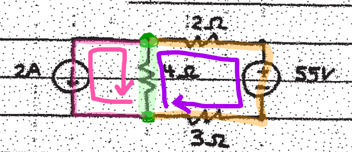
Ref: Nilsson & Riedel, Rev 6, Prob 4.2

ex:



Use Node-Voltage method to find how much power the 2A source extracts from circuit.

First we use terminology:

nodes = 4 (two or more circuit elements join)

essential nodes = 2 (three or more circuit elements join; they are nodes for 4Ω resistor, top & bottom)

paths: 2A → 4Ω, 4Ω → 2Ω → 55V → 3Ω, 2Ω → 55V, 2A → 2Ω → 55V are a few examples (trace of connected circuit elements without passing thru any element twice)

branch: (path that connect 2 nodes) 2A, 4Ω, 2Ω, 3Ω, 2Ω → 55V, | 55V → 3Ω, 3Ω → 55V (either direction ok), 55V, 2Ω → 55V → 3Ω

essential branch: (path connecting essential node w/o passing thru essential node) 2A, 4Ω, 2Ω → 55V → 3Ω, or 3Ω → 55V → 2Ω

loops: (path with last node = start node) 2A → 4Ω, 2A → 2Ω → 55V → 3Ω, 4Ω → 2Ω → 55V → 3Ω

mesh: (loop not enclosing any other loop) 2A → 4Ω, 4Ω → 2Ω → 55V → 3Ω but not 2A → 2Ω → 55V → 3Ω

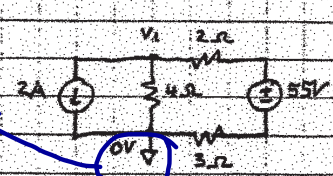
planar circuit: (can draw circuit w/o crossover branches) is planar

For Node-V method, we use all but 1 essential nodes after we define a ref node.

Choose node at bottom of 4Ω as ref node (i.e. 0V.)

symbol ≡ 0V

most branches joining



Node at top of 4Ω is the other essential node. Label it V_1

Although we call it the Node-V method, (because we get an equation that we solve for voltage), we are writing an equation for sum of currents out of node = 0.

$$2A + \frac{v_1 - 0V}{4\Omega} + \frac{v_1 - 55V}{2\Omega + 3\Omega} = 0A$$

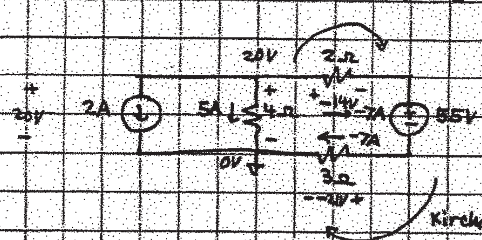
Note that current thru 2Ω is equal to total V-drop (i.e. $v_1 - 55V$) across the 2Ω and 3Ω R's.

$$\text{or} \quad v_1\left(\frac{1}{4\Omega} + \frac{1}{5\Omega}\right) = -2A + \frac{55V}{5\Omega}$$

$$\text{or} \quad \frac{v_1}{4\Omega \| 5\Omega} = 11A - 2A = 9A$$

$$\text{or} \quad v_1 = 9A \cdot 4\Omega \| 5\Omega = 9A \cdot \frac{4\Omega \cdot 5\Omega}{4\Omega + 5\Omega} = 20\,V$$

$$\text{check:} \quad \frac{v_1}{4\Omega} = \frac{20V}{4\Omega} = 5A \qquad \frac{v_1 - 55V}{2\Omega + 3\Omega} = -\frac{35V}{5\Omega} = -7A$$



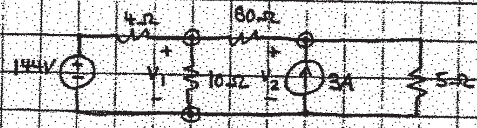
$-55V - -21V + 20V - -14V$
$= -55V + (21 + 20 + 14)V$
$= 0V$ ✓

check: current out of top node for 4Ω is $2A + 5A - 7A = 0$ ✓

Calculate power for 2A source: $p = i \cdot v = 2A \cdot 20V = 40W$

$p > 0 \Rightarrow$ power absorbed.

ex:

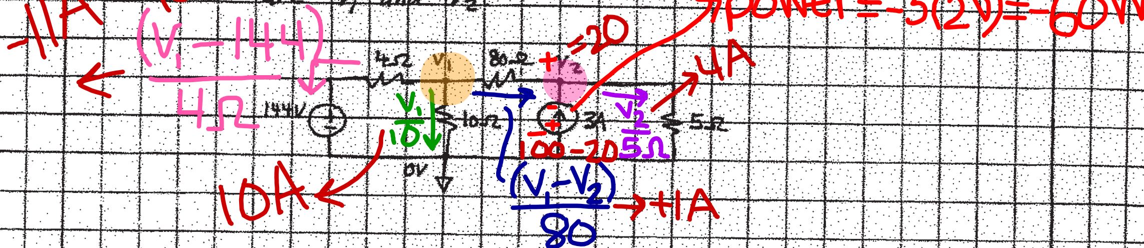


Use Node-V method to find v_1 & v_2.

sol'n: nodes are marked by dots. Essential nodes are marked by ⊙. Note that node under 3A source is considered to be part of essential node under 10Ω Resistor, (because they are connected by a wire.)

3 essential nodes ⇒ 3 - 1 = 2 equations needed.

Put ref V of 0V, (i.e. ⏚) at bottom so top nodes are v_1 and v_2



Node-V eq'ns give sum of currents out of node = 0.

node 1: (v_1)
$$\frac{v_1 - 144V}{4\Omega} + \frac{v_1 - 0V}{10\Omega} + \frac{v_1 - v_2}{80\Omega} = 0A$$

node 2: (v_2)
$$\frac{v_2 - v_1}{80\Omega} + -3A + \frac{v_2 - 0V}{5\Omega} = 0A$$

or node 1:
$$v_1\left(\frac{1}{4\Omega} + \frac{1}{10\Omega} + \frac{1}{80\Omega}\right) - \frac{v_2}{80\Omega} = \frac{144V}{4\Omega}$$

node 2:
$$-v_1\frac{1}{80\Omega} + v_2\left(\frac{1}{80\Omega} + \frac{1}{5\Omega}\right) = 3A$$

or node 1:
$$\frac{v_1}{4\Omega \| 10\Omega \| 80\Omega} - \frac{v_2}{80\Omega} = \frac{144V}{4\Omega}$$

node 2:
$$-\frac{v_1}{80\Omega} + \frac{v_2}{80\Omega \| 5\Omega} = 3A$$

$$4\,\Omega \parallel 10\,\Omega \parallel 80\,\Omega = (4\,\Omega \parallel 10\,\Omega) \parallel 80\,\Omega$$

$$= \frac{40}{14}\,\Omega \parallel 80\,\Omega$$

$$= 40\,\Omega \cdot \frac{1}{14} \parallel 2$$

$$= 40\,\Omega \cdot \frac{2/14}{2 + 1/14}$$

$$= 40\,\Omega \cdot \frac{2}{29} \quad \text{(mult top \& bottom by 14)}$$

$$= \frac{80}{29}\,\Omega$$

$$80\,\Omega \parallel 5\,\Omega = 5\,\Omega \cdot 16 \parallel 1 = 5\,\Omega \cdot \frac{16}{17} = \frac{80}{17}\,\Omega$$

Thus, we have: node 1 $\quad v_1 \dfrac{29}{80\,\Omega} - \dfrac{v_2}{80\,\Omega} = 36\text{ A}$

node 2 $\quad \dfrac{-v_1}{80\,\Omega} + v_2 \dfrac{17}{80\,\Omega} = 3\text{ A}$

or node 1 $\quad 29v_1 - v_2 = 2880\text{ V}$

node 2 $\quad -v_1 + 17v_2 = 240\text{ V}$

From node 1: $-v_2 = 2880\text{V} - 29v_1$; substitute this into Node 2:

node 2: $\quad -v_1 - 17(2880\text{V} - 29v_1) = 240\text{V}$

$$492v_1 = 240\text{V} + 48960$$

$$\boxed{v_1 = 100\text{ V}}$$

$$v_2 = \frac{240\text{V} + v_1}{17} = \frac{340\text{V}}{17} \boxed{= 20\text{V}}$$

