Operational Amplifiers

An *operational amplifier* is basically a complete high-gain voltage amplifier in a small package. Op-amps were originally developed to perform mathematical operations in analog computers, hence the odd name. They are now made using integrated circuit technology, so they come in the typical multi-pin IC packages. With the proper external components, the operational amplifier can perform a wide variety of "operations" on the input voltage. It can multiply the input voltage by nearly any constant factor, positive or negative, it can add the input voltage to other input voltages, and it can integrate or differentiate the input voltage. The respective circuits are called *amplifiers*, *summers*, *integrators*, and *differentiators*. Op-amps are also used to make active frequency filters, current-to-voltage converters, voltage-to-current converters, current amplifiers, voltage comparators, etc. etc.. These little parts are so versatile, useful, handy, and cheap that they're kind of like electronic Lego blocks — although somewhat drably colored.

Op-amp characteristics

Operational amplifiers have several very important characteristics that make them so useful:

1. An op-amp has *two* inputs and it amplifies the *voltage difference* between those two inputs. These two inputs are known as the noninverting input, labeled (+), and the inverting input, labeled (-), as shown in Fig. 1. The output voltage is a function of the noninverting input voltage minus the inverting input voltage.
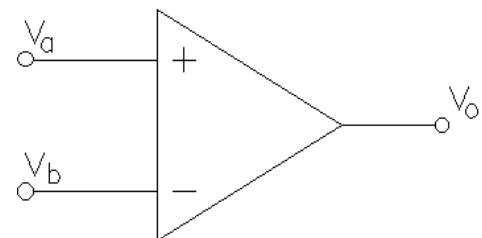
Figure 1 Op-amp symbol

That is: $v_o = G(v_a - v_b)$ Where G = voltage gain of the op-amp.

2. The op-amp must be connected to external sources of power (not shown on the drawing above). The output voltage (v_O) cannot be more positive than the positive power source or more negative than the negative power source. The gain (G) is very high, typically more than 100,000. Together that means that if the output (v_o) is in the *active* range (somewhere between its physical limits, often called "rails"), then $v_a - v_b \approx 0$, and $v_a \approx v_b$. This is a very important point. If you don't see this, look back at the equation above, v_O is limited, G is very big, so ($v_a - v_b$) must be very small.

If the output is:	The inputs must be:
In active range	$v_a \approx v_b$
- rail < v_O < +rail	

If the inputs are:	The output must be:
$v_a > v_b$	+ rail
$v_a < v_b$	- rail

3. In fact, $v_a - v_b$ must be so small that it's very difficult to make v_a & v_b close enough

without using some *negative feedback*. Negative feedback makes the op-amp maintain $v_a \approx v_b$ for itself. With the proper negative feedback the op-amp keeps $v_a \approx v_b$ so close that you can assume that $v_a = v_b$. Without this negative feedback the op-amp output will almost certainly be at one of its limits, either high or low, i.e. NOT in its active, or *linear*, range. Incidentally, circuits without negative feedback are also useful, but then the output is either high or low (digital) and not linearly related to the input. These types of circuits are called *nonlinear* circuits.

4. Op-amps amplify DC as well as AC.

5. The input currents are almost zero. In more technical terms, the op-amp has very high input impedance. As long as you use reasonable resistor values in your circuits (say $\leq$ 1 MΩ), you can neglect the input currents.
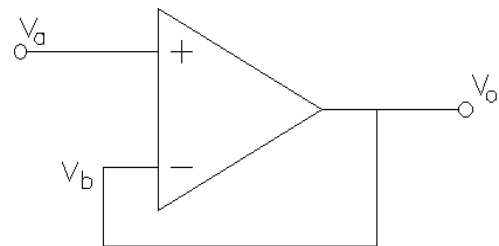
Simple, isn' t it? OK, so it doesn't sound so simple yet, but the application of these characteristics really isn't hard. Let's look at some circuits.

Linear Circuits
Linear circuits employ negative feedback to keep $v_a \approx$ v_b. If a circuit has a connection from the output to the inverting (-) input, then it has negative feedback.

