ECE 2100  Lecture notes  Wed, 1/22/03

Stuff
Problem Sessions:
W, 11:50 - 12:40 am, WBB 212 (tall brick geology building)
F, 10:45 - 11:35 am, MEB 1208 (by SW entrance)

Clean up some loose ends
Bode Plots
The Bode plots that we've covered in class are the simplest types and will do for our purposes. We are primarily interested in the design of amplifiers and analysis of the low and high corner frequencies. The poles and zeroes are usually far apart.

When poles and zeroes are too close to each other they can interact or even result in complex poles. Complex poles result in resonance effects like those shown at right.

If poles and zeroes are simply close but not complex, the effects of each simply add or subtract. That can throw off your assumptions (like the -3dB) around the corner frequency, etc..

Clipping
All real amplifiers will clip the output signal if the input is too big. Only one part in the model can possible account for the clipping-- a nonlinear $A_{vo}$.

Clipping level at the output ($v_{out}$) is less than that of $A_{vo}$ ($\pm$ 10V) because of the $R_o$, $R_l$ voltage divider.

The maximum allowable $v_{in} = \frac{10\cdot V}{A_{vo}}$

The maximum allowable $v_s$ is greater than this because of the $R_s$, $R_{in}$ voltage divider.

Operational Amplifiers  (Op amp)  Chapter 2 of the textbook  Start with lecture for F, 1/17

It's very difficult to make $v_a$ & $v_b$ close enough without using some negative feedback. Negative feedback makes the op-amp maintain $v_a \approx v_b$ for itself. With the proper negative feedback the op-amp keeps $v_a \approx v_b$ so close that you can usually assume that $v_a = v_b$. Without this negative feedback the op-amp output will almost certainly be at one of its limits, either high or low, i.e. NOT in its active, or linear, range. Incidentally, circuits without negative feedback are also useful, but the output is either high or low (digital) and not linearly related to the input. These types of circuits are called nonlinear circuits.
The op-amp has very high input impedance. The input currents are almost zero. As long as you use reasonable resistor values in your circuits (say 100Ω to 1MΩ), you can neglect the input currents.

The output impedance of IC op amps is not that great, but negative feedback makes it look much smaller than it really is, and it's often neglected.

Op-amps amplify DC as well as AC.

**Active (Linear) Op amp Circuits**

**Voltage follower**

Negative feedback, the entire \( v_0 \) is fed back.

\[
\begin{align*}
    v_a & \approx v_b = v_o \\
    v_o & = v_a = v_{in}
\end{align*}
\]

Normally the approximate equal sign is left out.

Also known as a buffer. This is very useful as a current amplifier. Very little (usually negligible) current flows in and significant current can flow out.

**Negative feedback**

What if: \( v_a = 1\text{ V} \) & \( v_o = 0\text{ V} \) because they're connected: \( v_b = 0\text{ V} \)

\[
100000 \cdot (v_a - v_b) = 100000 \cdot 0 = v_o
\]

So the op amp wants to make the output 100000V, and will "turn up" the output voltage as fast as it can. Very soon the output will reach 1V and the process will stop.

What if: \( v_a = 1\text{ V} \) & \( v_o = 3\text{ V} \) \( v_b = v_o \)

\[
100000 \cdot (v_a - v_b) = -200000 \cdot 0 = v_o
\]

Now the op amp wants to make the output -200000V, and will "turn down" the output voltage as fast as it can. Again, the output will soon reach 1V and the process will stop. This is how negative feedback works.

Negative feedback is an important concept. It is used in almost all systems, including all natural systems. A very simple example is the heating system in your house. If the air temperature is too low the thermostat detects a difference between its setting and the air temperature and turns on the heater. When the air temperature reaches the set temperature the thermostat turns off the heater—negative feedback.

Incidentally, a real op amp does have a limit as to how fast it can change its output voltage. This maximum is called the "slew rate" (SR) of the op amp and is generally specified in V/μs.

**Noninverting Amplifier**

Same type of negative feedback, but now only a fraction of \( v_o \) is fed back.

\[
\begin{align*}
    v_a & \approx v_b = \frac{R_1 \cdot v_o}{R_1 + R_f} \\
    v_o & = \frac{R_1 + R_f}{R_1} \cdot v_{in}
\end{align*}
\]

You can get whatever gain you want just by selecting the right resistor values.