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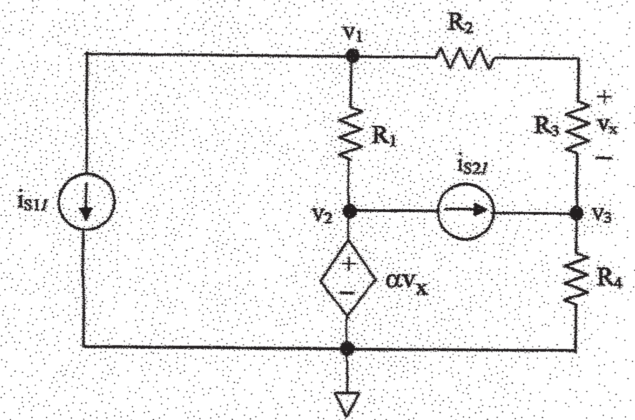
Node Voltage Example

1. a.



For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

b. Make a consistency check on your equations for problem 1 by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a).)

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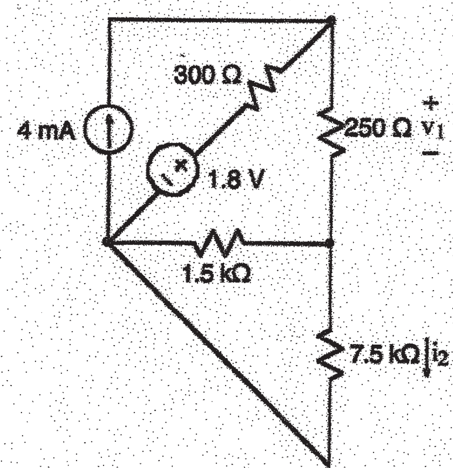
Node Voltage Example

Ex:



a) Use the node-voltage method to calculate v_1 and i_2.

b) Calculate the power in the 300 Ω resistor.

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HOMEWORK #3 Solution Prob 3

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Ex:

$0 = +100\mu + \frac{(V_1-V_3)}{50K} + \frac{V_2}{30K} + 40\mu$

$+100\mu + I_1 + \frac{(V_1-V_3)}{50K} = 0$

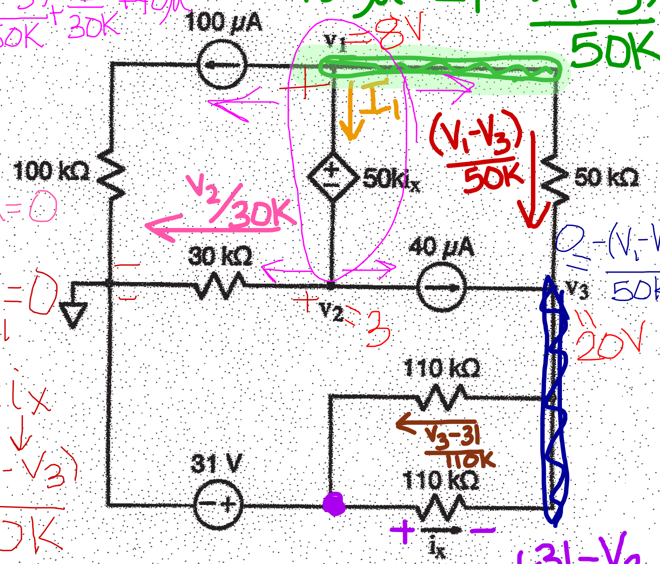
$I_1 = \frac{V_2}{30K} + 40\mu$

$+\frac{V_2}{30K} - I_1 + 40\mu = 0$

$+V_2 + 50Ki_x - V_1 = 0$

$(V_1 - V_2) = 50Ki_x$

$\frac{(31-V_3)}{110K}$



$0 = \frac{-(V_1-V_3)}{50K} - 40\mu + \frac{(V_3-31)}{110K} - \frac{(31-V_3)}{110K}$

$\frac{+31-V_3}{110K}$

Use the node-voltage method to find v_1, v_2, and v_3.

sol'n: We first write the variable for the dependent source, i_x, in terms of node V's.

$$i_x = \frac{31V - v_3}{110k\Omega}$$

We have a super node for v_1 and v_2 since these nodes are connected only by a V src. Thus, we sum all the currents flowing out of a bubble drawn around v_1, v_2, and the V-src between them.

$$v_1, v_2 \text{ node: } 100\mu A + \frac{v_1 - v_3}{50k\Omega} + \frac{v_2}{30k\Omega} + 40\mu A = 0A$$

We also write a voltage eq'n for v_1 & v_2:

$$v_1 = v_2 + 50k\Omega\, i_x = v_2 + 50k\Omega\left(\frac{31V - v_3}{110k\Omega}\right)$$

(Remember to use only node V's in eq'ns.)

