The switch has been closed for a long time before opening at time \( t=0 \).

Note: Whenever a circuit configuration has existed "for a long time," it means forever—from \( t=-\infty \) to \( t=0 \). Thus, all currents and voltages have achieved final values. In other words,

\[
\frac{di}{dt} = 0 \quad \text{for all currents}, \quad \frac{dv}{dt} = 0 \quad \text{for all voltages}.
\]

a) Find \( i_1(t=0^-) \) and \( i_2(t=0^-) \).

**Solu'n:** At \( t=0^- \), the circuit has had the same configuration "for a long time," and \( \frac{di_1}{dt} = 0 \), \( \frac{di_2}{dt} = 0 \).

For the \( L \) we have \( \frac{v_L}{dt} = L \cdot \frac{di}{dt} = 0 \Rightarrow v_L = 0V \).

Thus, the \( 30mH \) \( L \) looks like a wire (as \( L \)'s always do for their final values) and our circuit model is:

We get \( v_1 \) from \( V \)-divider:

\[
v_1 = 9V \cdot \frac{15k\Omega}{15k\Omega + 15k\Omega} = \frac{3V}{15k\Omega} = 0.2\ mA
\]

\( i_1(t=0^-) = i_2(t=0^-) = \frac{v_1}{15k\Omega} = \frac{3V}{15k\Omega} = 0.2\ mA \)
b) Find \( i_1(t=0^+) \) and \( i_2(t=0^+) \).

\[ i_1(t=0^+) \text{ and } i_2(t=0^+) \]

\( \text{Solution:} \) With the switch open, the 9V and 15kΩ before the switch no longer affect \( i_1 \) and \( i_2 \). Our model (for \( i_1 \) and \( i_2 \)) becomes:

\[ \begin{align*}
   & i_1(t) \rightarrow 15k\Omega \\
   & i_2(t) \rightarrow 30m\Omega \\
   & \text{since the current through the } L \text{ cannot change instantly, we must have } \ i_1(t=0^+) = i_1(t=0^-). \\
   \text{But now we also have the two 15kΩ } R \text{'s in series. Thus, } \ i_2 = -i_1 = -i_1(t=0^-). \\
   \therefore \ i_1(t=0^+) = 0.2mA \quad i_2(t=0^+) = -0.2mA.
\end{align*} \]

\( \text{Note: The current in an } R \text{ can change instantly, as happens here (with } i_2 \text{ changing sign).} \)

\( \text{c) Find } i_1(t) \text{ for } t \geq 0 \)

\( \text{Solution:} \) Our circuit model (with 15kΩ R's summed) is:

\[ \begin{align*}
   & 30k\Omega \\
   & 30m\Omega \quad V_L
\end{align*} \]

\( V_L = L \frac{di_1}{dt} \) and \( V_L = -i_1 \cdot 30k\Omega \) by Ohm's law

\[ \therefore -i_1 \cdot 30k\Omega = L \frac{di_1}{dt} \quad \text{or} \quad i_1 \cdot R + L \frac{di_1}{dt} = 0 \ V \\
\]

\( \therefore -i_1 \cdot 30k\Omega = L \frac{di_1}{dt} \quad \text{or} \quad i_1 \cdot \frac{R}{dt} + L \frac{di_1}{dt} = 0 \ V \\
\]

\( \text{Note: we used sum of } V\text{'s around loop } = 0V \)

\( \text{From text p. 279, } \ i_1(t) = i(0^+) e^{-t \frac{R}{L}} = 0.2mA \)

\[ \therefore i_1(t) = 0.2mA \cdot e^{-t \cdot \frac{R}{L}} \]
d) Find \( i_2(t) \) for \( t \geq 0^+ \)

Sol'n: From part (a) we have \( i_2(t) = -i_1(t) \) for \( t \geq 0^+ \)

\[
  i_2(t \geq 0^+) = -0.2 \text{ mA} \cdot e^{-\frac{t}{0.4}}
\]

e) Explain why \( i_2(0^-) \neq i_2(0^+) \).