Figure 2 Voltage follower

Voltage follower
The voltage follower shown in Fig. 2 is probably the simplest linear op-amp circuit. Notice the feedback from the output to the inverting (-) input. If we were to hook the circuit input (v_a) up to some voltage source, say 2 volts DC, what would happen? If the output was lower than 2 V, then the input voltage difference (v_a - v_b) would be positive and the huge gain of the op-amp would drive the output higher. If the output was higher than 2 V, then v_a - v_b would be negative the output would go down. Very quickly the output voltage v_o would change until v_a - v_b becomes very small. Or basically, until v_o = 2 V. This is the concept of negative feedback! A fraction (in this case all) of the output voltage is "fed back" to the input in order to control the gain of the op-amp. The op-amp works very hard to maintain a very small difference between the voltages on its inputs. This circuit is known as a voltage follower because the output "follows" the input.

Negative feedback is an important concept. It is used in almost all systems, including all natural systems. A very simple example is the heating system in your house. If the air temperature is too low the thermostat detects a difference between its setting and the air temperature and turns on the heater. When the air temperature reaches the set temperature the thermostat turns off the heater—negative feedback. The servo system that you've seen in lab is another example of negative feedback. When the motor position sensor senses a different position than the input position sensor the circuit makes the motor turn in such a way that the difference is minimized and the positions line up.

Noninverting amplifier

Now suppose we feed back only a fraction of the output voltage rather than all of it. The method used for this is shown in Fig. 3. R_1 and R_f constitute a voltage divider. Remember, the current flowing into an op-amp input is virtually nil, so we can neglect its effect on the voltage divider. This is one of the very nice features of an op-amp. In this circuit, as in the voltage follower, the op-amp works very hard to keep v_a - v_b very small. Only now v_b is a fraction of v_o and the op-amp has to make v_o that much larger.
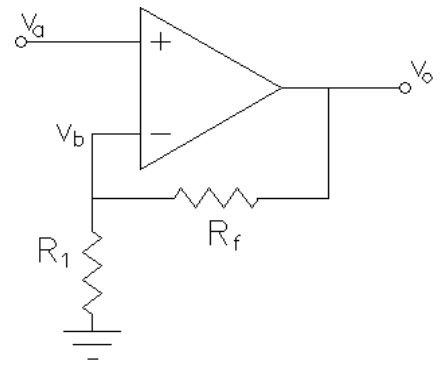
Figure 3 Noninverting amplifier

$$v_{in} = v_a \approx v_b = \frac{R_1}{R_1 + R_f} v_o$$

For all practical purposes:

$$v_o = \frac{R_1 + R_f}{R_1} v_{in} = (1 + \frac{R_f}{R_1}) v_{in}$$

Notice that by adjusting the ratio of R_f and R_1, we can make the gain of the op-amp circuit almost anything we want. Isn' t that neat? The circuit in Fig. 3 is called a noninverting amplifier because the output voltage is in phase with the input voltage; that is, it is *not* inverted. When the input voltage increases, the output voltage will also increase and vice-versa. Yes, noninverting is a double negative and kind-of a dumb name.

Inverting amplifier

Before going on, observe that I've swapped the positions of the two inputs(- & +) on my op-amp symbol. Either way of drawing the op-amp is OK, whatever makes the whole schematic look better. The noninverting input is on the bottom in this case because it's hooked to ground. Draw the op-amp so that the surrounding circuitry is clear.
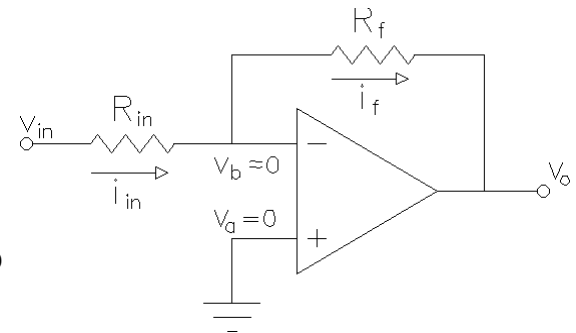
Figure 4 Inverting amplifier

The op-amp output is still connected to the inverting (-) input, so again we have negative feedback. If $v_b > v_a$ then the output will go down, taking v_b with it, until $v_b \approx v_a$. If $v_b < v_a$ then the output will go up until $v_b \approx v_a$. Negative feedback makes the op-amp do its best to equalize its inputs. In this circuit $v_a = 0$, which means that the op-amp will try to keep $v_b \approx 0$ as well. The current into the op-amp is zero, so i_{in} and i_f must be the same ($i_f = i_{in}$). Using these two ideas together:

$$i_{in} = \frac{v_{in} - 0}{R_{in}} = i_f = \frac{0 - v_o}{R_f} \qquad v_o = -\frac{R_f}{R_{in}} v_{in}$$