For the v_3 node, we only have a current sum.

$$v_3 \text{ node: } \frac{v_3 - v_1}{50k\Omega} + -40\mu A + \frac{v_3 - 31V}{\underbrace{110k\Omega \| 110k\Omega}_{55k\Omega}} = 0A$$

Now we solve the 3 eq'ns. We put terms multiplying v_1, v_2, and v_3 on the left side and constant terms on the right side.

$$v_1 \frac{1}{50k\Omega} + v_2 \frac{1}{30k\Omega} + v_3\left(\frac{-1}{50k\Omega}\right) = -100\mu A - 40\mu A$$

$$v_1 - v_2 + v_3 \frac{50k\Omega}{110k\Omega} = 31V \cdot \frac{50k\Omega}{110k\Omega}$$

$$v_1\left(\frac{-1}{50k\Omega}\right) + v_3\left(\frac{1}{50k\Omega} + \frac{1}{55k\Omega}\right) = 40\mu A + \frac{31V}{55k\Omega}$$

We multiply both sides of the 1st eq'n by 150 kΩ to clear the denominators.

$$v_1 \cdot 3 + v_2 \cdot 5 + v_3(-3) = -140\mu A \cdot 150k\Omega = -21V$$

We multiply both sides of the 2nd eq'n by 11Ω to clear the denominators.

$$v_1(11) + v_2(-11) + v_3(5) = 31V(5) = 155V$$

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We multiply the 3^{rd} eq'n by $1100\,k\Omega = 1.1\,M\Omega$

$$v_1(-22) + v_3(22+20) = 40\cancel{\mu}A \cdot 1.1\cancel{M}\Omega + 31V(20)$$

$$\text{or} \quad v_1(-22) + v_3(42) = 44V + 620V = 664V$$

Now we start eliminating variables. From the 1^{st} eqn, we have

$$v_2 = \frac{-21V - 3v_1 + 3v_3}{5}$$

Our 2^{nd} and 3^{rd} eq'ns with this v_2 become

$$11v_1 - 11\left(\frac{-21V - 3v_1 + 3v_3}{5}\right) + 5v_3 = 155V$$

and

$$-22v_1 + 42v_3 = 664V$$

Solving the last eq'n for v_3 gives

$$v_3 = \frac{664V + 22V_1}{42}$$

Substituting into the 1^{st} eq'n (and collecting terms multiplying v_3) yields the following eq'n:

$$\left(11 + \frac{33}{5}\right)v_1 + \left(-\frac{33}{5} + 5\right)\left(\frac{664V + 22v_1}{42}\right) = 155V - \frac{21(11)V}{5}$$

or, after multiplying both sides by 5,

$$88v_1 - 8\left(\frac{664V + 22v_1}{42}\right) = 775V - 231V = 544V.$$

Dividing both sides by 4 and moving constant term:

$$22v_1 - \frac{2(22)}{42}v_1 = 136V + \frac{2(664)}{42}V.$$

Multiplying both sides by 42 gives

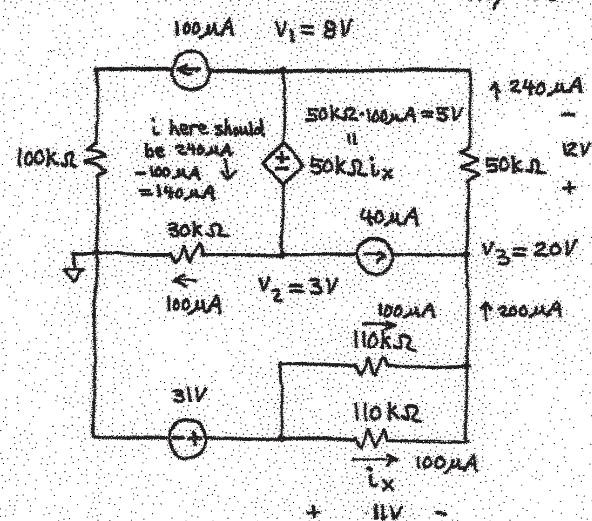
$$[42(22) - 44]v_1 = 136V(42) + 2(664)V$$

or $$v_1 = \frac{5712 + 1328}{924 - 44}V = \frac{7040}{880}V = \boxed{8V}$$

Then $$v_3 = \frac{664V + 22(8V)}{42} = \boxed{20V}$$

and $$v_2 = \frac{-21V - 3(8V) + 3(20V)}{5} = \boxed{3V.}$$

Consistency check: Calculate currents from these voltages and verify that currents sum to zero at nodes... They do!



