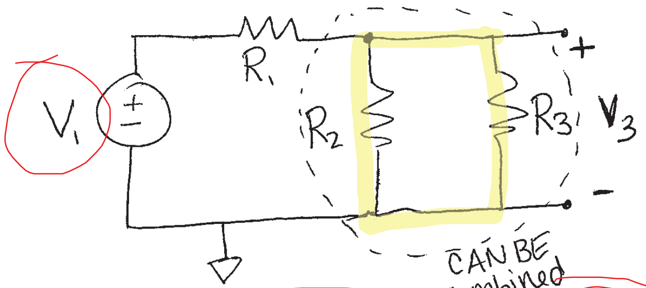
Requirements to use voltage divider:

- Current is <u>same</u> through all R's in loop.

Ex: Able to use V-divider

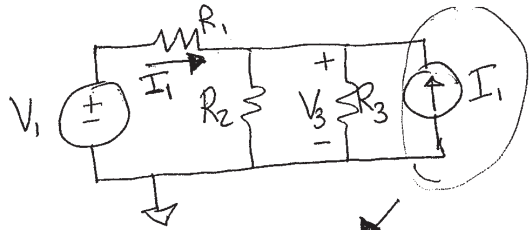


CAN BE combined

$$V_3 = \frac{V_1 (R_2 \| R_3)}{(R_2 \| R_3) + R_1}$$

$$\left[\frac{\text{Voltage Src} * R\_\text{desired V}}{\text{total R}}\right]$$

Not valid to use



A voltage or current source will alter the current so that I_1 does not split equally through R_2 & R_3

Requirement to use current divider:

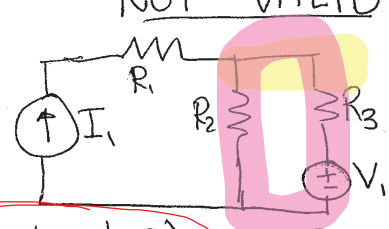
- Same current going into R's comes out.
- voltage is <u>equal</u> across all R's.

Ex: <u>VALID</u>

$$I_3 = \frac{I_1 (R_2)}{R_2 + R_3}$$

$$\left[\frac{\text{I src} * (\text{opposite branch R})}{\text{total R (in branches)}}\right]$$

<u>NOT VALID</u>



note that R_1 does nothing in this circuit

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HOMEWORK #1 Examples

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1.

$-12 + i_1 + i_2 = 0$

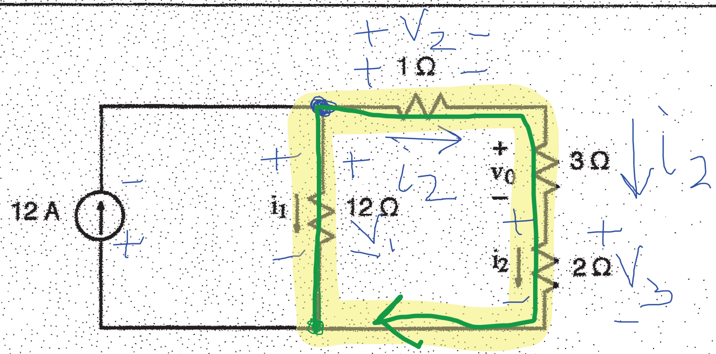
$+v_1 - v_2 - v_0 - v_3 = 0$

$v_1 = i_1(12)$

$v_2 = i_2(1)$

$v_3 = i_2(2)$

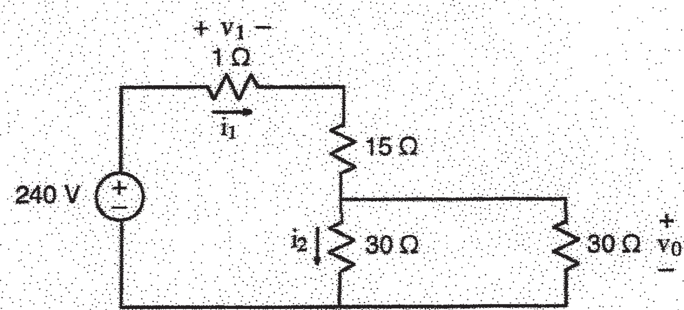
$v_0 = i_2(3)$



a) Calculate i_1, i_2, and v_0.