The minus sign means that v_o will be inverted with respect to v_{in}, hence the name of this amplifier. When v_{in} is positive, v_o is negative, and when v_{in} is negative, v_o is positive. The gain of the inverting amplifier, like that of the noninverting amplifier, is completely dependent on our choices of R_f and R_{in}.
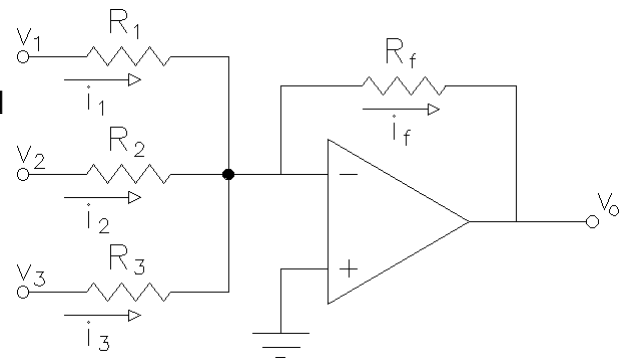
Summer

The inverting amplifier can also be used as a summing amplifier; that is, it can be made to add the effects of several input voltages together. Look at the circuit in Fig. 5.

Figure 5 Summer

$$i_f = i_1 + i_2 + i_3$$

$$i_f = -\frac{v_o}{R_f} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

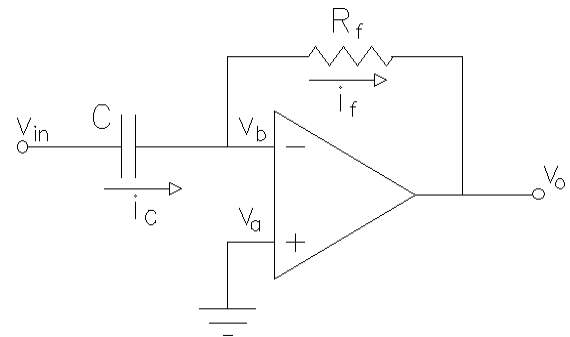$$v_o = -\frac{R_f}{R_1}v_1 - \frac{R_f}{R_2}v_2 - \frac{R_f}{R_3}v_3$$

The summer can be expanded to any number of inputs. See? This is getting easier.

Differentiator

The differentiator looks an awful lot like the inverting amplifier, and is analyzed in a very similar way.

$$i_C = C\frac{dv_{in}}{dt} = i_f = -\frac{v_o}{R_f}$$

$$v_o = -CR_f\frac{dv_{in}}{dt}$$

Figure 6 Differentiator

Integrator

Another useful op-amp circuit is the integrator, shown in Fig. 7. For this circuit:

$$i_{in} = \frac{v_{in}}{R_{in}} = i_C = -C\frac{dv_o}{dt}$$

$$v_o = -\frac{1}{CR_{in}}\int v_{in}dt$$
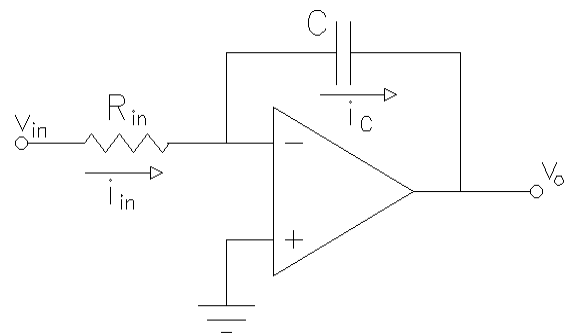
Figure 7 Integrator