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HOMEWORK #3 Solution Prob 4

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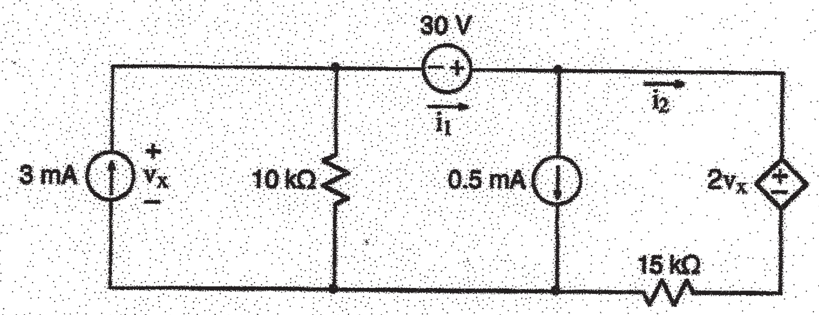
Ex:



a) Use the mesh-current method to find i_1 and i_2.

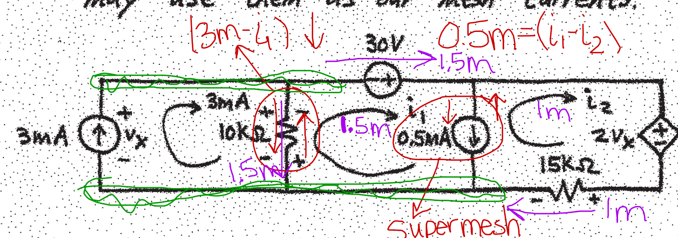
b) Find the power dissipated by the dependent source.

sol'n: a) We follow a step-by-step procedure:

1) We define mesh currents. If, however, we have any current sources on outside edges of the circuit, the mesh currents for those loops will be the same as the current source.

In this circuit, we have a current source on the left edge. Thus, the mesh current for the left loop is 3mA.

Since i_1 and i_2, as defined, are on the outside edge of the circuit, we may use them as our mesh currents.



2) We define the voltage from the dependent src, v_x, in terms of mesh currents. Here, we observe that v_x is across the $10k\Omega$ resistor, too. For the $10k\Omega$ resistor, we have

$$v_x = 3\,mA \cdot 10k\Omega - i_1 \cdot 10\,k\Omega$$

3) We look for loops with a current source in between, meaning we have a super mesh. This is the case for the i_1, i_2 loops. For the i_1, i_2 supermesh, we take a v-loop around the outside edge of the i_1 and i_2 loops, (bypassing the $0.5\,mA$ src).

$$i_1 i_2 \text{ v-loop: } -i_1 \cdot 10\,k\Omega + 30V - 2\overbrace{(3mA - i_1)\,10k\Omega}^{v_x} + 3mA \cdot 10k\Omega$$

$$- i_2 \cdot 15\,k\Omega = 0V$$

Add a current eq'n for the $0.5\,mA$ src between the loops:

$$i_1 - i_2 = 0.5\,mA = \tfrac{1}{2}\,mA$$

Note: we have $-i_2$ for current measured opposite the arrow in the current src.

4) We solve our eq'ns for i_1 and i_2.

We group i_1 and i_2 terms on the left and move constant to the right side.

HOMEWORK #3 Solution Prob 4 (cont.)

