Unit 3

95

Capacitor (C in Farads)

$\boxed{i(t) = C\frac{dv(t)}{dt}}$ Voltage

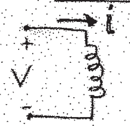
parallel plates dielectric between
V causes i to flow
· stores charge on plates like a small battery

V can not change instantly for C

V constant $\Rightarrow i = 0$ open $\left(\frac{dv}{dt} = 0\right)$

Energy $= \boxed{\frac{1}{2}CV^2}$ Voltage

Inductor (L in Henry)



$\boxed{V(t) = L\frac{di(t)}{dt}}$ (current src)

i cannot change instantly for L

i constant $\Rightarrow V = 0$ $\left(\frac{di}{dt} = 0\right)$ short

Energy $= \boxed{\frac{1}{2}Li^2}$

Cap (opp. R):
Series: $C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$ C_1, C_2
parrallel: $C_{eq} = C_1 + C_2$ C_1, C_2

L (same as R):
Series: $L_{eq} = L_1 + L_2$ L_1, L_2
parrallel: $L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$ L_1, L_2

General Equation:

$$X(t) = \text{Final} + [\text{Initial} - \text{Final}]\, e^{-(t - t_0)/\tau}$$

start time at switch (usually set to zero)

procedure:

1. Find Initial value on cap/inductor before switch $(t = 0^-)$
   $\{ i_L(t=0^-)$ OR $V_c(t=0^-) \}$

   $\Rightarrow$ Note: voltage in C cannot change instantaneously
   $V_c(t=0^-) = V_c(t=0^+)$ {V src.}

   $\Rightarrow$ Note: current in L cannot change instantaneously
   $i_L(t=0^-) = i_L(t=0^+)$ {I src.}

2. Find final value on C/L $(t \to \infty)$
   $\Rightarrow$ Note: use $V_L(t) = 0$ (wire) ; $i_c(t) = 0$ (open)

3. Find τ (use switch in <u>FINAL</u> position)
   $\tau = R_{eq}\, C$ OR $\tau = L/R_{eq}$ where R_{eq} is R seen by element

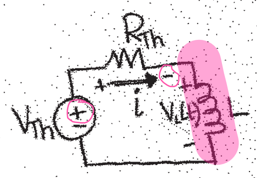
4. Plug into general equation
   Note: If finding other variables, relate to $V_c(t)$ or $i_L(t)$ eq.

Derivation

Recall #1
$$\int \frac{du}{au+b} = \frac{1}{a}\ln|au+b| + C$$

Recall #2
$$\ln a - \ln b = \ln\left(\frac{a}{b}\right)$$

(96)



V-Loop ⇒

$$+V_{Th} - iR_{Th} - V_L(t) = 0$$

$$\boxed{V_{Th} - iR_{Th} - L\frac{di(t)}{dt} = 0}$$

$$L\frac{di(t)}{dt} = +V_{Th} - iR_{Th}$$

$$\frac{di(t)}{dt} = \frac{-(iR_{Th} - V_{Th})}{L}$$

$$\int_{i(t_0)}^{i(t)} \frac{di(t)}{i(t)R_{Th} - V_{Th}} = \int_{t_0}^{t} -\frac{1}{L}\,dt$$

$$\frac{1}{R_{Th}}\left[\ln(R_{Th}i(t) - V_{Th}) - \ln(R_{Th}i(t_0) - V_{Th})\right] = \frac{-(t-t_0)}{L}$$

$$e^{\ln\left[\frac{R_{Th}i(t) - V_{Th}}{R_{Th}i(t_0) - V_{Th}}\right]} = e^{\frac{-(t-t_0)R_{Th}}{L}}$$

$$R_{Th}i(t) - V_{Th} = (R_{Th}i(t_0) - V_{Th})\,e^{\frac{-(t-t_0)R_{Th}}{L}}$$

$$i(t) = \frac{V_{Th}}{R_{Th}} + \left(i(t_0) - \frac{V_{Th}}{R_{Th}}\right)e^{-\frac{(t-t_0)R_{Th}}{L}}$$

Note: after a long time $(t\to\infty)$
V_{Th} = constant and so $V_L(t\to\infty) = 0$ (wire)
so $i_L(t\to\infty) = \frac{V_{Th}}{R_{Th}}$ (final value)

$i(t_0)$ = initial value

$$\boxed{\tau = L/R_{Th}}$$

$$\therefore\ i(t) = \text{Final} + (\text{initial} - \text{Final})\,e^{-\frac{(t-t_0)}{\tau}}$$

$$+V_{Th} - i_c(t)\cdot R_{Th} - V_c(t) = 0$$

$$V_{Th} - C\frac{dV_c(t)}{dt}\cdot R_{Th} - V_c(t) = 0$$

$$\frac{dV_c(t)}{dt} = \frac{V_{Th} - V_c(t)}{R_{Th}C}$$

$$\int_{V_c(t_0)}^{V_c(t)} \frac{dV_c(t)}{V_c(t) - V_{Th}} = \int_{t_0}^{t} \frac{-1}{R_{Th}C}\,dt$$

$$\ln(V_c(t) - V_{Th}) - \ln(V_c(t_0) - V_{Th}) = \frac{-(t-t_0)}{R_{Th}C}$$

$$e^{\ln\left[\frac{(V_c(t) - V_{Th})}{V_c(t_0) - V_{Th}}\right]} = e^{\frac{-(t-t_0)}{R_{Th}C}}$$

$$V_c(t) - V_{Th} = (V_c(t_0) - V_{Th})\,e^{\frac{-(t-t_0)}{R_{Th}C}}$$

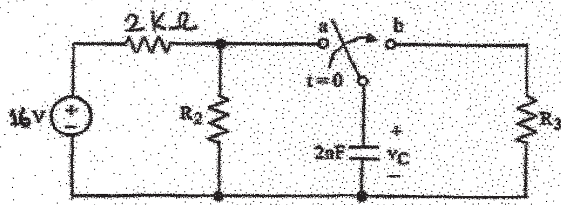
$$V_c(t) = V_{Th} + (V_c(t_0) - V_{Th})\,e^{\frac{-(t-t_0)}{R_{Th}C}}$$