Answer: As noted in the solution to (b), the current in an \( R \) can change instantly. It does so here to keep \( i_1(t) \) the same when the switch opens.

Note: The \( R \) current can change instantly because an \( R \) stores no energy. It takes time to change the energy stored in an \( L \), and energy \( = \frac{1}{2}L_i^2 \).
The switch in the circuit has been in position 'a' for a long time before it changes to position 'b' at $t=0$.

a) Find $i$, $v_1$, and $v_2$ for $t \geq 0^+$.

At $t=0^-$ we have $\frac{dv}{dt} = 0$ both $C_1$.

$	herefore$ $i = 0$ for both $C_1$.  $\therefore$ $C_1$'s look like open circuits

Since no current flows on the left side, there is no $V$-drop across the 4.7kΩ R. Thus, $v_C(t) = 75V$.

When we throw the switch at $t=0$, the left side 75V and 4.7kΩ no longer affect $i$, $v_1$, and $v_2$.

We combine the two $C_1$'s in series into one $C_{eq} = \frac{2\mu F \parallel 8\mu F}{10\mu F}$

$C_{eq} = \frac{2\mu F \cdot 8\mu F}{16\mu F} = 1.6\mu F.$

The initial voltage on $C_{eq}$ is $v_1(0^+) = v_2(0^+) = v_C(0^-) - v_2(0^-)$, since $V$ across $C_1$'s cannot change instantly.

Now our equivalent circuit is an RC with initial voltage $v_C(t) - v_2 = 75V - 0 = 75V$. 

...
We sum currents flowing out of node between $R$ & $C$.

Instead of summing $V$'s around loop as we did for $RL$ problems.

In other words, we equate the $i$ in the $R$ & $C$.

\[-i = C \frac{dv}{dt} \quad \text{or} \quad i = \frac{v_2 - v_1}{R}\]

\[- \frac{v}{R} = -C \frac{dv}{dt} \quad \text{or} \quad \frac{v}{R} + C \frac{dv}{dt} = 0A\]

As shown on p. 287 of text, the solution is

\[v(t) = v(t=0^+) \cdot e^{-t/RC_{eq}} = 75V \cdot e^{-t/5kΩ \cdot 1.6μF} = 75V \cdot e^{-t / 8 \text{ms}}\]

\[i(t) = \frac{v(t)}{R} = \frac{75V \cdot e^{-t / 8 \text{ms}}}{5kΩ} = 15 \text{mA} \cdot e^{-t / 8 \text{ms}}\]

Now that we have $i(t)$, we can integrate $i(t)$ to find $v_1(t)$ and $v_2(t)$.

\[-i(t) = C_1 \frac{dv_1}{dt} \quad \Rightarrow \quad i(t) = C_2 \frac{dv_2}{dt}\]

\[-\int i(t')dt' = C_1 \int v_1(t')dt' = C_2 \int v_2(t')dt' \quad \Rightarrow \quad v_1(t=0^+)\quad \text{and} \quad v_2(t=0^+)\]

\[
\begin{align*}
\text{Given:} & \quad V_1(t) = 60V \left( e^{-t/8\text{ms}} - 1 \right) + 75V \\
& \quad V_2(t) = -75V \left( e^{-t/8\text{ms}} - 1 \right)
\end{align*}
\]
Switch has been closed for a long time before opening at time \( t = 0 \).

\( \text{a) Find initial value of } i_0(t). \)

\( \text{sol'n: If switch was closed for a long time, then the capacitor will discharge through } 6.8 \text{k}\Omega. \)

\[
\begin{array}{c}
\text{(Right side of circuit, see 'note' below.)}
\end{array}
\]

Note: The closed switch creates a short circuit. The left and right sides of the circuit may then be treated as though they are totally independent. Why? Because the currents flowing through the short create no V drop. Thus, the mesh currents on the sides of the short do not interact.

Thus, at \( t = 0^- \) the \( C \) has no charge, and \( v(t) = 0 \).

