3.\hspace{1cm}

Homework #6/Review Examples

3. \hspace{1cm}

\begin{center}
\begin{circuitikz}
\draw (0,0) to [v sources] (0,1) to [r] (-1,0) to [c] (0,0);
\end{circuitikz}
\end{center}

a) Calculate the value of $R_L$ that would absorb maximum power.
b) Calculate that value of maximum power $R_L$ could absorb.
Using superposition, derive an expression for $v$ that contains no circuit quantities other than $i_s$, $v_s$, $R_1$, $R_2$, and $\beta$. Note: $\beta > 0$.

**Solution case I:** $i_s$ on, $v_s$ off = wire

Current summation at center node gives

$$-i_s + i_{x1} + \frac{\beta i_{x1}}{R_2} = 0A$$

or

$$i_{x1} (1 + \frac{\beta}{R_2}) = i_s$$

$$i_{x1} = \frac{i_s}{1 + \frac{\beta}{R_2}} = i_s \cdot \frac{R_2}{R_2 + \beta}$$

$$v_1 = \beta i_{x1} = \beta i_s \cdot \frac{R_2}{R_2 + \beta} = i_s \cdot R_2 \parallel \beta$$
Solt: 5, cont. case I: $i_S$ off, $V_S$ on

\[ V \text{ loop around outside gives} \]

\[ \beta \cdot i_{x_2} - i_{x_2} \cdot R_2 - V_S = 0 \]

\[ i_{x_2} (\beta + R_2) = V_S \]

\[ i_{x_2} = \frac{V_S}{\beta + R_2} \]

\[ V_2 = -i_{x_2} \cdot R_2 = -V_S \cdot \frac{R_2}{\beta + R_2} = -V_S \cdot \frac{R_2}{R_2 + \beta} \]

\[ V = V_1 + V_2 = \beta \cdot \frac{i_S \cdot R_2}{R_2 + \beta} - \frac{V_2}{R_2 + \beta} = (\beta i_S - V_S) \cdot \frac{R_2}{R_2 + \beta} \]

\[ V = (\beta i_S - V_S) \cdot \frac{R_2}{R_2 + \beta} \]
Using superposition, derive an expression for $v_l$ that contains no circuit quantities other than $i_s$, $v_s$, $R_1$, $R_2$, and $\beta$, where $\beta > 0$. 
The short created by the switch creates a voltage loop on top left with OV across C and across the 20kΩ resistor.

Thus $V_C(t^-) = 0V = V_C(t^+)$.

Thus $V_C(t^+) = 0V$.

Note: The units for energy are Joules.

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Write a numerical expression for $V_C(t)$ for $t > 0$.

Solution: We use general form of solution for RC circuits.

$$v(t^+) = v(t^-) = v(t^-) - v(t^-) e^{-t/RC}$$

We find $v(t^+)$, $v(t^-)$, and $R_{eq}$.

We start at $t = 0^-$ to find voltage on $C$ at $t = 0^+$.

$t = 0^-$: $C$ acts like an open circuit.

Switch is closed.

Switch creates a short circuit, and $V_C(t^-) = 0V$.
\( v(t) = 6V \cdot \frac{12}{11} \)

\( t = \infty: \) Switch is open; \( C = \) open circuit.

\[ v(\infty) = \frac{6V \cdot 12}{11} = 6.99V \]

No current can flow through the 240 kΩ resistor.

\[ v(t=\infty) = 6V \]

\( \text{R}_{12}: \) We look in from the terminals where \( C \) is connected, and we turn off the current source.

\[ R_{12} = 20kΩ \]

\( R_{13} = 120kΩ + 240kΩ = 360kΩ \)

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b) For \( i_c(t=0^+) \), we use the general form of solution for RL problems:

\[ i_c(t) = i_c(t=0^+) e^{-t/\tau} \]

\( t = 0^+ \): We model \( L \) as an ideal with

\[ v_2 = 0 \]

Switch is closed.

\[ \text{Since the quantity we are looking for is } i_c(0^+), \text{ we do not have to solve the circuit, but this is the circuit we would use.} \]

\[ t = \infty: \text{ L acts like wire} \]

Switch is closed.
Then we use a voltage divider formula:

\[ V_1 = \frac{V_0 R_1}{R_1 + R_2 + R_3} \]

\[ V_1 = \frac{V_0 R_1}{R_1 + R_2 \left(1 + \frac{R_0}{R_2}\right)} \]

We divide \( V_1 \) by \( R_1 \) to find \( i_L(t+\omega) \):

\[ i_L(t+\omega) = \frac{-V_0}{R_1 + R_2 \left(1 + \frac{R_0}{R_2}\right)} \]

This answer is equivalent to our previous answer.