Unfortunately, The simple integrator does have one little practical problem. Notice that if the input voltage has any dc component, the output voltage will soon try to run off to infinity. (Actually it will stop when the op-amp reaches one of its output limits, either negative or positive.) A resistor is usually placed in parallel with the capacitor to eliminate this rather annoying effect. The circuit in Fig. 8 has such a resistor. This is a *running-average* or *Miller* integrator.
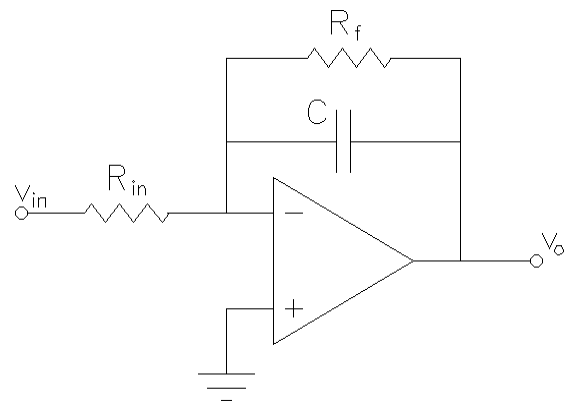
Figure 8 Practical integrator

Active Filters

If you replace the resistors in the inverting and noninverting amplifiers with frequency dependant impedances (capacitors and/or inductors), you can make all sorts of frequency dependant circuits, including filters. In fact, the differentiator and integrator circuits can be thought of as filters.

One of the main advantages of active filters is that you don't need to use inductors. Real inductors are far from ideal, as you've no doubt observed in lab. Real capacitors are much closer to ideal capacitors and they're cheaper than inductors. Entire books are devoted to these *active* filters and we won't cover them any further here.

Differential amplifier

This circuit amplifies only the difference between the two inputs. In this circuit there are two resistors labeled R_{in}, which means that their values are equal. Same goes for the two R_f's.

$$v_a = \frac{R_f}{R_{in}+R_f}v_2 = v_b = \frac{R_f}{R_{in}+R_f}(v_1-v_o) + v_o$$

$$\frac{R_f}{R_{in}+R_f}v_2 = \frac{R_f}{R_{in}+R_f}(v_1-v_o) + \frac{R_{in}+R_f}{R_{in}+R_f}v_o$$

$$R_f v_2 = R_f v_1 - R_f v_o + R_{in} v_o + R_f v_o$$

$$R_f v_2 = R_f v_1 + R_{in} v_o$$
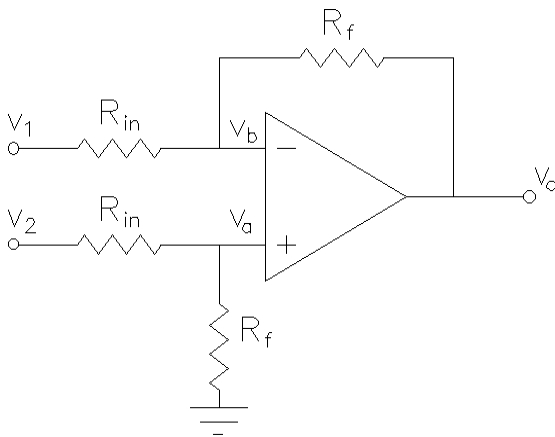
$$v_o = \frac{R_f}{R_{in}}(v_2 - v_1)$$

Figure 10 Differential amplifier

Don't confuse the differential amplifier with the differentiator. The differential amplifier amplifies the <u>difference</u> of two inputs while the differentiator amplifies the <u>slope</u> of an input.

Instrumentation Amplifier

The differential amplifier isn't really very practical. The current that flows into the top input depends on the voltage applied to the bottom input. This may not seem that bad, but it is. It means that the input characteristics of this circuit are not constant. One way to get around this would be to place a voltage follower on each input, as shown here.
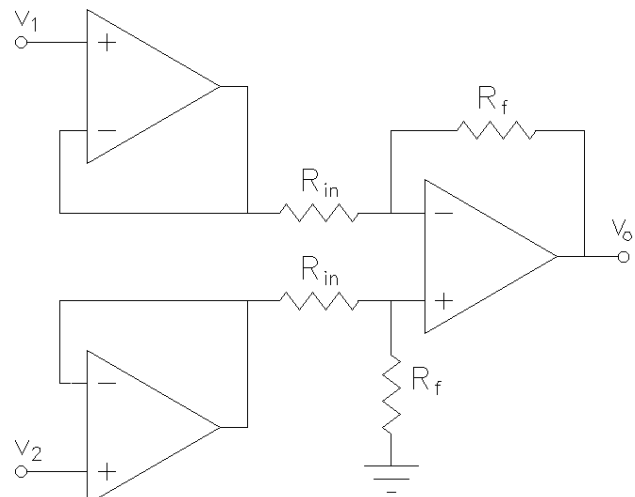
$$v_o = \frac{R_f}{R_{in}}(v_2 - v_1)$$

