

**TOOL:** The following step-by-step procedure may be used to solve RC or RL circuit problems.

- i) Every voltage or current in an RC or RL circuit after time  $t = 0$  may be expressed in the following general form:

$$v(t) = [v(t = 0^+) - v(t \rightarrow \infty)]e^{-t/\tau} + v(t \rightarrow \infty)$$

or

$$i(t) = [i(t = 0^+) - i(t \rightarrow \infty)]e^{-t/\tau} + i(t \rightarrow \infty)$$

where  $\tau \equiv R_{Th}C_{eq}$  or  $\tau \equiv L_{eq}/R_{Th}$  as appropriate, and

$R_{Th} \equiv$  Thevenin equivalent resistant for circuit (after  $t = 0$ ) seen from terminals where  $C$  or  $L$  is connected (without the  $C$  or  $L$  present)

$C_{eq} \equiv$  equivalent capacitance of capacitors in series or parallel

$L_{eq} \equiv$  equivalent inductance of inductors in series or parallel

- ii) At time  $t = 0^-$ , assume the circuit has been stable for a long time, causing derivative values to become zero:

$$\frac{dv_C(t)}{dt} = 0 \quad \text{and} \quad \frac{di_L(t)}{dt} = 0$$

This means that  $i_C(t = 0^-) = 0$  A and  $v_L(t = 0^-) = 0$  V, which in turn means that a  $C$  looks like an open circuit and an  $L$  looks like a wire. Thus, the circuit model for  $t = 0$  has  $C$ 's replaced with open circuits and  $L$ 's replaced with wires.

- iii) Find  $v_C(t = 0^-)$  or  $i_L(t = 0^-)$ . These are the energy variables that will not change as a switch or source changes state at time  $t = 0$ , and they must be known in order to determine initial conditions for the general solution of the circuit. No other values in the circuit are guaranteed to stay the same at time  $t = 0^+$ , so find only the value of  $v_C(t = 0^-)$  or  $i_L(t = 0^-)$  at time  $t = 0^-$ .
- iv) Set  $v_C(t = 0^+) = v_C(t = 0^-)$  or  $i_L(t = 0^+) = i_L(t = 0^-)$ . At  $t = 0^+$ , model a capacitor as a voltage source with value  $v_C(t = 0^-)$ , and model an inductor as a current source with value  $i_L(t = 0^-)$ .
- v) Using the circuit model for  $t = 0^+$  with the  $C$  or  $L$  modeled as described in (iii), find the value of  $v(t = 0^+)$  or  $i(t = 0^+)$ .

- vi) For  $t \rightarrow \infty$ , assume the circuit has been stable for a long time, as in (iii), and replace  $C$ 's with open circuits and  $L$ 's with wires. Find  $v_C(t \rightarrow \infty)$  or  $i_L(t \rightarrow \infty)$ .
- vii) To find  $R_{Th}$ , remove the  $C$  or  $L$  from the circuit and find the Thevenin resistance of the circuit seen looking into the circuit from the terminals where the  $C$  or  $L$  was attached.