\( \therefore \) \( C \) acts like short at \( t = 0^- \). Since \( v(t) \) cannot change instantly, \( v(t = 0^+) = 0 \text{V}, \) too.

Also, we replace the 75V and 4k\Omega and 16k\Omega with a Thévenin equivalent. \( V_{\text{Th}} \) from open-circuit \( V \)-divider is \( 75V \cdot \frac{16k\Omega}{16k\Omega + 4k\Omega} = 60V = V_{\text{Th}}. \)

Turn 75V down to 0V and connect 1V source to output to get \( R_{\text{Th}} = 1V/1V/(4k\Omega || 16k\Omega) = 4k\Omega || 16k\Omega \).
\[ R_{TH} = 4 \text{k}\Omega \cdot \frac{1}{4} = 4 \text{k}\Omega \cdot 4 = 3.2 \text{k}\Omega \]

Circuit model for \( t=0^+ \) is:

\[ \begin{align*} & \text{60V} \quad \uparrow \\ & \text{3.2k}\Omega \quad 6.8k\Omega \\
\hline
& + \\
& \frac{v_0(t)}{i_0(t)} \\
\hline
& \text{0V (C like short circuit)} \end{align*} \]

\[ i_o(t=0^+) = \frac{60V}{3.2k\Omega + 6.8k\Omega} = \frac{60V}{10k\Omega} = 6mA \]

b) Find \( i_o(t \to \infty) \).

\[ \text{sol'n: When the C is charged, it looks like open circuit.} \]

Note: \( i_o(t \to \infty) = 0 \text{A} \) for all \( C \)'s in all switching prob.

\[ V_c(t=0^-) = V_c(t=0^+) \quad \text{in in all switching prob.} \]

But \( V_c(t=0^-) = 0 \text{V} \) only if circuit discharges \( C \) completely for \( t<0 \). Otherwise, \( V_c(t=0^-) \) is some nonzero value.

If \( C \) is open circuit, then \( i_o(t \to \infty) = 0 \text{A} \).

c) Find time constant for \( t \geq 0 \).

\[ \text{sol'n: } \tau = \text{Reg} \cdot C \quad \text{Reg} = 10\text{k}\Omega = 3.2\text{k}\Omega + 6.8\text{k}\Omega \]

\[ = 10\text{k}\Omega \cdot 0.2\mu\text{F} \quad C = 0.2\mu\text{F} \]

\[ = 2 \text{ms} \]

d) Find expression for \( i_o(t) \) when \( t \geq 0^+ \)

\[ \text{sol'n: General sol'n for } i_o(t) \text{ is:} \]

\[ i_o(t) = i_o(t=0^+) \frac{1 - e^{-t/RC}}{} \]

\[ = 6 \text{mA} + [0 - 6\text{mA}] [1 - e^{-t/2\text{ms}}] \]

\[ = 6 \text{mA} e^{-t/2\text{ms}} \quad t \geq 0^+ \]
e) Find expression for \( v_0(t) \) when \( t \geq 0^+ \).

**Sol'n:**

\[ v_0(t) = 60V - i_0(t) \cdot 3.2k\Omega = 60V - 6mA \cdot e^{-\frac{t}{2ms}} \cdot 3.2k\Omega \]

\[ = 60V - 19.2V e^{-\frac{t}{2ms}} \quad t \geq 0^+ \]
ex:

No energy stored in \( L_1 \) and \( L_2 \) when switch opens.

a) Find \( i_1(t \geq 0) \) and \( i_2(t \geq 0) \).

\[ i_1(t \geq 0) = I_g \frac{L_2}{L_1 + L_2} \left(1 - e^{-\frac{t}{\tau}}\right) \]

where \( \tau = \frac{L_1 + L_2}{R_g} \).

\[ i_2(t \geq 0) = I_g \frac{L_1}{L_1 + L_2} \left(1 - e^{-\frac{t}{\tau}}\right) \]

b) \( i_1(t \to \infty) = I_g \frac{L_2}{L_1 + L_2} \)

\( i_2(t \to \infty) = I_g \frac{L_1}{L_1 + L_2} \)

soln: a) Take Thévenin equivalent of \( I_g \) and \( R_g \) on left.

Solve for \( v \) across \( L' \)s by replacing \( L' \)s with equivalent \( L \):

\[ v(t \geq 0) = \frac{I_g}{R_g} \left(1 - e^{-\frac{t}{\tau}}\right) \]

\( L' \)s in parallel give \( L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \).

(\( L' \)s in parallel are like \( R' \)s in parallel in terms of the formula we use.)

Now we use the general solution for \( v(t \geq 0) \):

\[ v(t \geq 0) = v(t \to \infty) + \left(v(0^+) - v(t \to \infty)\right) e^{-\frac{t}{\tau}} \]

To find \( v(0^+) \), we use \( i_1(0^+) = i_1(0^-) \) and \( i_2(0^+) = i_2(0^-) \). But \( i_1(0^-) = i_2(0^-) = 0 \) since no energy is stored in \( L_1 \) and \( L_2 \) at \( t = 0 \).
a) cont.

Since \(i_1(0^+)=0\) and \(i_2(0^+)=0\), we must have no current through \(R_g\) at \(t=0^+\).

\[\therefore \text{At } t=0^+, \text{ we have no } v \text{ drop across } R_g.\]

\[\therefore v(t=0^+) = V_{Th} = I_g R_g\]

For \(v(t \to \infty)\) we observe that the L's act like wires, and \(v(t \to \infty) = 0\).

Plugging into the general soln gives

\[v(t \geq 0) = I_g R_g \ e^{-t/(\text{Log}/R_g)}\]

Note: The time constant for circuit with \(L\) and \(R\) is \(\text{Log}/R_{Th}\). Taking the Thenen equivalent always gives the needed \(R\).

Now we can also write down a formula for \(i(t) = i_1(t) + i_2(t)\) for \(t \geq 0\):

\[i(t \geq 0) = i(t \to \infty) + [i(0^+) - i(t \to \infty)] \ e^{-t/(\text{Log}/R_g)}\]

Note: All \(i\)'s and \(v\)'s have same time constant.

We know \(i(0^+) = i_1(0^+) + i_2(0^+) = i_1(0^-) + i_2(0^-) = 0\).

At \(t \to \infty\), the L's act like wires, giving \(i = I_g\).

\[\therefore i(t \geq 0) = I_g \left[ 1 - e^{-t/(\text{Log}/R_g)} \right]\]

Now we determine how \(i(t \geq 0)\) is divided between the two L's to give \(i_1(t \geq 0)\) and \(i_2(t \geq 0)\).
a) cont.

Since both L's have same V across them, we have

\[ v = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt} \]

\[ \therefore \frac{di_1}{dt} = \frac{L_2}{L_1} \frac{di_2}{dt} \]

Now we calculate currents:

\[ i_1(t) = \int \frac{di_1}{dt} \, dt = \int \frac{L_2}{L_1} \frac{di_2}{dt} \, dt = \frac{L_2}{L_1} \int di_2 \]

or \[ i_1(t) = \frac{L_2}{L_1} i_2(t) \]

Also, \[ i_1(t) + i_2(t) = i(t) \]

Solving these two eqns gives

\[ i_1(t) = \frac{L_2}{L_1 + L_2} i(t) \]

\[ i_2(t) = \frac{L_1}{L_1 + L_2} i(t) \]

Thus, \[ i_1(t \geq 0) = I_0 \frac{L_2}{L_1 + L_2} \left( 1 - e^{-t/(\text{Log}/R_g)} \right) \]

\[ i_2(t \geq 0) = I_0 \frac{L_1}{L_1 + L_2} \left( 1 - e^{-t/(\text{Log}/R_g)} \right) \]

b) At \( t \to \infty \) we have \( e^{-t/(\text{Log}/R_g)} \to 0 \)

\[ \therefore i_1(t \to \infty) = I_0 \frac{L_2}{L_1 + L_2} \]

\[ i_2(t \to \infty) = I_0 \frac{L_1}{L_1 + L_2} \]