Figure 9 Buffered differential amplifier

Now this is a perfectly good circuit. If the two R_fs are closely matched and the two R_{in}s are also closely matched, then this circuit will amplify differential voltages very well and reject *common* voltages (a voltage that is common to both inputs should subtract out of the equation. In EE terms, it has a good Common-Mode-Rejection-Ration (CMRR).

But what if you want to change the gain? You'd have to change two resistors at the same time. By adding two more matched resistors and variable resistor we'll get the instrumentation amplifier shown at right. The equation for this circuit is:

$$v_o = (1 + \frac{2R_2}{R_1}) \frac{R_4}{R_3} (v_2 - v_1)$$

This is an important circuit and you will probably see it again many times. For instance, if you had to amplify the output of a wheatstone bridge of stain gages, this would be the amp for the job.
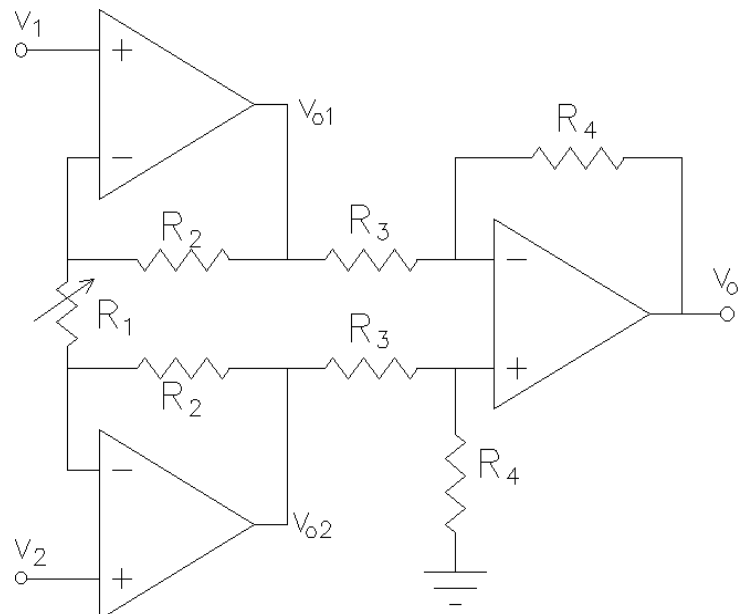
Figure 11 Instrumentation amplifier

Op-amp with extra current amplification
Most op-amps cannot supply much current to the load. They are often limited to 10 or 20 mA, about enough to light an LED, but not much more. That can be very limiting. The circuit at right shows a quick and dirty way to use two transistors to greatly increase the load current (at a small cost in output voltage swing). Notice that the feedback is taken from the output of the transistors, so they sort-of become part of the op-amp and the op-amp will do a pretty good job of eliminating the "crossover" dead-zone that occurs as one transistor turns off and the other turns on.
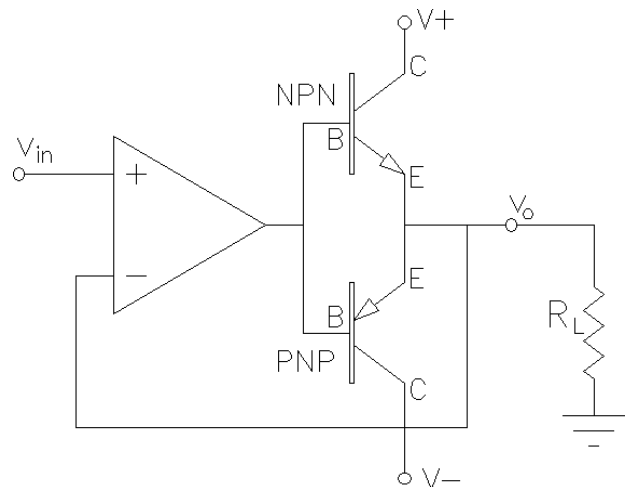
Figure 12 How to get more current

This particular circuit is a simple voltage follower. You can adapt this same current amplification to most of the other op-amp circuits that we have discussed. A few words of warning, however. The extra delay in the feedback can result in instabilities. Try it with the parts you intend to use before you depend on this design. Also, if you use a low quality op-amp (with a slow slew rate) you can get significant crossover distortion.

