

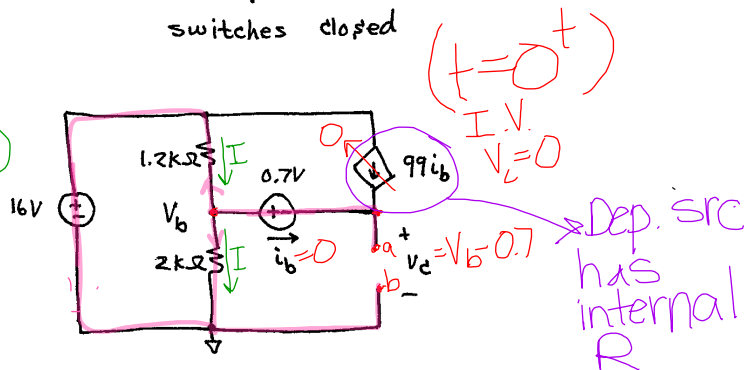
- Find $v_c(t)$ for $t > 0$ for circuit on left
 - Find v_s and C' values that make circuit on right have same v_c as circuit on left.
- Assume $v_c(0^-) = 0V$.

Sol'n: a) $v_c(0^+) = v_c(0^-) = 0V$ since v_c can't change instantly

Now find $v_c(t \rightarrow \infty)$.

$t \rightarrow \infty$: $C = \text{open}$
switches closed

$$\frac{V_b}{2K} + \frac{V_b - 0.7}{1.2K} = 0$$



If we sum currents at node above C , we have $-i_b - 99i_b = 0A \Rightarrow i_b = 0$.

Thus, $99i_b$ src = open.

$$\therefore v_c = V_b - 0.7V$$

sol'n: 4.a) cont.

We use v-divider formula to find V_b :

$$V_b = 16V \cdot \frac{2k\Omega}{1.2k\Omega + 2k\Omega} = 16V \cdot \frac{2k\Omega}{3.2k\Omega}$$

$$V_b = 16V \cdot \frac{20k\Omega}{32k\Omega} = \frac{1}{2} \cdot 20V$$

$$V_b = 10V$$

$$V_c(t \rightarrow \infty) = 10V - 0.7V = 9.3V$$

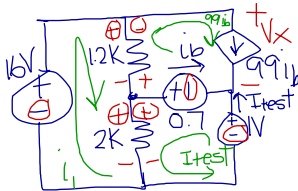
Now we find R_{Th} . First, we find

V_{Th} = V across terminals where C connected
but with C removed

But this is the same as $V_c(t \rightarrow \infty)$.

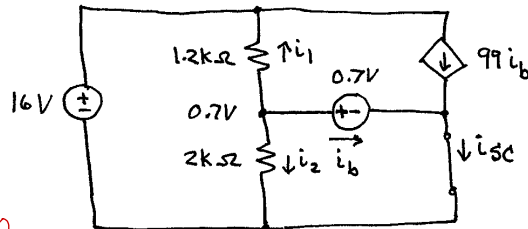
$$V_{Th} = 9.3V$$

Now short the terminals where C connected
and find i_{sc} .



$$\begin{aligned} -I_{test} - i_b - 99i_b &= 0 \\ I_{test} &= -100i_b \\ -16 + 2k(i_b + I_{test}) + 1.2k(i_b - 99i_b) &= 0 \\ +2k(i_b + I_{test}) - 1.7 &= 0 \end{aligned}$$

$$\begin{aligned} I_{test} &= \frac{1}{7.5} \\ R_{Th} &= 7.5\Omega \end{aligned}$$



From current summation at node
above C, $i_{sc} = i_b + 99i_b = 100i_b$.