Note: After a long time $(t\to\infty)$
V_{Th} = constant and so $i_c(t) = 0$ (open)
so $V_c(t\to\infty) = V_{Th}$ (final value)
$V_c(t_0)$ = initial value

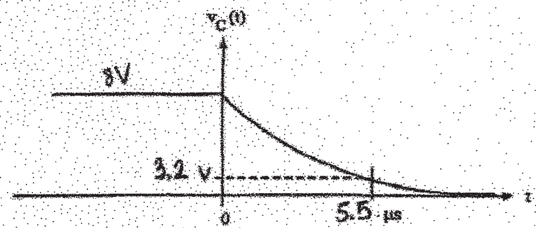
$$\boxed{\tau = R_{Th}\cdot C}$$

$$\therefore\ V_c(t) = \text{Final} + [\text{initial} - \text{Final}]\,e^{-\frac{(t-t_0)}{\tau}}$$

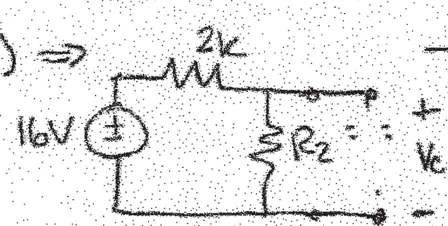
1.



After being in position a for a long time, the switch moves to position b at t = 0. Find R_2 and R_3 that give the following plot for $v_C(t)$:



1. $(t=0^-) \Rightarrow$



$$V_c = \frac{16(R_2)}{R_2+2k}$$

$(t=0^+) \Rightarrow$ because cap V can not change instantaneou

$$V_c(t=0^-) = V_c(t=0^+) = \frac{16(R_2)}{R_2+2k}$$

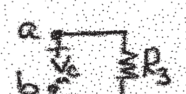
· from graph →

$$V_c(t=0^+) = 8V$$

$$8 = \frac{16(R_2)}{R_2+2k} \Rightarrow 8R_2 + 16k = 16R_2 - 8R_2$$

$$\boxed{R_2 = 2k\Omega}$$

$(t=\infty)$:



$V_c = 0$

$\tau = R_3(2n)$

$$V_c(t) = 8 + (0-8)(1 - e^{-t/R_3(2n)}) = +8e^{-t/R_3(2n)}$$

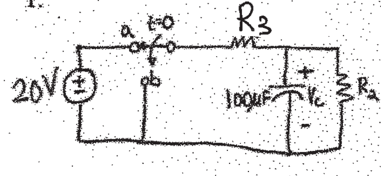
from graph $V_c(5.5\mu) = 3.2V$

$$3.2 = 8e^{-5.5\mu/R_3(2n)}$$

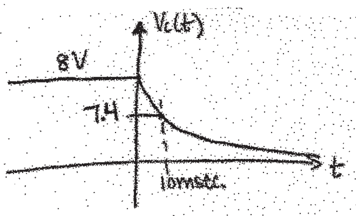
$$\ln\frac{3.2}{8} = \frac{-5.5\mu}{R_3(2n)} \Rightarrow R_3 = \frac{-5.5\mu}{2n(\ln(\frac{3.2}{8}))} \doteq \boxed{3k\Omega}$$

Homework #5 Example

1.



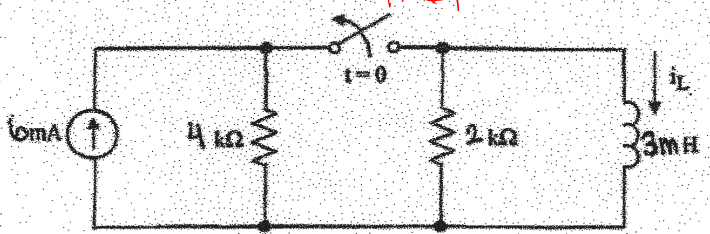
After being in position a for a long time, the switch moves to position b at t=0. Find R_2 and R_3 that gives the plot below for $V_c(t)$.



Homework #5 Examples

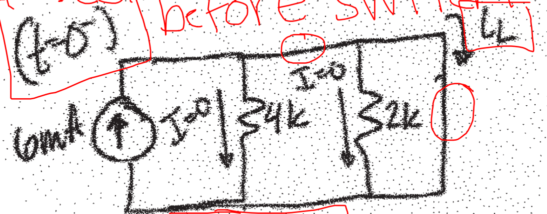
2.

at t=0 : it opens



After being closed for a long time, the switch opens at t = 0. Find $i_L(t)$ for $t > 0$.

closed $\boxed{(t=0^-)}$ before switch



$\boxed{i_L = 6mA}$

$\boxed{(t=0^+)}$: ∵ inductor current remains the same

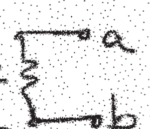
open $i_L(t=0^-) = i_L(t=0^+) = \boxed{6mA \leftarrow \text{(initial value)}}$

$(t=\infty)$:

$\boxed{i_L = 0 \text{ final}}$

I_L(t)=0+(6-0)e^(-t/tau)

(switch at $t=\infty$):



∴ $\boxed{i_L(t) = +6m\,e^{-t/(3m/2k)} \text{ A}}$

$\tau = \frac{L}{R_{th}}$

No class on Thursday.
Watch videos on class webpage.