$$i_1\big(\underbrace{-10k\Omega + 2\cdot 10k\Omega}_{=10k\Omega}\big) + i_2(-15k\Omega) = -60V + 60V$$

$$i_1 \qquad - i_2 = \tfrac{1}{2}\,mA$$

Solving the 2nd eq'n for i_1, we have

$$i_1 = i_2 + \tfrac{1}{2}\,mA$$

Substituting into 1st eq'n, we have

$$\left(i_2 + \tfrac{1}{2}\,mA\right) 10k\Omega + i_2(-15k\Omega) = 30V$$

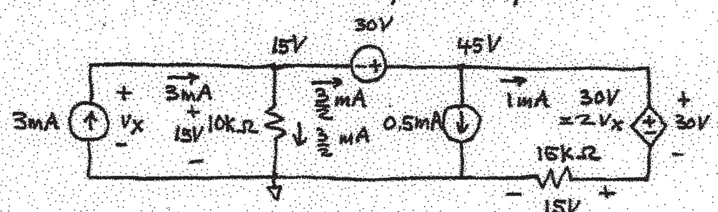
or $\quad i_2\,(10k\Omega - 15k\Omega) = 0V - \tfrac{1}{2}\,mA\cdot 10k\Omega$

or $\quad -i_2\,(5k\Omega) = -5V$

or $\quad \boxed{i_2 = 1\,mA}$

Then $\quad i_1 = 1\,mA + \tfrac{1}{2}\,mA = \boxed{\tfrac{3}{2}\,mA.}$

Consistency check: calculate v-drops for i_1, i_2 and verify v-loops.



$$v_x = \tfrac{3}{2}\,mA\cdot 10k\Omega = 15\,V$$

All v-loops sum to 0V, and all current sums at nodes = 0A. ✓

b) We know $v_x = (3\,mA - i_1)\,10k\Omega$

$" = \frac{3}{2}\,mA \cdot 10k\Omega$

$v_x = 15V$

The current for the dependent src is i_2.

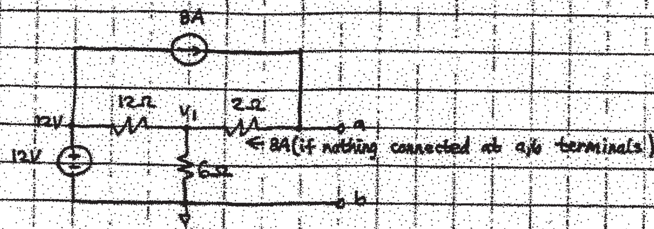
$i_2 = 1\,mA$

Thus, power for the dependent src is

$p = v \cdot i = 2v_x\,i_2 = 2(15V)\cdot 1mA$

or $p = 30\,mW$.

ex:



Find the Thevenin equivalent with respect to terminals a,b.
(In other words, create a circuit with a V-source and internal R that has same i and v at its terminals as above circuit has at a,b terminals.)

Use Node-V method to find v_1, then use $v_a = v_1 + 8A \cdot 2\Omega = v_{Th}$.
(We are finding v_{Th} by calculating voltage at a,b terminals with nothing connected to a,b terminals.)

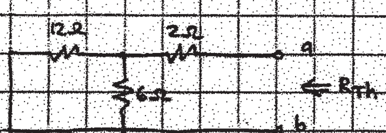
Node v_1 eqn: $\dfrac{v_1 - 12V}{12\Omega} + \dfrac{v_1}{6\Omega} - 8A = 0A$

or $\dfrac{v_1}{12\Omega \| 6\Omega} = 8A + \dfrac{12V}{12\Omega} = 9A$

or $v_1 = 9A \cdot 12\Omega \| 6\Omega = 9A \cdot 6\Omega (2\|1) = 54V \cdot \frac{2}{3} = 36V$

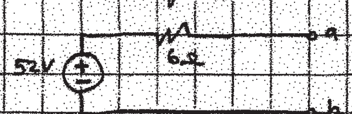
$V_{Th} = V_a = v_1 + 8A \cdot 2\Omega = 36V + 16V = 52V$

To find R_{Th} we turn independent sources to zero and find R looking into a,b terminals:



$$R_{Th} = (6\Omega \| 12\Omega) + 2\Omega = 6\Omega \cdot (1\|2) + 2\Omega = 6\Omega \cdot \frac{2}{3} + 2\Omega = 6\Omega$$

Thevenin equivalent:



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Homework #4 Examples

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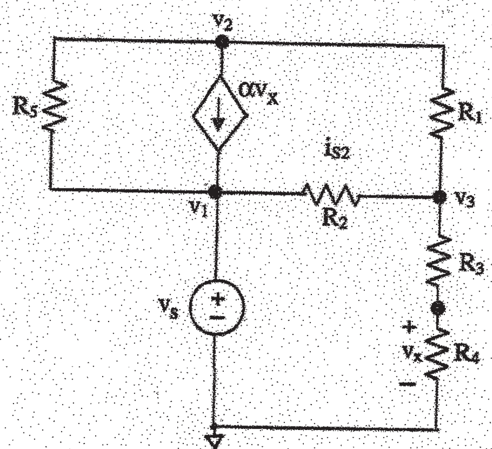
1.