Because of 0.7V src, we 0.7V at
center node. We get i_b from
current summation at this node.

sol'n: 4.a) cont.

$$i_1 = \frac{0.7V - 16V}{1.2k\Omega} = -\frac{15.3V}{1.2k\Omega}$$

$$i_2 = \frac{0.7V}{2k\Omega}$$

$$i_1 + i_2 + i_b = 0A$$

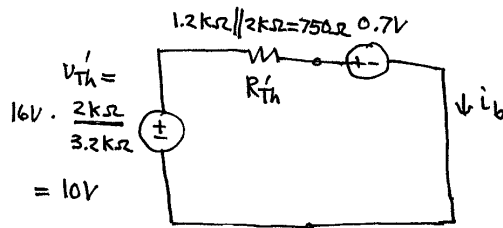
$$i_b = -(i_1 + i_2) = \frac{15.3V}{1.2k\Omega} - \frac{0.7V}{2k\Omega}$$

$$i_{sc} = 100 i_b = 100 \left(\frac{15.3V}{1.2k\Omega} - \frac{0.7V}{2k\Omega} \right)$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{9.3V}{100 \left(\frac{15.3V}{1.2k\Omega} - \frac{0.7V}{2k\Omega} \right)}$$

$$R_{Th} = 7.5\Omega$$

This worked, but a better approach is to use a Thevenin equiv of 16V, 1.2k Ω , and 2k Ω .



$$\text{We have } i_b = \frac{V'_{Th} - 0.7V}{R'_{Th}} = \frac{9.3V}{750\Omega}$$

$$i_{sc} = 100 i_b = \frac{9.3V \cdot 100}{750\Omega}$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}} = \frac{9.3V}{\frac{9.3V \cdot 100}{750\Omega}} = \frac{750\Omega}{100} = 7.5\Omega$$

sol'n: 4a) cont.

Plug values into general sol'n.

$$v_C(t > 0) = v_C(t \rightarrow \infty) + [v_C(0^+) - v_C(t \rightarrow \infty)] e^{-t/R_{Th}C}$$

$$R_{Th}C = 7.5 \Omega \cdot 15 \mu F = 112.5 \mu s$$

$$v_C(t > 0) = 9.3V [1 - e^{-t/112.5 \mu s}]$$

4.b)

For circuit on right, we have $v_C(0^+) = v_C(0^-) = 0V$ (given).We also have $v_C(t \rightarrow \infty) = v_S$ and $RC' = 750 \Omega \cdot C'$.

$v_C = v_S + (0 - v_S) e^{-t/RC'}$
 $v_C = 9.3V + (0 - 9.3V) e^{-t/112.5 \mu s}$

To make the v_C 's the same for the two circuits, we use $v_S = 9.3V$ and

$$C' = \frac{RC}{750 \Omega} = \frac{7.5 \Omega \cdot 15 \mu F}{750 \Omega} = 150 \text{ nF}$$

$$C' = 150 \text{ nF}$$

needs to
be equal
 $112.5 \mu = 750 \cdot C'$

3.

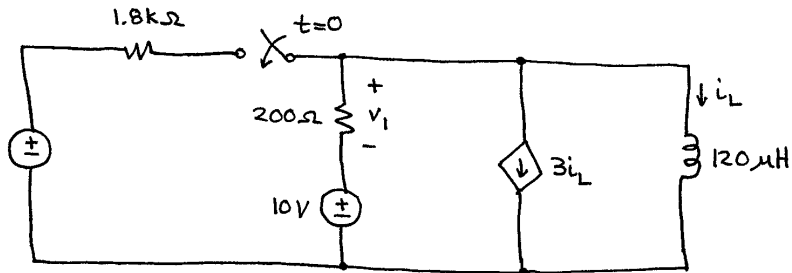
$(t=0^-)$ L \rightarrow wire
Solve for i_L

$(t=0^+)$ L \rightarrow $I_{src} = i_L(0^-)$

I.V.

$(t \rightarrow \infty)$ L \rightarrow wire

FINAL



Find $v_1(t)$ for $t > 0$.

sol'n: Find i_L for $t=0^-$.