These calculations are only valid if the op amp is in its **active** region. If \( v_{in} \) is too big, the output will be clipped and \( v_a \) will NOT equal \( v_b \). It is only reasonable to assume that the circuit might be in the active region if there is **negative** feedback. Look for a connection from the output to the inverting (-) input.

This amplifier is called "noninverting" because the output is not inverted with respect to the input. You can tell this at a glance if you see that the input is connected to the noninverting (+) input.

**Inverting Amplifier**

You still have negative feedback, but now the \( v_{in} \) is also connected to the same op-amp input.

\[
\begin{align*}
    v_b & \approx v_a = 0\text{ V} \text{, it's connected to ground} \\
    i_{in} & = \frac{v_{in}}{R_{in}} \text{ But this current can't flow into the op amp, so it must flow up through } R_f \text{ (the feedback resistor).}
\end{align*}
\]
Again, you can get whatever gain you want just by selecting the right resistor values.

Usually the noninverting input is hooked to ground through a resistor such that:

\[ R_1 \approx \frac{1}{1 + \frac{1}{R_{in}}} \]

This doesn’t affect our ideal calculations above, but does make real op amps appear more like ideal ones. We’ll revisit this again when we talk about the non-ideal effects and how to deal with them.

Summer

This is a variation of the inverting amp.

\[ \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3} = i_f = \frac{v_b - v_o}{R_f} = \frac{-v_o}{R_f} \]

OR: \[ v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) \]

Differentiator

Another variation of the inverting amp.

\[ i_C = C \frac{d}{dt} v_{in} = \frac{-v_o}{R_f} \]

OR: \[ v_o = -R_fC \frac{d}{dt} v_{in} \]

Integrator

Another variation of the inverting amp.

\[ i_{in} = \frac{v_{in}}{R_{in}} = i_C = C \frac{d}{dt} (-v_o) \]

Integrate both sides

\[ \frac{1}{R_{in}} \int v_{in} dt = C (-v_o) \]

OR: \[ v_o = -\frac{1}{R_{in} C} \left( \int v_{in} dt \right) \]

This is a nice circuit, but impractical in real life.

The slightest DC voltage at \( v_{in} \) will make the output drift to one of the rail voltages.

Even if you hooked \( v_{in} \) to ground, imperfections in the op amp would cause the same result.

A practical variant is the running average or Miller Integrator.

\( R_f \) allows the capacitor charge to leak off.

This and all the previous circuits could also be looked at in terms of steady-state AC analysis:

\[ i_{in} = \frac{v_{in}}{Z_{in}} = i_f = \frac{-v_o}{Z_f} \]

OR: \[ v_o = \frac{Z_f}{Z_{in}} v_{in} \]

The - sign means a 180° phase change.
Active Filters
By using various networks of resistors and capacitors in place of $Z_1$ and $Z_f$ you can make all sorts of excellent filters without using inductors. That’s good because real inductors are expensive and not nearly as ideal as real capacitors.

In the case of the Miller integrator:

$$v_o = \frac{Z_f}{Z_1} v_{in} = -\frac{1}{R_f + j\omega C} v_{in} = -\frac{1}{\frac{R_f}{R_{in}} + j\omega C R_f} v_{in} = \frac{R_f}{R_{in}} \frac{1}{1 + j\omega C R_f} v_{in}$$

A single-pole low pass filter with: $\omega C = \frac{1}{C R_f}$

A good reference: Active Filter Cookbook by Don Lancaster, published by Howard W. Sams & Co..

Differential amplifier

$$v_a = \frac{R_2}{R_1 + R_2} v_2 = v_b$$

$$v_b = \frac{R_2}{R_1 + R_2} (v_1 - v_o) + v_o$$

They must be equal:

$$\frac{R_2}{R_1 + R_2} (v_1 - v_o) + v_o = \frac{R_2}{R_1 + R_2} v_2$$

$$R_2 (v_1 - v_o) + v_o (R_1 + R_2) = R_2 v_2$$

$$R_2 v_1 - R_2 v_o + v_o R_1 + v_o R_2 = R_2 v_2$$

$$R_2 v_1 + v_o R_1 = R_2 v_2$$

OR:

$$v_o = \frac{R_2}{R_1} (v_2 - v_1)$$

Or, Calculated by superposition:

$$v_o = \frac{R_2}{R_1 + R_2} v_2$$

What remains is just a noninverting amplifier

$$v_{o2} = \frac{R_1 + R_2}{R_1} v_a = \frac{R_1 + R_2}{R_1} \frac{R_2}{R_1 + R_2} v_2 = \frac{R_2}{R_1} v_2$$