For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

2. Make a consistency check on your equations for problem 1 by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem 1(a) are satisfied for these values. (In other words, plug the values into your equations for problem 1(a).)

sol'n: 1. 1) $v_1 = v_s$ (Connected to ref node by v-src)

2) $$\frac{v_2 - v_1}{R_5} + \alpha \boxed{\frac{v_3 R_4}{R_3 + R_4}} + \frac{v_2 - v_3}{R_1} = 0\,\text{A}$$

This is v_x in terms of node voltage. We are using a v-divider.

3) $$\frac{v_3 - v_2}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - 0\,\text{V}}{R_3 + R_4} = 0\,\text{A}$$

Note: we only have to write the eq'ns, not solve them.

Homework #4 Examples

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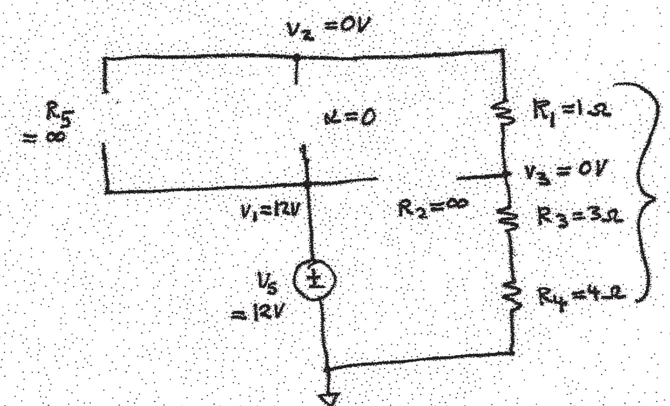
2. Many consistency checks are possible.
Choose values of R's and src's such that values of v_1, v_2, v_3 are obvious from inspection.

My choices:



no current flow on right side
$\therefore iR = 0V$ for R_1, R_3, R_4

Plug all the component values and v's into eq'ns in prob 1 to verify that eq'ns are satisfied.

1) $12V = 12V$ ✓
$v_1 = v_s$

2) $$\frac{0-12V}{\infty}^{0} + 0 \cdot \frac{0V \cdot 4\Omega}{3+4\Omega} + \frac{0-0V}{1\Omega} = 0A \checkmark$$

$$\frac{v_2 - v_1}{R_5} + \alpha \frac{v_3 R_4}{R_3 + R_4} + \frac{v_2 - v_3}{R_1} = 0A$$

3) $$\frac{0-0V}{1\Omega} + \frac{0-12V}{\infty} + \frac{0V-0V}{3+4\Omega} = 0A \checkmark$$

$$\frac{v_3 - v_2}{R_1} + \frac{v_3 - v_1}{R_2} + \frac{v_3 - 0V}{R_3 + R_4} = 0A$$

$i_{s2} = (i_2 - i_1)$ +(same) -opposite

$\uparrow (i_{s1} + i_1)$ +

(82)

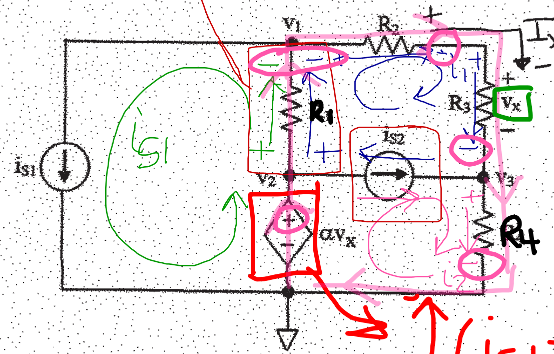
Homework #4 Example

$0 = +\alpha V_x - (i_{s1} + i_1)R_1 + \ldots$

$\ldots - i_1 R_2 - V_x - i_2 R_4$

$V_x = i_1 R_3$

1.



$\uparrow (i_{s1} + i_2)$ +

(a) For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

(b) Make a consistency check on your equations for (a) by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem (a) are satisfied for these values. (In other words, plug the values into your equations for problem (a).)