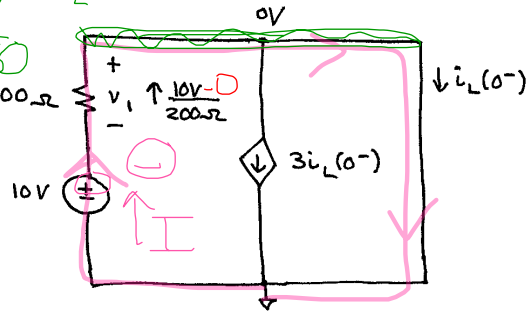
$t=0^-$: Switch is open. L = wire.

$$\sum I: -\frac{1}{20} + 4i_L = 0$$

$$4i_L = \frac{1}{80}$$

$$+10 - I(200) = 0$$

$$I = \frac{10}{200}$$



Because L = wire, we have 10V across 200Ω (so $10V_{src} + v_{200\Omega} = 0V$).

Thus, current $\frac{10V}{200\Omega}$ flows upward thru

200Ω . This current equals $3i_L(0^-) + i_L(0^-)$.

$$\therefore \frac{10V}{200\Omega} = 3i_L(0^-) + i_L(0^-) = 4i_L(0^-)$$

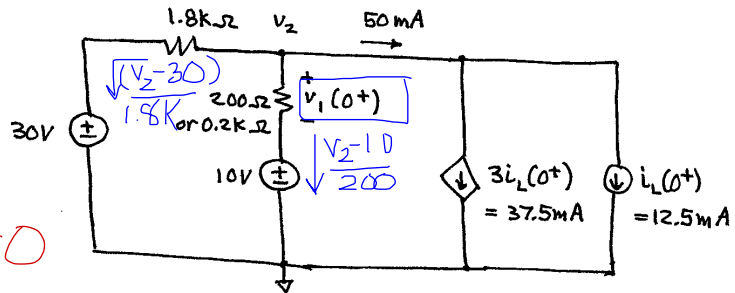
$$50mA = 4i_L(0^-)$$

$$i_L(0^-) = \frac{50mA}{4} = 12.5mA$$

sol'n: 3. cont.

Now find $v_1(t=0^+)$. $t=0^+$: Switch is closed.L = current source at $t=0^+$ $i_L(0^+) = i_L(0^-)$ since i_L doesn't

" = 12.5 mA change instantly

 $(t=0^+)$ 

$$v_2 - v_1 - 10 = 0$$

OR

$$v_1 = IR = \frac{(v_2 - 10)200}{200}$$

Use node voltage to find v_2 .

$$\frac{v_2 - 30V}{1.8k\Omega} + \frac{v_2 - 10V}{0.2k\Omega} + 50mA = 0A$$

$$v_2 \left(\frac{1}{1.8k\Omega} + \frac{1}{0.2k\Omega} \right) = \frac{30V}{1.8k\Omega} + \frac{10V}{0.2k\Omega} - \cancel{50mA}$$

" $\frac{50mA}{10}$

$$v_2 \left(\frac{1}{1.8k\Omega} + \frac{1}{0.2k\Omega} \right) \cdot 1.8k\Omega = 30V$$

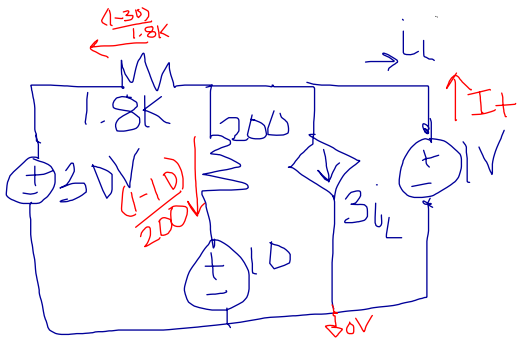
$$v_2 (1 + 9) = 30V$$

$$v_2 = \frac{30V}{10} = 3V$$

$$10V + v_1(0^+) = v_2 = 3V$$

$$v_1(0^+) = -7V \quad \text{I.V.}$$

sol'n: 3. cont.