We turn off the \( V_0 \) source and look in from the terminals where \( L_1 \) is connected. Switch is closed.

\[ R_{th} = R_1 + R_2 + R_3 \]

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**Example:**

- **a)** \( R_1 \approx R_{th} \) for max power transfer.

We find \( R_{th} \) by looking into the terminals where \( R_L \) is connected (but without \( R_L \)) with the two independent sources turned off.

\[ R_{th} = 750 \Omega + 150 \Omega \]

\[ R_{th} = 150 \Omega \]

\[ \therefore R_L \approx 150 \Omega \]

- **b)** max power = \( \frac{(V_{th}/2)^2}{R_{th}} \)

We find \( V_{th} \) as the open circuit voltage across the terminals where \( R_L \) is connected.

We find \( V_{th} \) by using superposition.

**Case 1: 30 V on, 180 mA off**

\[ V_{th} = 30 \text{V} \]

Since no current flows, there is no voltage across the \( R_L \).

\[ V_{th} = -30 \text{V} \cdot (-150 \Omega) \]

Notes: Since we will ignore \( V_{th} \), the polarity we choose for measuring \( V_{th} \) doesn’t matter.
Case 1: 3V off, 150mA on

\[
\begin{align*}
\text{No current flows in the} & \quad 750\Omega \text{ and } 100\Omega. \\
\text{Thus, there is no voltage across these resistors.} & \\
\text{The voltage drop across the } 100\Omega \text{ is equal to } V_{100}. & \\
V_{100} = 150\text{mA} \cdot 100\Omega & = 15\text{V}.
\end{align*}
\]

We can now use KVL to find \( V_{100} \).

\[
\begin{align*}
V_{100} &= V_{100} + V_{150} & \\
V_{100} &= -3\text{V} + 15\text{V} & = 12\text{V}. \\
\text{max power} &= \left(\frac{V_{100}}{R_{100}}\right)^2 & = \frac{12\text{V}}{1\text{k}} & = 36\text{mW.}
\end{align*}
\]

\[
\begin{align*}
\text{Case 1: } V_{100} \text{ on, } i_{100} \text{ off.} & \\
\text{We turn on one source at a time.} & \\
\text{(Never turn off dependent source.)} & \\
\text{Case 1: } V_{100} \text{ on, } i_{100} \text{ off.} & \\
\text{The dependent source is equivalent} & \\
to \quad R_{eq} = \frac{V}{i} = \frac{V_{100}}{i_{100}} &= \frac{12\text{V}}{150\text{mA}} & = \frac{1}{12}. \\
\text{or } i_{12} &= i_{100} \frac{R_{100}}{R_{100} + R_{1}} \frac{R_{1}}{R_{1} + 1\text{M}} & \\
\text{or } i_{12} &= i_{100} \frac{R_{100}}{R_{100} + R_{1}(R_{1} + 1\text{M})} & \\
\text{or } i_{12} &= i_{100} \frac{R_{100}}{R_{1} + R_{100}(R_{1} + 1)} & \\
\text{We can find } i_{11} \text{ and } i_{12} \text{ to get } i_{11} & \\
i_{11} = i_{10} + i_{12} & \\
or i_{1} = -\frac{V_{100}}{R_{1} + R_{100}} + i_{100} \frac{R_{100}}{R_{1} + R_{100}(R_{1} + 1\text{M})} & \\
\text{Note: When a current source is off it becomes an open circuit.} & \\
\text{When a voltage source is off it becomes a wire.} &
\end{align*}
\]