Input resistance problem

Input resistance for $v_2$ is: $R_1 + R_2$

Input resistance for $v_1$ is: $R_1$ but may change if $v_2$ changes.

What if $v_a = v_1$? Then $v_b = v_1$ and then the input resistance for $v_1$ is $\infty$.

One cure for this variable input resistance is to buffer both inputs, and this will indeed make a good differential amplifier.

However, varying the gain of this amplifier would involve changing two resistors at a time.
Instrumentation amplifier

The circuit below is only slightly more complex, but offers a simple way to change the gain.

To find the gain of the left part of the circuit:

\[
\frac{v_{o1} - v_{o2}}{v_{1} - v_{2}} = I \left( \frac{R_2 + R_1 + R_2}{R_1} \right)
\]

and:

\[
v_{1} - v_{2} = I \cdot R_1
\]

so:

\[
\frac{v_{o1} - v_{o2}}{v_{1} - v_{2}} = \frac{I \left( R_2 + R_1 + R_2 \right)}{I \cdot R_1} = 1 + \frac{2 \cdot R_2}{R_1} = \frac{v_{o2} - v_{o1}}{v_{2} - v_{1}}
\]

\[
v_{o} = \left( 1 + \frac{2 \cdot R_2}{R_1} \right) R_4 \left( v_{2} - v_{1} \right)
\]

Uses

So where might you use one of these instrumentation amplifiers? Here’s an example of where I personally used one. The objective was to add force sensing to a robot gripper. The sensors were called "load cells" and are based on a full wheatstone-bridge of strain gages. Let’s look at strain and strain gages first:

Stain is the stretch of a material under stress (forces)

\[
\frac{\Delta L}{L} = \text{strain}
\]

\[
\text{stress} = \frac{F}{A}
\]

\[A = \text{cross-sectional area}\]

A strain gage is a thin backing material with a wire and two contacts.

So, as the wires in a strain gage are stretched, the resistance goes up

4 strain gages are usually wired into wheatstone bridges, like the one shown below, where each of the resistors is a strain gage attached to a beam. This helps cancel temperature dependencies.
A load cell generally contains 4 strain gages in a full bridge. The load cells we used were shaped like washers. They react to an axial force.

Two load cells were placed under the gripper "fingers". Robot gripper movement

If the gripper has a part, the load-cell force goes down. If the part hits something as it’s being inserted, the force goes up.

I used an instrumentation amplifiers to amplify the differential voltage (V_{ab}) from each load cell.

**Nonlinear Circuits**

You may get the idea from the previous circuits that v_a = v_b in all cases, but that is not always true-- the op amp is not always in its active region. Not only that, but you can make useful circuits even if the op amp can only switch between the two rail voltages.

**Comparator**

Without negative feedback the op amp is simply a comparator.

If v_a > v_b then v_o = L+, the positive output limit (+ rail).

If v_a < v_b then v_o = L-, the negative output limit (- rail).

You now have a digital output that tells you which voltage is greater.

If v_a = v_b the output will switch at the slightest noise on the inputs. The next circuit will fix that.

**Schmitt Trigger**

Now we’ve got some positive feedback. This causes a “toggle” action or hysteresis in the switching. As soon as the output switches, the switching level changes as well.

The switching level is:

\[
\frac{R_1}{R_1 + R_2} V_o + \text{ if } v_o \text{ is } +, \text{ if } v_o \text{ is } -
\]

There are other ways to make Schmitt triggers, but all have some positive feedback.

**Other Nonlinear Circuits**

You’ll see more in the Nonlinear Op Amp Circuits lab.

**References** for practical circuits


*The Art of Electronics* by Horowitz and Hill published by Cambridge University Press

ECE 2100 Lecture notes Wed, 1/22/03  p6 Next time: Non-ideal effects