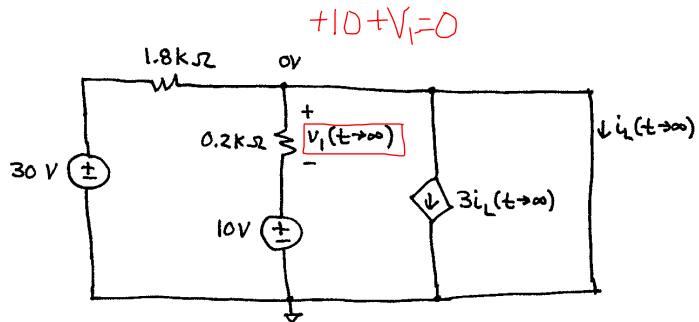
Now find $v_1(t \rightarrow \infty)$. $t \rightarrow \infty$: Switch is closed. $L = \text{wire}$ 

$$I_t = -i_L$$

$$\frac{(1-30)}{1.8k} + \frac{(1-10)}{200} + 4i_L = 0$$

$$i_L = \frac{1}{720}$$

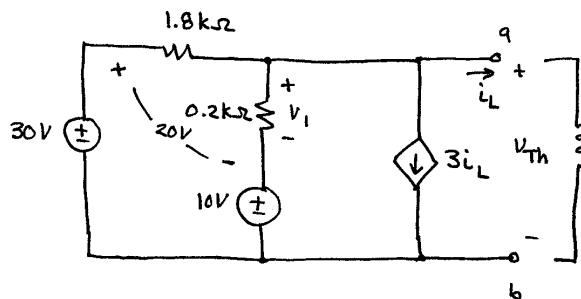
$$R_{Th} = |-720|$$



The voltage across 10V src and 0.2k Ω is 0V. $\therefore v_1(t \rightarrow \infty) = -10V$

Now find time constant $\frac{L}{R_{Th}}$. R_{Th} is for

circuit where L is connected.



$$V_{Th} = V_{a,b} \text{ open circ}$$

$i_L = 0$ so we treat $3i_L$ src as open circuit.

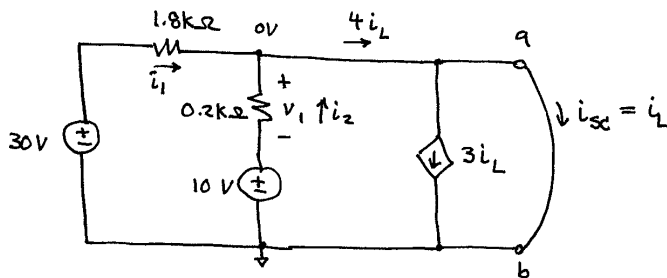
We could use node voltage method to find $v_{a,b}$, but a simpler approach is to observe that we have 20V across 1.8k Ω and 0.2k Ω in series.

soln: 3. cont.

Using a voltage divider, $V_1 = 20V \cdot \frac{0.2k\Omega}{0.2k\Omega + 1.8k\Omega} = 2V$.

$$\therefore V_{Th} = V_{ab \text{ open circ}} = 10V + V_1 = 12V$$

Now short a, b terminals to find i_{sc} .



This is the same circuit as for $t \rightarrow \infty$.

Using the value of 0V on the top node,

we can find i_1 and i_2 :

$$i_1 = \frac{30V}{1.8k\Omega} \quad i_2 = \frac{10V}{0.2k\Omega}$$

Summing currents, we have

$$-i_1 - i_2 + 4i_L = 0A$$

$$\text{or } i_L = i_{sc} = \frac{i_1 + i_2}{4} = \frac{\frac{30V}{1.8k\Omega} + \frac{10V}{0.2k\Omega}}{4}$$

$$i_{sc} = \frac{\frac{30V}{1.8k\Omega} + \frac{9 \cdot 10V}{1.8k\Omega}}{4} = \frac{120V}{4(1.8k\Omega)}$$

$$i_{sc} = \frac{30V}{1.8k\Omega}$$

$$R_{Th} = V_{Th}/i_{sc} = \frac{12V}{\frac{30V}{1.8k\Omega}} = 1.8k\Omega \frac{12V}{30V}$$

$$R_{Th} = 1.8k\Omega \cdot \frac{2}{5} = \frac{3.6k\Omega}{5} = 720\Omega$$

sol'n: 3. cont. Our time constant is $\frac{L}{R_{Th}} = \frac{120 \mu\text{H}}{720 \Omega} = \frac{1}{6} \mu\text{s}$.