Nonlinear Circuits

In all cases so far, the feedback signal (voltage) has been applied to the inverting (-) input of the op-amp. This means that the feedback is negative. Negative feedback tends to reduce the difference between the v_a and v_b voltages and make linear circuits. Without negative feedback the op-amp cannot minimize the difference between v_a and v_b and the very high sensitivity of the op-amp results in *switching*, or *nonlinear* circuits.

Comparator

Now look at Fig. 14. This circuit will **not** work as a linear circuit. If $v_a > 0$ the output will be as high as the op-amp can make it, usually a volt or two below the positive power supply. If $v_a < 0$ the output will be as low as the op-amp can make it, usually a volt or two above the negative power supply. The output is no longer linearly related to the input– it's more like a digital signal, high or low depending on how v_{in} compares to ground (0 V). The comparator is a *nonlinear* circuit.
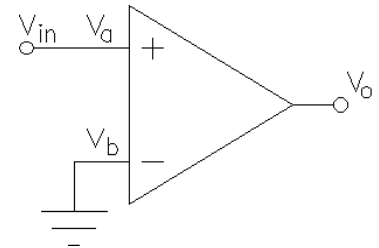
Figure 13 Comparator
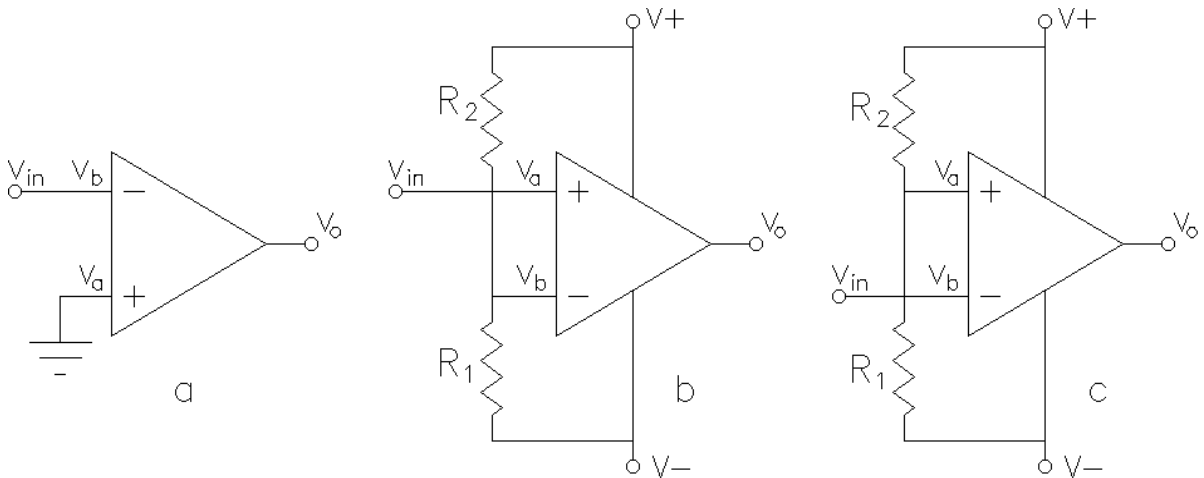
Figure 14 Other comparators