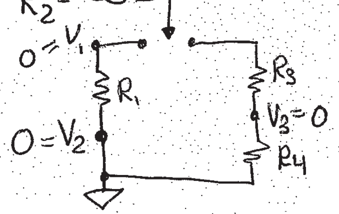
(a) node voltage: $\frac{(V_1 - V_3)R_3}{R_3 + R_2} = V_x$ (V-divider)

OR $V_x = I_x R_3 = \underbrace{\left[\frac{(V_1 - V_3)}{R_2 + R_3}\right]}_{I_x} R_3$ (same

$V_2 = \alpha V_x = \boxed{\frac{\alpha (V_1 - V_3) R_3}{R_2 + R_3} = V_2}$

$\boxed{i_{s1} + \frac{(V_1 - V_2)}{R_1} + \frac{(V_1 - V_3)}{R_2 + R_3} = 0}$ $\boxed{\frac{V_3 - V_1}{R_2 + R_3} - i_{s2} + \frac{V_3}{R_4} = 0}$

(b) Let $\alpha = 0$, $i_{s1} = 0$, $i_{s2} = 0$ [Redraw Circuit]



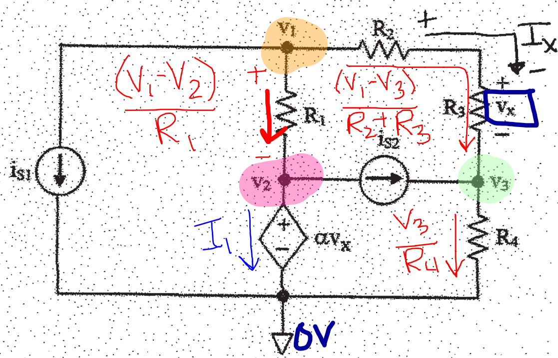
From eq: $V_2 = \frac{0\,(V_1 - V_3)R_3}{R_2 + R_3} = 0$ //

$0 + \frac{(V_1 - 0)}{R_1} + \cancel{\frac{(V_1 - V_3)}{\infty}} = 0 \Rightarrow V_1 = 0$ //

$\frac{V_3 - 0}{R_2 + R_3} - 0 + \frac{V_3}{R_4} = 0 \Rightarrow V_3 = 0$ //

Homework #4 Example

1.



$V_2 - 0 = \alpha V_x$

(a) For the circuit shown, write three independent equations for the node voltages v_1, v_2, and v_3. The quantity v_x must not appear in the equations.

(b) Make a consistency check on your equations for (a) by setting resistors and sources to values for which the values of v_1, v_2, and v_3 are obvious. State the values of resistors, sources, and v_1, v_2, v_3 for your consistency check, and show that your equations for problem (a) are satisfied for these values. (In other words, plug the values into your equations for problem (a).)

(a) node voltage: $\underbrace{\frac{(V_1-V_3)}{R_3+R_2}}_{I} R_3 = V_x$ (V-divider)

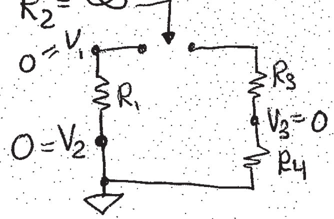
OR $V_x = I_x R_3 = \underbrace{\left[\frac{(V_1-V_3)}{R_2+R_3}\right]}_{I_x} R_3$ (same

$V_2 = \alpha V_x = \boxed{\frac{\alpha (V_1-V_3)R_3}{R_2+R_3} = V_2}$

$\boxed{i_{s1} + \frac{(V_1-V_2)}{R_1} + \frac{(V_1-V_3)}{R_2+R_3} = 0}$ $\boxed{\frac{V_3-V_1}{R_2+R_3} - i_{s2} + \frac{V_3}{R_4} = 0}$

(b) Let $\alpha = 0$, $i_{s1} = 0$, $i_{s2} = 0$ [Redraw Circuit]

$R_2 = \infty$



From eq: $V_2 = \frac{0\,(V_1-V_3)R_3}{R_2+R_3} = 0$ //

$0 + \frac{(V_1-0)}{R_1} + \cancel{\frac{(V_1-V_3)}{\infty}}^{0} = 0 \Rightarrow V_1 = 0$ //

$\frac{V_3-0}{R_2+R_3} - 0 + \frac{V_3}{R_4} = 0 \Rightarrow V_3 = 0$ //