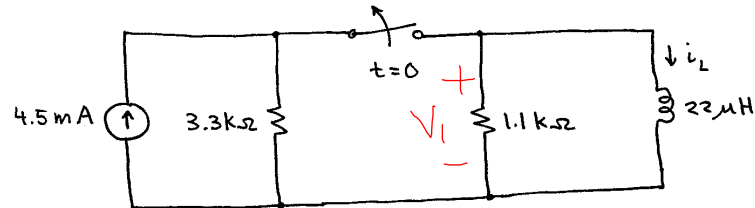
Plug values into general sol'n:

$$v_1(t > 0) = v_1(t \rightarrow \infty) + [v_1(0^+) - v_1(t \rightarrow \infty)] e^{-t/L/R_{Th}}$$

$$v_1(t > 0) = -10\text{V} + [-7\text{V} - (-10\text{V})] e^{-t/1/6 \mu\text{s}}$$

$$v_1(t > 0) = -10\text{V} + 3\text{V} e^{-t/1/6 \mu\text{s}}$$

2.



Find $i_L(t)$ for $t > 0$. Find $V_L(t)$

sol'n: We use general sol'n:

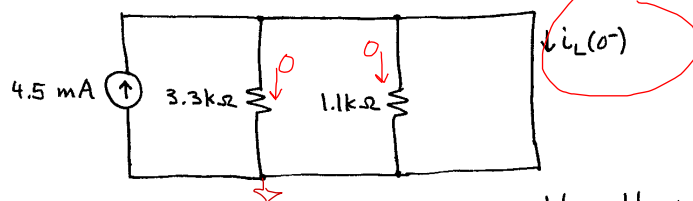
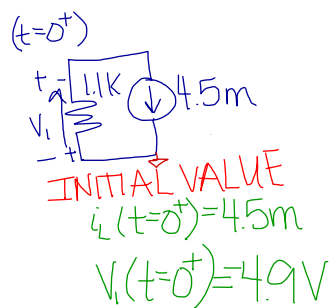
$$i_L(t > 0) = i_L(t \rightarrow \infty) + [i_L(t=0^+) - i_L(t \rightarrow \infty)] e^{-t/L/R_{Th}}$$

To find $i_L(t=0^+)$, we start at $t=0^-$.

$t=0^-$: Treat L as wire when circuit has been sitting for a long time, (so $\frac{di_L}{dt} = 0 \Rightarrow V_L = L \frac{di_L}{dt} = 0$).

Find $i_L(0^-)$ because $i_L(0^+) = i_L(0^-)$.

Switch is closed at $t=0^-$.



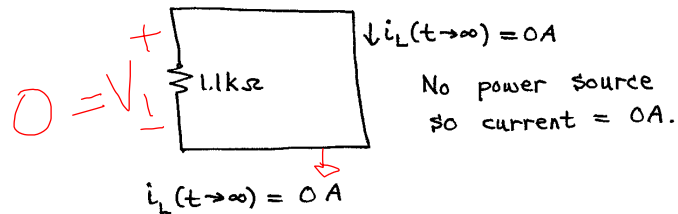
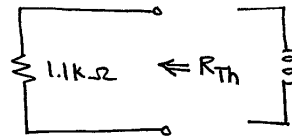
All the current will flow thru the wire.

$$i_L(0^-) = 4.5 \text{ mA}$$

Since the energy stored by the inductor cannot change instantly, $i_L(0^+) = i_L(0^-)$.

$$\therefore i_L(0^+) = 4.5 \text{ mA}$$

sol'n: 2. cont.

To find $i_L(t \rightarrow \infty)$, we again treat L as wire. $t \rightarrow \infty$: Switch open. 4.5mA and $3.3\text{k}\Omega$ not connected.The time constant is $\frac{L}{R_{Th}}$ where R_{Th} is fromthe Thevenin equivalent of circuit (for $t > 0$)where L is connected.

$$R_{Th} = 1.1\text{k}\Omega$$

$$\frac{L}{R_{Th}} = \frac{22\mu\text{H}}{1.1\text{k}\Omega} = 20\text{ns}$$

Plug quantities into general sol'n:

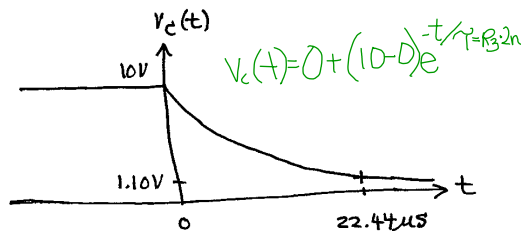
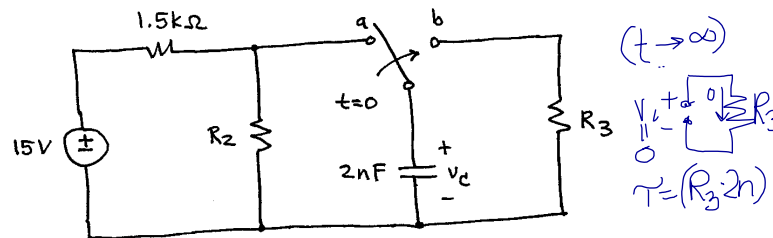
$$i_L(t > 0) = 0\text{A} + [4.5\text{mA} - 0\text{A}] e^{-t/20\text{ns}}$$

or

$$i_L(t > 0) = 4.5\text{mA} e^{-t/20\text{ns}}$$

$$V_L(t > 0) = 0 + [4.9\text{V} - 0] e^{-t/20\text{ns}}$$

1.

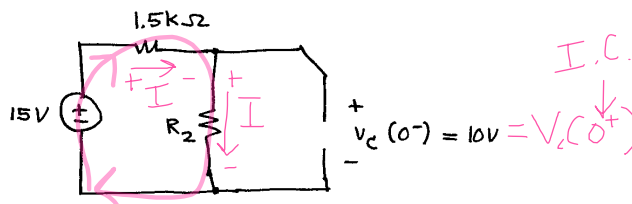


Find R_2 and R_3 to give above plot.

sol'n: R_2 determines init voltage on C.

From plot, $v_c(t=0^-) = 10V$.

Circuit model at $t=0^-$: C acts like open circuit.



Use v-divider. $v_c(0^-) = 15V \cdot \frac{R_2}{R_2 + 1.5k\Omega} = 10V$

$$\therefore \frac{R_2}{R_2 + 1.5k\Omega} = \frac{2}{3} \quad \text{or} \quad 3R_2 = 2(R_2 + 1.5k\Omega)$$

$$\boxed{R_2 = 3k\Omega}$$

R_3 determines the time constant $= R_3 C$

Using the general sol'n for $v_c(t)$ we have:

Sol'n: 1. cont.

$$v_c(t > 0) = v_c(t \rightarrow \infty) + [v_c(t=0^+) - v_c(t \rightarrow \infty)] e^{-t/R_3 C}$$

$$\begin{array}{ccc} \parallel & \parallel & \parallel \\ 0V & v_c(0^-) & 0V \\ & \parallel & \\ & 10V & \end{array}$$

$$v_c(t > 0) = 10V e^{-t/R_3 C} \quad \text{where } C = 2nF$$

From the plot, we have $v_c(22.44\mu s) = 1.10V$.

$$\therefore 1.10V = 10V e^{-22.44\mu s / R_3 C}$$

Solve for R_3 :

$$\ln \frac{1.10V}{10V} = -22.44\mu s / R_3 C$$

$$R_3 = \frac{-22.44\mu s / 2nF}{\ln \frac{1.10V}{10V}}$$

$$R_3 \doteq 5.1k\Omega$$