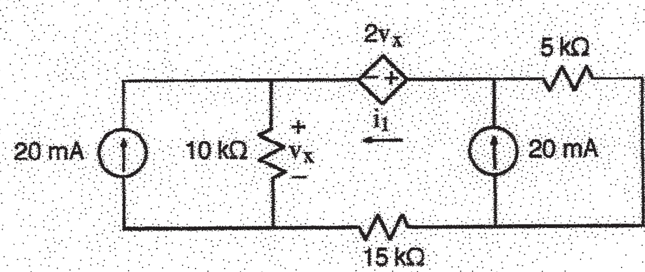
b) Find the power dissipated for every component, including the current source.

2.



Calculate i_1, i_2, and v_0.

3.



Find v_x, i_1, and the power dissipated by the dependent source.

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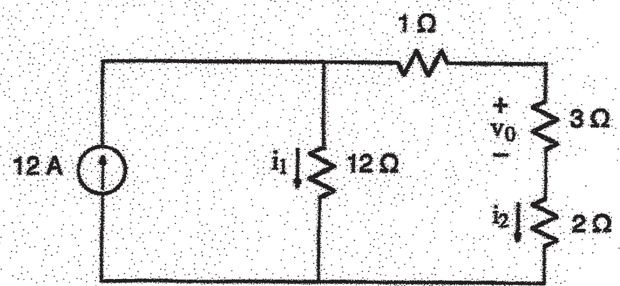
HOMEWORK #1 Prob 1. sol'n

Ex:

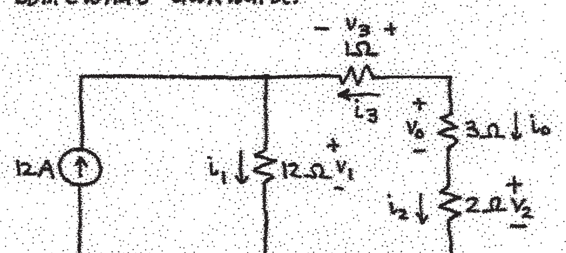


a) Calculate i_1, i_2, and v_0.

b) Find the power dissipated for every component, including the current source.

sol'n: a) We first label current and voltage for each resistor. We follow the passive sign convention: the arrow for the direction of current measurement points toward the − sign of the voltage measurement.

For the 1Ω resistor, we may define the current measurement in either direction. For the sake of illustration, we define the direction of current measurement in a way that is somewhat awkward.



Now we write eq'ns for voltage loops. We try to write v-loop eq'ns for inner loops, but we avoid loops that include a current source. (The reason we do so is to avoid defining a new variable that requires another eq'n.) In this problem, there is only one v-loop without a current source. Going around the inner loop on the right side in a clockwise direction and using the sign where we exit a component, we have

$$+v_1 + v_3 - v_o - v_2 = 0V$$

Next, we write eq'ns for current summations at nodes. We sum the currents measured flowing away from the top center node.

$$-12A + i_1 - i_3 = 0A$$

There is always one redundant node. So we only need this one eq'n.

Now we look for components in series that carry the same current.

$i_3 = -i_o$ (minus sign because currents measured in opposite directions)

$i_o = i_2$

Our last set eq'ns comes from Ohm's Law for each resistor.

$v_1 = i_1 \cdot 12\Omega$

$v_0 = i_0 \cdot 3\Omega$

$v_2 = i_2 \cdot 2\Omega$

To solve the simultaneous eq'ns, we substitute i_2 for i_0 and $-i_2$ for i_3. Then we substitute for v's using the Ohm's law eq'ns.

Our v-loop eq'n becomes

$$i_1 \cdot 12\Omega - i_2 \cdot 1\Omega - i_2 \cdot 3\Omega - i_2 2\Omega = 0V.$$

Our i-sum eq'n becomes

$$-12A + i_1 + i_2 = 0A.$$

Solving the second eq'n, we have, for i_2,

$$i_2 = 12A - i_1.$$

Substituting for i_2 in the v-loop eq'n:

$$i_1 \cdot 12\Omega - (12A - i_1)(\underbrace{1\Omega + 3\Omega + 2\Omega}_{6\Omega}) = 0V$$

or $i_1 (12\Omega + 6\Omega) = 12A \cdot 6\Omega$

or $i_1 = 12A \cdot 6\Omega / 18\Omega = 4A$

Using an earlier eqn:

$$i_2 = 12A - i_1 = 12A - 4A = 8A$$

From earlier eqn:

$$v_o = i_o \cdot 3\Omega = i_2 \cdot 3\Omega = 8A \cdot 3\Omega = 24V$$

b) Power dissipated: $p = i \cdot v = i^2 R$ for R's

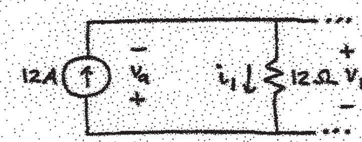
12Ω: $p = i_1^2 \cdot 12\Omega = 4A^2 \cdot 12\Omega = 192W$

1Ω: $p = i_3^2 \cdot 1\Omega = (-8A)^2 \cdot 1\Omega = 64W$

3Ω: $p = i_o^2 \cdot 3\Omega = (8A)^2 \cdot 3\Omega = 192W$

2Ω: $p = i_2^2 \cdot 2\Omega = (8A)^2 \cdot 2\Omega = 128W$

For the current source, we find the voltage drop from a v-loop on the left side.



We have $-v_a - v_1 = -v_a - i_1 \cdot 12\Omega = 0V$

$$v_a = -i_1 \cdot 12\Omega = -4A \cdot 12\Omega = -48V$$

Power for 12A src: $p = 12A \cdot v_a = 12A(-48V)$

$$p = -576W$$

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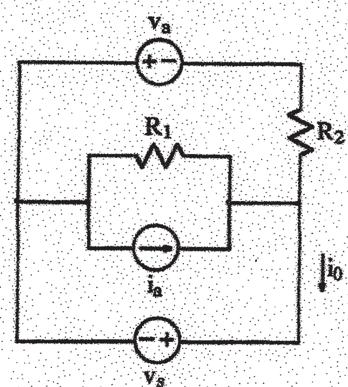
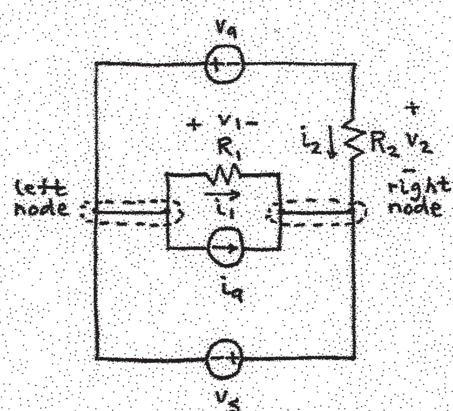
HOMEWORK #2 Example #3

EX:



Derive an expression for i_0. The expression must not contain more than the circuit parameters v_a, v_s, i_a, R_1, and R_2.

sol'n: Label R's first.



v-loops: (v_a and R_1, R_2; v_s and R_1)

$$-v_a - v_2 + v_1 = 0V$$

$$-v_1 - v_s = 0V.$$

We observe that we can solve these 2 eq'ns in 2 unknowns without proceeding further.

$v_1 = -v_s$ from 2nd eq'n

$v_2 = -v_a + v_1 = -v_a - v_s$ from 1st eq'n

Note: If we try to write current-sum eq'ns, we find that the left node and right node and connected by only v-src v_s. Thus, we should not write i-sum eq'ns. (And we don't need them!)

Note: We also have no components in series that carry the same current, (except v-src v_a and R_2).

We now use Ohm's law:

$v_1 = i_1 R_1$

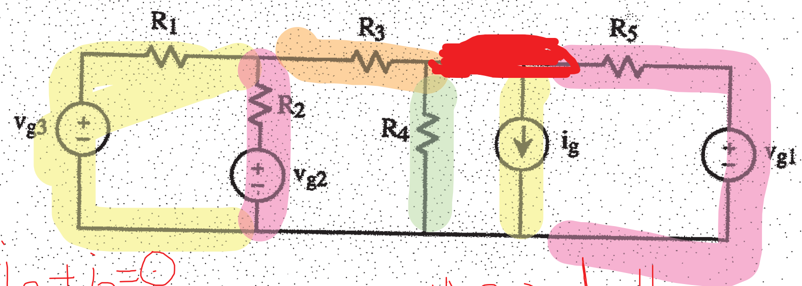
$v_2 = i_2 R_2$

Using v-eq'ns:

$$i_1 = \frac{v_1}{R_1} = \frac{-v_s}{R_1}$$

$$i_2 = \frac{v_2}{R_2} = \frac{-v_a - v_s}{R_2}$$

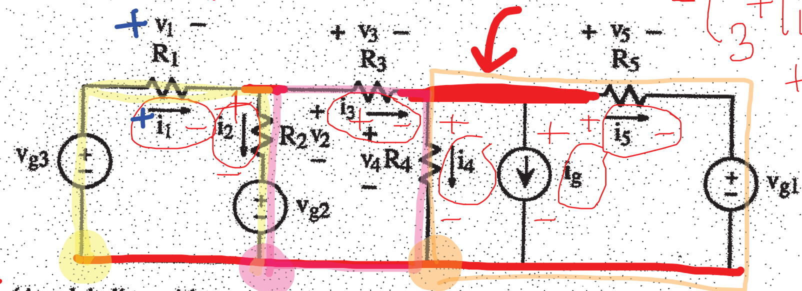
EX: In the circuit below, label currents in each element and then label potential differences according to the passive sign convention.

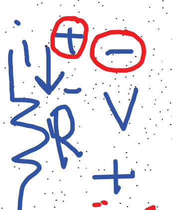


$-i_1+i_2+i_3=0$ junction "node"

$-i_3+i_4+i_g+i_5=0$

ANS:





$V=-iR$

(Any labeling with current measurement arrows pointing from + signs to - signs of voltage measurements is valid.)

$+V_{g3}-V_1-V_2-V_{g2}=0$

$V_1=+i_1R_1$

SOL'N: The answer is not unique. We may choose either direction for the + and - of the voltage measurement for each resistor and then label the current measurement with an arrow pointing from the + sign to the - sign. Or we may choose either direction for the current measurement arrow for each resistor and then label the voltage measurement with the + and - sign such that the current measurement arrow points toward the - sign. What matters is the consistency of voltage and current measurements with each other, not the actual direction of current flow or the sign of actual voltage.

When we deal with a source, we normally leave it as is and avoid adding labels to it. Sometimes, however, we may wish to calculate the power dissipation for a source. In that case, we take the + and - sign or the arrow as the direction of one measurement, and we make the other measurement consistent with it.

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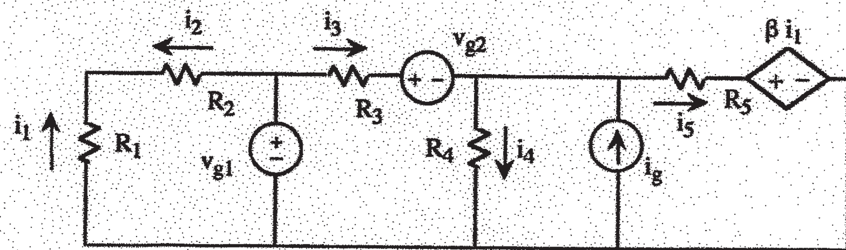
Thus, if we were to measure current in the voltage sources for the above circuit, the arrows would all point down. If we were to measure voltage in the current source for the above circuit, the + sign would be on top and the - sign would be on the bottom.

It often turns out to be convenient to label resistors with the current measurement arrow pointing to the right or pointing down. This often matches the direction of the physical current flow and is often the direction we would measure the output voltage for a circuit. Later on, we will usually designate the bottom wire on a circuit diagram as the "reference" which is where the minus sign of every voltage measurement will be located.

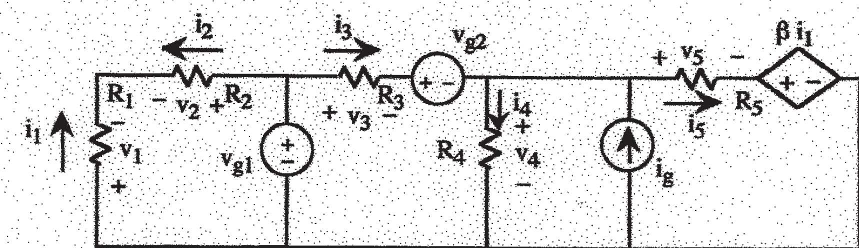
Note: we normally add voltage and current measurement labels for only the resistors in a circuit.

5

EX: In the circuit below, the currents in each element are labeled as shown. Using the passive sign convention, label the potential difference across each element and show its polarity.



ANS:



(Any labeling with current measurement arrows pointing from + signs to - signs of voltage measurements is valid.)

SOL'N: The passive sign convention dictates that the current measurement arrow always points away from the + sign and toward the - sign of the voltage measurement. Note that the current arrow and + and - sign for voltage indicate only the <u>Polarity Of</u> current and voltage <u>Measurements</u>—<u>NOT</u> the <u>direction</u> of the <u>physical flow</u> of current <u>or</u> the <u>sign</u> of the <u>actual voltage</u>. In other words, the arrows and + and - sign tell us how to connect the leads of a multimeter to make a measurement. We do not know, (nor do we have to know), in advance which direction current is flowing or what the actual sign of the voltage will be.

6

DEF: A loop is any continuous path (that may even cross gaps across open space) that ends where it starts.

NOTE: The goal of writing current summation and voltage loop equations is to obtain n equations in n unknowns that we can solve to find all the currents and voltages in a circuit.

TOOL: Loops that cross over themselves may always be treated as two smaller loops. (Smaller loops yield simpler equations.)

NOTE: We may proceed in either direction around a loop when we write a voltage loop equation, (but we must continue in the same direction all the way around the loop).

TOOL: We set the sum of voltage drops around a loop to zero. If we exit a circuit element from the + sign of the voltage measurement as we proceed around the loop, then that voltage appears with a plus sign in the loop equation. If we exit a circuit element from the – sign of the voltage measurement as we proceed around the loop, then that voltage appears with a minus sign in the loop equation.

NOTE: Using a + sign for a voltage term if we enter a circuit element from the + sign of the voltage measurement, and using a – sign for a voltage term if we enter a circuit element from the – sign of the voltage measurement yields an equation equivalent to using the opposite sign convention (as stated in the preceding tool). Multiplying one equation by –1 on both sides yields the other equation.

TOOL: We skip voltage loops where we would be forced to define a voltage for a current source. Writing an equation for such a loop adds a new variable and a new equation. Thus, we merely create more equations in more unknowns rather than moving closer to the goal of writing n equations in n unknowns.

TOOL: We write voltage loops for all inner loops, if appropriate. If we are able to write an equation for each inner loop, we have all the voltage loop equations we need. When we must skip an inner loop equation (because we would have to define a voltage for a current source), we write a voltage-loop equation for the next larger voltage loop containing some portion of that inner loop. If we must skip that next larger loop, we proceed to the next larger loop, and so on recursively. If we must skip even the largest voltage loop, then that voltage loop is unnecessary. (In that case, the other voltage-loop and current-summation equations will be sufficient to solve the circuit.)

TOOL: When necessary, we supplement voltage-loop equations with equations that equate voltages across circuit elements that are in parallel.

7

DEF: A circuit node is any point to which three or more circuit elements are attached.

NOTE: The goal of writing current summation and voltage loop equations is to obtain n equations in n unknowns that we can solve to find all the currents and voltages in a circuit.

TOOL: Nodes connected by wires are considered to be a single node.

TOOL: We set the sum of currents measured as flowing out of a node to zero. If the current measurement arrow points toward a node, that current appears with a minus sign in the current-sum equation.

TOOL: We skip current sums for nodes where we would be forced to define a current for a voltage source. Writing an equation for such a node adds a new variable and a new equation. Thus, we merely create more equations in more unknowns rather than moving closer to the goal of writing n equations in n unknowns.

TOOL: There is always at least one node we may skip when writing current-sum equations. The equation for that node would be redundant.

TOOL: When necessary, we supplement current sum equations with equations that equate currents flowing in circuit elements that are in series.

NOTE: Summing the currents flowing into a node yields an equation that is equivalent to the equation for summing the currents flowing out of that node. Multiplying one equation by -1 on both sides yields the other equation.

NOTE: Summing currents measured as flowing into a node and setting that sum equal to currents measured as flowing out of that node yields an equation equivalent to setting the sum of all currents measured as flowing out of the node to zero.

