Problem Session #1 Problems:

1. Calculate $v_1$.

Solution: $v_1 = 16V$

2. Calculate $i_1$.

Solution: $i_1 = 48A$

3. Derive an expression for $i_1$. The expression must not contain more than the circuit parameters $v_0$, $i_2$, $R_1$, $R_2$, and $R_3$.

Solution: $i_1 = \frac{v_0}{R_1 + R_2} + i_2 \frac{R_2}{R_1 + R_2}$

4. Derive an expression for $i_4$. The expression must not contain more than the circuit parameters $\beta$, $v_1$, $R_1$, and $R_2$.

Solution: $i_4 = \frac{v_1}{R_1 + R_2}$

5. The op-amp operates in the linear mode. Using an appropriate model of the op amp, derive an expression for $v_0$ in terms of not more than $v_0$, $i_2$, $R_1$, and $R_3$.

Solution: $v_0 = v_0 - i_2 R_2$
1. a. (5 points)
Calculate $v_1$.

**Solution:**
The 6A source across the wire may be ignored. Its current flows through the wire but produces no V-drop. Without the 6A src we have a V-divider:

$$v_1 = \frac{36V \times 12Ω}{12Ω + 10Ω + 24Ω} = 6V, \quad \frac{v_1}{36V} = \frac{1}{6}$$

b. (5 points)
Calculate $i_1$.

**Solution:**
If we redraw the circuit, we see a current divider:

$$i_1 = \frac{80A \times 4Ω}{8Ω + 24Ω} = \frac{28A}{3}$$

2. (30 points)

Derive an expression for $i_4$. The expression must not contain more than the circuit parameters $v_4$, $v_3$, $R_1$, $R_2$, and $R_3$.

**Solution:**
Redraw with top as one node:

Current sum at top or bottom node? No, because we would have to define a current for source $v_4$.

Current at center node: $i_4 - i_3 + i_5 = 0A$

V-loop around left inner loop: $v_4 - i_3R_1 - i_2R_2 = 0V$

No v-loop for other inner loops because we would have to define V-drop for $i_4$.

Next larger loop is $R_1, R_4, R_3$: $i_3R_1 + 1R_4 - 1R_3 = 0V$

Now we have 3 eqns in 3 unknowns, and we want to find $i_3$. We observe, however, that the first two eqns have only two unknowns. So we don't actually need the 3rd eqn. Use 1st eqn to find $i_3 = i_2 = i_4$.

Substitute into 2nd eqn: $v_4 - i_4R_1 - (i_4 - i_3)R_3 = 0V$

or $i_1(\frac{R_3}{R_3} - R_2) = -v_4 - i_4R_2$ or $i_1 = \frac{v_4 + i_4R_2}{R_3}$.
3. (30 points)

a. Derive an expression for \( i_a \). The expression must not contain more than the circuit parameters \( \alpha, v_a, R_1, \) and \( R_2 \).

![Circuit Diagram]

b. Make at least one consistency check (other than a units check) on your expression. Explain the consistency check clearly.

**Solution:**

Redraw circuit

No current sums at nodes because of \( v_a \).

**V-loop on left:** \( v_a - i_a R_2 = 0 \) \( \Rightarrow i_a = \frac{v_a}{R_2} \)

**V-loop in middle:** \( i_{12} R_3 + i_{12} R_1 = 0 \) \( \Rightarrow i_{12} = -\frac{v_a}{R_1} \)

we could also just observe that \( v_a \) is across \( R_1 \) and \( R_2 \).

Now that we have found \( i_1 \) and \( i_2 \), we use a current at top node to find \( i_a \):

\[ i_a + i_2 = i_i - i_1 \quad \text{GA} \quad \text{or} \quad i_a = \frac{v_a}{R_2} + \frac{v_a}{R_1} \]

or

\[ i_a = \frac{-v_a}{R_1 + R_2} \quad \text{or} \quad i_a = -\frac{v_a}{\frac{1}{R_1} + \frac{1}{R_2}} \]

**Solution:** 3.b.

Many possible answers.

**Example:**

Suppose \( \alpha = 0 \). Choose other simple values:

\[ v_a = -12V, \quad R_1 = 2\Omega, \quad R_2 = 2\Omega, \quad R_3 = 1\Omega \]

We see that \( i_a \) is current thru \( R_1 R_2 \)

\[ R_1 R_2 = 1\Omega \cdot 2\Omega = \frac{12V}{3A} = \frac{2}{3} \Omega \]

\[ i_a = -\frac{v_a}{R_1 R_2} = -\frac{12V}{\frac{2}{3} \Omega} = -18A \]

Use formula from (2) with these component values:

\[ i_a = -12V \left( \frac{\frac{1}{2\Omega} + \frac{1}{2\Omega} + \frac{1}{1\Omega}}{2\Omega + 2\Omega + 1\Omega} \right) = -12V \cdot \frac{3}{2} \]

\[ i_a = -18V \checkmark \]

agrees with obvious soln for the simple case.
4. (30 points)

The op-amp operates in the linear mode. Using an appropriate model of the op-amp, derive an expression for \( v_o \) in terms of no more than \( v_i \), \( i_2 \), \( R_1 \), and \( R_2 \).

**Solution:** Redraw without op-amp and cv drop across + and - inputs:

V-loop on left thru \( R_1 \) and cv drop:
\[ i_1 R_1 + v_0 = 0 \quad \text{or} \quad i_1 = 0 \]

Current sum at node above \( R_1 \):
\[ -i_1 + i_2 + i_3 = 0 \quad \text{or} \quad i_2 = i_3 \]

V-loop on right thru cv drop, \( v_2 \), \( R_2 \), and \( v_o \):
\[ -v_2 + i_2 R_2 - v_o = 0 \quad \text{or} \quad i_2 = \frac{v_o - v_2}{R_2} \]

Now use \( i_3 = i_2 \).
Thus \( v_o - v_2 = i_2 R_2 \)
\[ i_3 = \frac{v_o - v_2}{R_2} \quad \text{or} \quad v_o = v_2 - i_3 R_2 \]