_3} = 0A$$



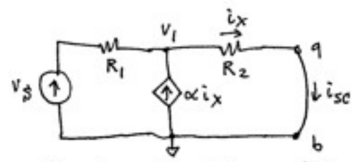
$$\text{or} \quad v_1 \frac{1-\alpha}{R_2+R_3} = v_s$$

$$v_1 = v_s \frac{R_2+R_3}{1-\alpha}$$

$$v_{Thev} = v_1 \cdot \frac{R_3}{R_2+R_3} = \boxed{v_s \frac{R_3}{1-\alpha}} \quad \text{v-divider}$$

Now short a,b terminals and find i_{sc} flowing from a to b.

Note that R_3 will carry no current; it all flows thru the short. ∴ we may ignore R_3.



Use node voltage to find v_1.

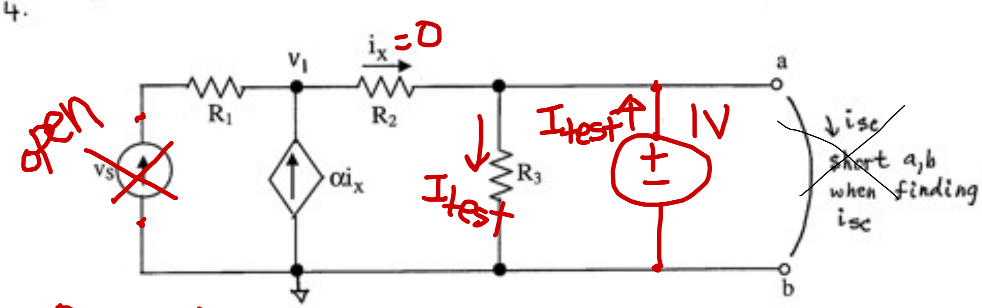
$$-v_s - \alpha \underbrace{\boxed{\frac{v_1}{R_2}}}_{=i_x} + \frac{v_1}{R_2} = 0A$$

$$v_1 = v_s \frac{R_2}{1-\alpha}$$

Now, $i_{sc} = i_x = \frac{v_1}{R_2} = \frac{v_s}{1-\alpha}$

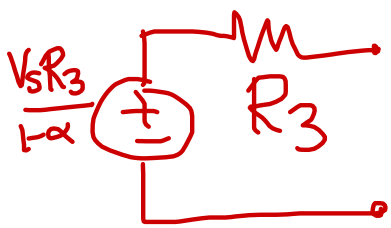
$R_{Thev} = v_{Thev}/i_{sc} = R_3$

4.

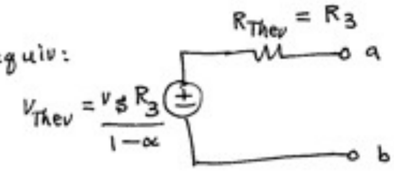


$$i_x - \alpha i_x = 0$$
$$i_x(1-\alpha) = 0$$
$$i_x = \frac{0}{1-\alpha} = 0$$

$$i_x = \alpha i_x \rightarrow i_x = 0$$
$$I_{test} = \frac{1}{R_3}$$
$$R_{th} = \frac{1V}{I_{test}} = R_3$$

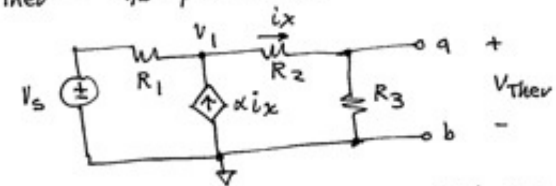


4. cont. Thevenin equiv:



Assuming V_S is <u>voltage src</u> V_S:

$V_{Thev} = V_{a,b}$ open circuit



Use node-voltage method to find V_1:

$$\frac{V_1 - V_S}{R_1} - \alpha \underbrace{\boxed{\frac{V_1}{R_2+R_3}}}_{= i_x} + \frac{V_1}{R_2+R_3} = 0A$$

$$V_1 \left(\frac{1}{R_1} + \frac{1-\alpha}{R_2+R_3} \right) = \frac{V_S}{R_1}$$

mult both sides by R_1 and re-arrange:

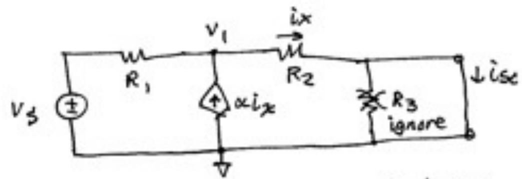
$$V_1 = \frac{V_S}{1 + \frac{(1-\alpha) R_1}{R_2 + R_3}}$$

$$V_{Thev} = V_1 \cdot \frac{R_3}{R_2+R_3} = \frac{V_S R_3}{\left(1 + \frac{(1-\alpha) R_1}{R_2+R_3} \right)(R_2+R_3)}$$

$$V_{Thev} = \frac{V_S R_3}{R_2+R_3+(1-\alpha)R_1}$$

Now short a,b terminals and find i_{sc} flowing from a to b. Note that R_3 will be bypassed. All current will flow thru short. ∴ we may ignore R_3.

4. cont.



use node-voltage to find v_1:

$$\frac{v_1 - V_s}{R_1} - \alpha \underbrace{\boxed{\frac{v_1}{R_2}}}_{= i_x} + \frac{v_1}{R_2} = 0A$$

$$v_1 \left(\frac{1}{R_1} + \frac{1-\alpha}{R_2} \right) = \frac{V_s}{R_1}$$

$$\text{or} \quad v_1 = \frac{V_s}{1 + \frac{(1-\alpha) R_1}{R_2}}$$

$$i_{sc} = i_x = \frac{v_1}{R_2} = \frac{V_s}{R_2 + (1-\alpha) R_1}$$

$$R_{The} = \frac{v_{Thev}}{i_{sc}} = \frac{R_3}{R_2 + R_3 + (1-\alpha) R_1} \cdot R_2 + (1-\alpha) R_1$$

$$R_{Thev} = R_3 \| [R_2 + (1-\alpha) R_1]$$

Thevenin equiv: