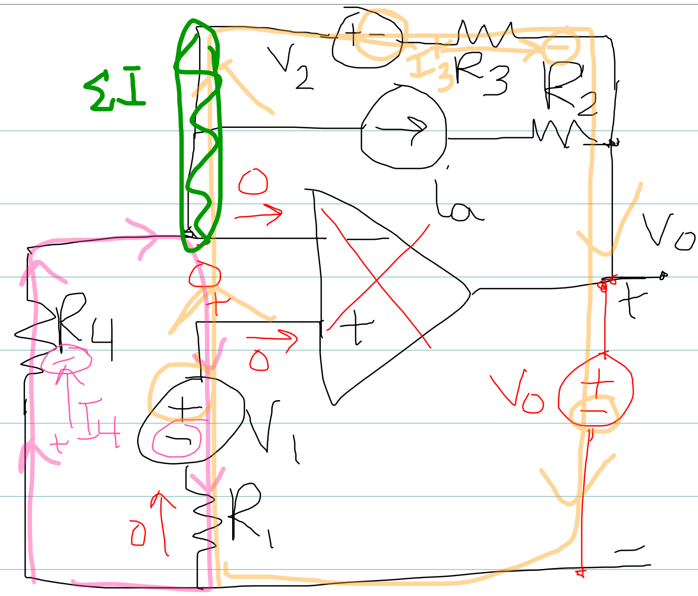


$+V_1 - V_2 - I_3 R_3 - V_o = 0$

$V_o = V_1 - V_2 - I_3 R_3$

$-I_4 R_4 - V_1 = 0 \rightarrow I_4 = \frac{-V_1}{R_4}$

$\Sigma I: +i_a + I_3 - I_4 = 0$

$I_3 = -i_a + I_4$

$$\boxed{V_o = V_1 - V_2 - \left(-i_a - \frac{V_1}{R_4}\right) R_3}$$

Review Solution

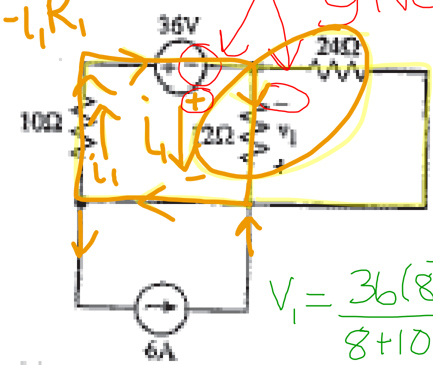
56

1. a. (5 points)

Calculate v_1.

$V_1 = -i_1 R_1$

same gives +



$24 \| 12 = \frac{24(12)}{24+12} = 8\,\Omega$

$V_1 = \frac{36(8)}{8+10} = 16V$

Sol'n: The 6A source across the wire may be ignored. Its current flows through the wire but produces no V-drop. Without the 6A src we have a V-divider:

$v_1 = 36V \cdot 12\Omega \| 24\Omega \,/\, 12\Omega \| 24\Omega + 10\Omega = 36V \cdot 8\Omega / 18\Omega = 16V$

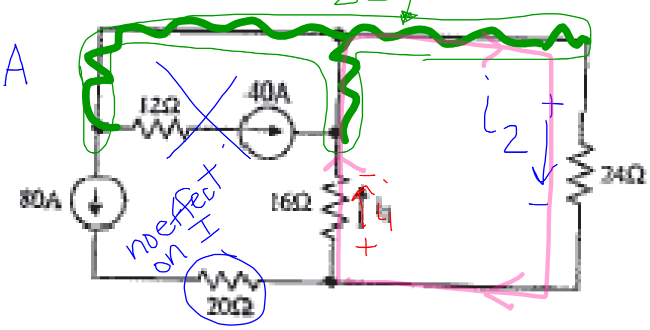
$\boxed{v_1 = 16V}$

b. (5 points)

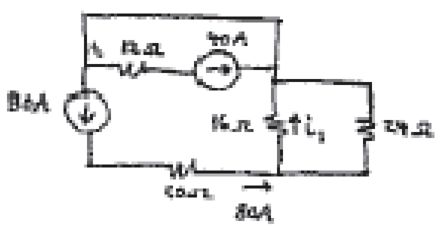
Calculate i_1.

$i_1 = \frac{+80(24)}{24+16} = 48A$

ΣI



Sol'n: If we redraw the circuit, we see a current divider:



$i_1 = 80A \cdot \frac{24\Omega}{16 + 24\Omega} = 80A \cdot \frac{24\Omega}{40\Omega}$

$\boxed{i_1 = 48A}$

Exam 1
Voltage divider
Current divider
Independent source(kirchoff's laws and ohms law)
Dependent source
Op amp

2. (30 points)

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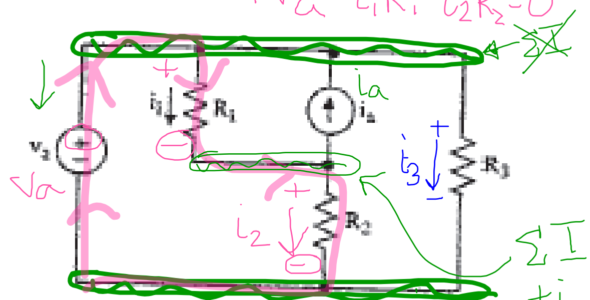
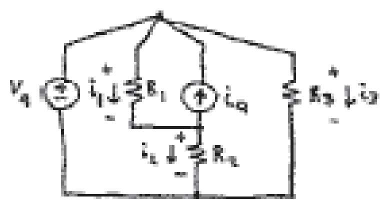
Derive an expression for i_1. The expression must not contain more than the circuit parameters v_a, i_a, R_1, R_2, and R_3. $+V_a - i_1 R_1 - i_2 R_2 = 0$



ΣI
$+i_a + i_2 - i_1 = 0$

soln: Redraw with top as one node:



Current sum at top or bottom node? No, because we would have to define a current for source V_a.

Current at center node: $i_a - i_1 + i_2 = 0\,A$

V-loop around left inner loop: $V_a - i_1 R_1 - i_2 R_2 = 0V$

No V-loop for other inner loops because we would have to define V-drop for i_a.

Next larger loop is R_1, R_2, R_3: $i_2 R_2 + i_1 R_1 - i_3 R_3 = 0V$

Now we have 3 eq'ns in 3 unknowns, and we want to find i_1. We observe, however, that the first two eq'ns have only two unknowns. So we don't actually need the 3rd eq'n. Use 1st eq'n to find $i_2 = i_1 - i_a$.

Substitute into 2nd eq'n: $v_a - i_1 R_1 - (i_1 - i_a) R_2 = 0V$

or $i_1(-R_1 - R_2) = -v_a - i_a R_2$ or $\boxed{i_1 = \dfrac{v_a + i_a R_2}{R_1 + R_2}}$

power $= IV = \alpha i_1 \cdot V_x$
?
$= \alpha i_1 \cdot (-V_a) = \alpha \left(\frac{-V_a}{R}\right)(-V_a)$
consuming

3. (30 points) $+V_a + V_x = 0$
$V_x = -V_a$

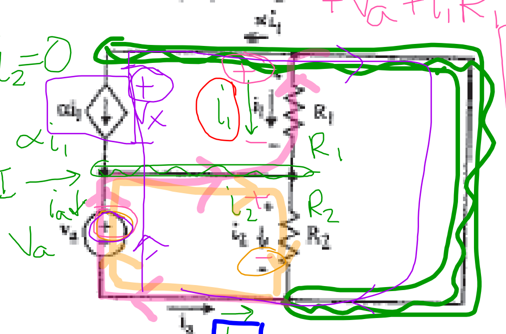
58

a. Derive an expression for i_a. The expression must not contain more than the circuit parameters α, v_a, R_1, and R_2.

$+V_a + i_1 R_1 = 0$

$i_1 = \frac{-V_a}{R}$

$-\alpha i_1 - i_1 + i_a + i_2 = 0$

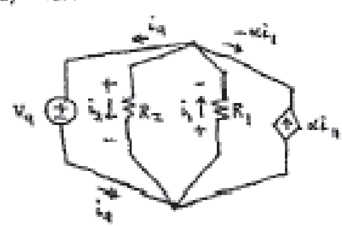


$+V_a - i_2 R_2 = 0$

$i_2 = \frac{V_a}{R_2}$

b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

soln: a) Redraw circuit



No current sums at nodes because of V_a.

V- loop on left: $V_a - i_2 R_2 = 0V \Rightarrow i_2 = \frac{V_a}{R_2}$

V- loop in middle: $\underbrace{i_2 R_2}_{= \frac{V_a}{R_2}} + i_1 R_1 = 0V \Rightarrow i_1 = -\frac{V_a}{R_1}$

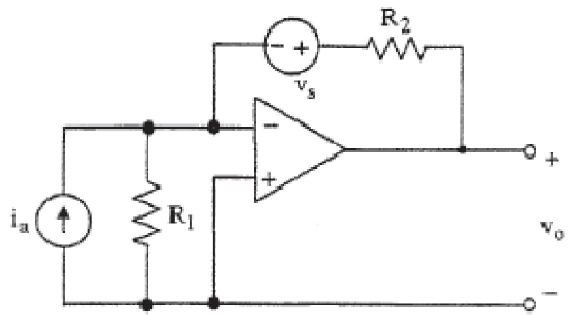
we could also just observe that V_a is across R_1 and R_2.

Now that we have found i_1 and i_2, we use a current at top node to find i_a:

$i_a + i_2 - i_1 - \alpha i_1 = 0A$ or $i_a + \frac{V_a}{R_2} + \frac{V_a}{R_1} + \alpha \frac{V_a}{R_1} =$

or $\boxed{i_a = -V_a\left(\frac{1}{R_2} + \frac{1}{R_1} + \frac{\alpha}{R_1}\right)}$ or $i_a = -\frac{V_a}{R_1 \| \frac{R_1}{\alpha} \| R_2}$

The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for v_o in terms of not more than v_s, i_a, R_1, and R_2.



sol'n: Redraw without op-amp and 0V drop across + and − inputs:

$+i_1R_1 = 0$

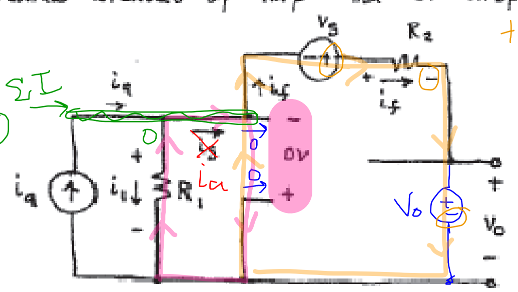
$i_1 = 0$

ΣI

$-i_a + i_f = 0$

$i_f = i_a$

$+V_S - i_f R_2 - V_o = 0$



V-loop on left thru R_1 and 0V drop:

$$i_1 R_1 + 0V = 0V \quad \text{or} \quad i_1 = 0$$

Current sum at node above R_1:

$$-i_a + \underset{0}{\underset{\shortparallel}{i_1}} + i_s = 0A \quad \text{or} \quad i_s = i_a$$

V-loop on right thru 0V drop, v_s, R_2, and v_o:

$$-0V + v_s - i_f R_2 - v_o = 0V \quad \text{or} \quad i_f = \frac{v_s - v_o}{R_2}$$

Now use $i_s = i_f$.

$$i_a = \frac{v_s - v_o}{R_2}$$

Thus $v_s - v_o = i_a R_2$

or $\boxed{v_o = v_s - i_a R_2}$