7a

Procedure Summary to find I and V:

1. a. Label every current through R's
   b. Label polarity with (-) at arrow side
2. Take a voltage loop for every possible path.

[NOTE!] → • Do not take a loop across a current source

3. Write all node current summations.

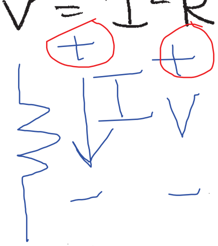
[Note!] → • Do not use a node with a branch having only a V src in it.

4. Use Ohm's law for every R.
5. Reduce equation set to solve for unknown's (i.e. use a set with common unknown variables)

---

power summary:

$$P = I * V = I^2 R \quad [W] \text{ Watts}$$

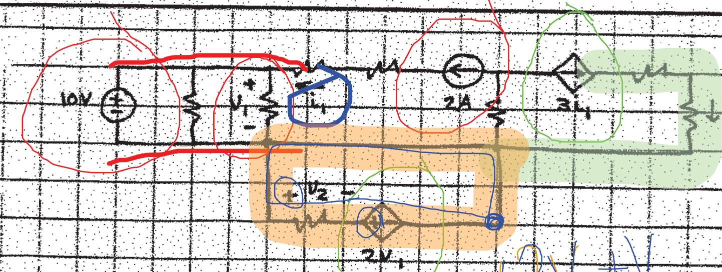


+ consuming

- generates

8

ex:



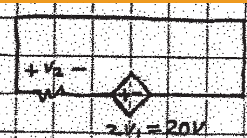
Find: v_1, v_2, i_1, i_2.

$0 = +2V_1 + V_2$

soln: $\boxed{v_1 = 10V}$ from closed loop w/ 10V source and R for v_1. True even with intervening R.

$10V - v_1 = 0\,V$

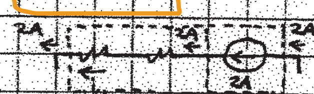
$\boxed{v_2 = -2v_1 = -20V}$ from closed loop involving only $2v_1$ source and R for v_2



$2v_1 + v_2 = 0V \quad 20 + v_2 = 0V$

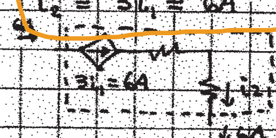
$v_2 = -20V$

$\boxed{i_1 = 2A}$



$i_1 = 2A$ because resistor for i_1 is in series with 2A source. 0A net current flows out of the dotted boxes.

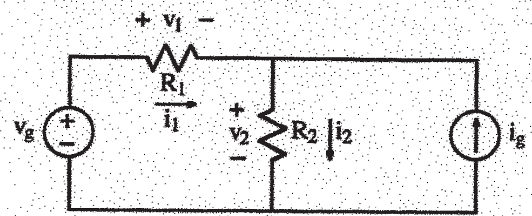
$\boxed{i_2 = 3i_1 = 6A}$



$i_2 = 3i_1$ because resistor for i_2 is in series with dependent source that creates $i = 6A$. Current flow into dotted box $= 6A$. ∴ current flow out of dotted box $= 6A$.

9

EX: In the circuit below, use Kirchhoff's voltage and current laws to write equations relating voltages and currents.



ANSWER: $-i_1 + i_2 - i_g = 0$

$v_g - v_1 - v_2 = 0$

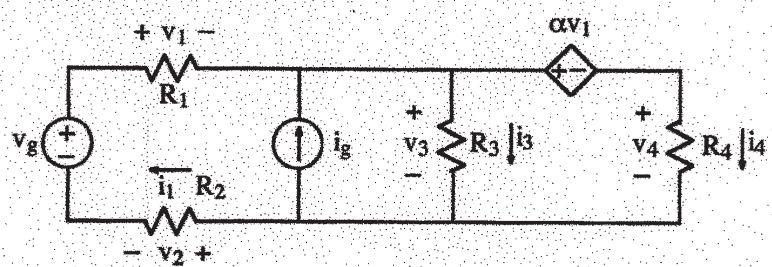
SOL'N: We sum the currents flowing out of the top center node. Writing an equation for the bottom node would be redundant. Recall that we always have one extra node.

Because writing a v-loop equation for the right inner loop would require defining a voltage for a current source, we write a v-loop equation for only the left loop. Note that the only larger loop containing the right inner loop would also require defining a voltage for the current source. Thus, a voltage loop equation for the right side is unnecessary.

Our voltage loop on the left starts from the lower left and proceeds in a clockwise direction. We may start voltage loops wherever we desire, but being consistent tends to improve accuracy.

10

EX: In the circuit below, use Kirchhoff's voltage and current laws to write equations relating voltages and currents.



ANSWER: $-i_1 - i_g + i_3 + i_4 = 0$

$+v_g - v_1 - v_3 - v_2 = 0$

$+v_3 - \alpha v_1 - v_4 = 0$

SOL'N: We avoid labels defining the current for a voltage source or the voltage for a current source. Thus, we look for nodes where every branch has a labeled current, (i.e., contains at least one resistor or current source, as opposed to only v-sources), and v-loops where the loop where every element has a labeled voltage, (i.e., without current sources).

We sum the currents flowing out of the top center node. (Writing an equation for the bottom node would be redundant.) Note that, because a wire connects them, we consider the two top-center nodes as a single node. (We may redraw the circuit with the wire collapsed to a point.) Note also that current i_1 continues around the loop to flow through R_1 from left to right, and current i_4 flows through the dependent source.

Because writing v-loop equations for the left or center inner loops would require defining a voltage for a current source, we write a v-loop equation for the next larger loop that goes around the current source. We also write a v-loop equation for the inner loop on the right.

Our voltage loops start from the lower left and proceed in a clockwise direction. We may start voltage loops wherever we desire, but being consistent tends to improve accuracy.

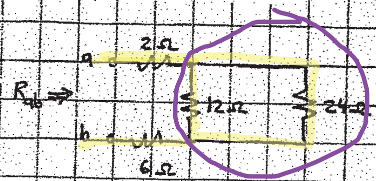
CONCEPTUAL TOOLS

By: Neil E. Cotter

Ref: Nilsson & Riedel, Rev 6, Prob 3.6a

CIRCUITS
OHM'S LAW
Resistor networks
EXAMPLE 1

11

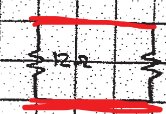


series and parallel R's

Find equivalent resistance R_{ab} looking into a,b.

sol'n:

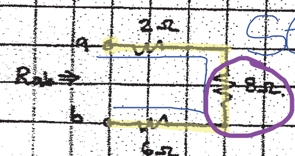
parallel



$$12\Omega \parallel 24\Omega = 12\Omega \cdot 1 \parallel 2 = 12\Omega \cdot \frac{1 \cdot 2}{1+2} \qquad \frac{1}{\frac{1}{12}+\frac{1}{24}}$$

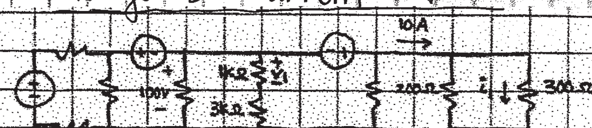
$$= 12\Omega \cdot \frac{2}{3} = 8\Omega$$

series



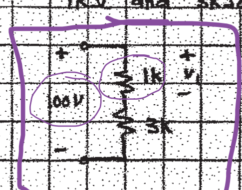
$$R_{ab} = 2\Omega + 8\Omega + 6\Omega = \boxed{16\Omega}$$

Voltage & Current Divider



a) Use V-divider to find v_1.

sol'n: We are given 100V drop across R to left of 1kΩ and 3kΩ. ∴ we have 100V drop across 1kΩ and 3kΩ in series.

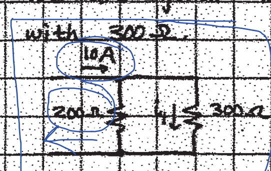


By V-divider formula,

$$v_1 = 100V \cdot \frac{1k\Omega}{1k + 3k\Omega} = 100V \cdot \frac{1}{4} = 25V$$

b) Use i-divider to find i_1.

sol'n: We are given 10A thru 200Ω in parallel with 300Ω.



By i-divider formula,

$$i_1 = 10A \cdot \frac{200\Omega}{200\Omega + 300\Omega} = 10A \cdot \frac{2}{5} = 4A$$