All the circuits above are also comparators. In the first circuit, the input is again compared to ground, but this time the output goes low when the input goes high and vise-versa. In the remaining circuits the input is compared not to 0 V, but to some voltage set by the voltage divider of R_1 and R_2.
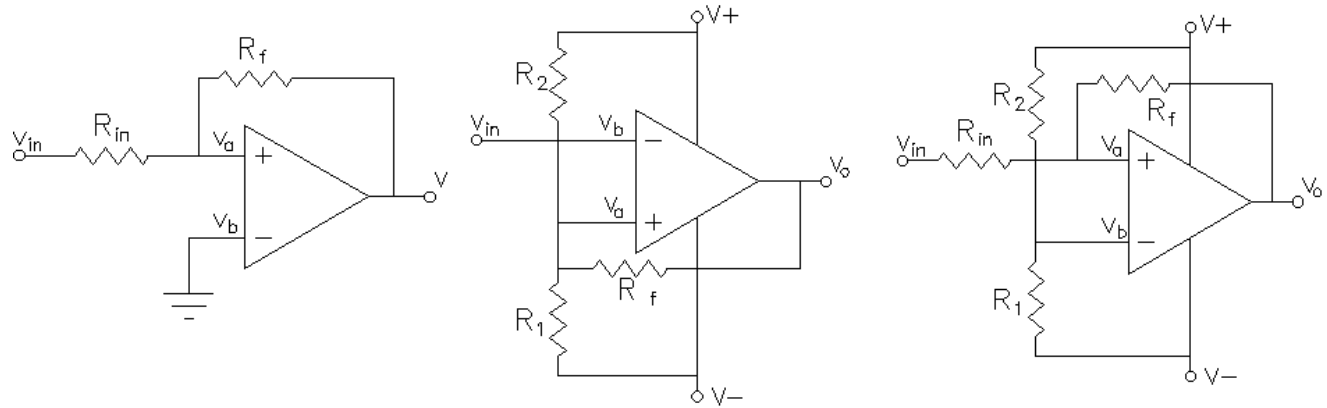
Schmitt trigger

The Schmitt trigger is a variation of the simple comparator which has hysteresis, that is, it has a toggle action. When the output is high, positive feedback makes the switching level higher than it is when the output is low. A little positive feedback makes a comparator with better noise immunity. Increase the positive feedback and the Schmitt trigger can be used in other switching applications.

Look at the Schmitt ttrigger circuit shown at right. Notice that $v_a = [R_1/(R_1 + R_f)]v_o$, it depends on the output. Lets say
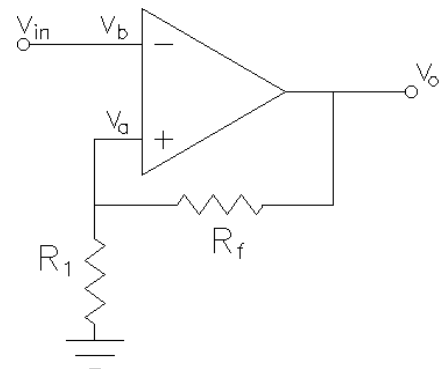
Figure 15 Schmitt trigger

the output is low and the input is decreasing. When $v_{in} < v_a$ the output goes high and suddenly v_a goes a little bit higher with it. That makes the difference between v_b and v_a even bigger. To make the circuit switch again v_{in} has to go back up beyond the original switching level. It has to reach the new v_a before the output will switch low. In this circuit the two switching levels are above and below ground by the same amount (unless you have nonsymmetric power supplies).


Figure 16 Other Schmitt triggers

The circuits above are variations of the Schmitt trigger. In the first circuit, the input is again compared to levels above and below ground, but this time the output goes high when the input goes high and vise-versa. In the remaining circuits the switching levels are not symmetric about 0 V, but about some voltage set by the voltage divider of R_1 and R_2.

Multivibrator (square wave generator)
The heart of the multivibrator is a Schmitt trigger with lots of positive feedback. Usually $R_2 = R_3$, which set the switching levels at about ½ V+ and ½ V-. When the output is high the capacitor charges through R_1 until it reaches the ½ V+ switching level, the output switches low and the capacitor discharges to zero and then charges up (down) until it reaches the ½ V- switching level. That makes the output switch high and the process repeats.
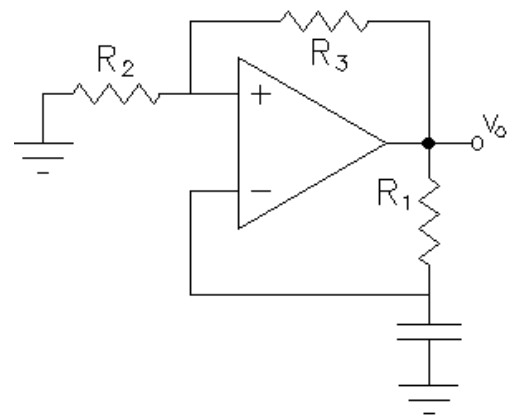
Figure 17 Multivibrator

Conclusion
In all of these circuits, with either negative or positive feedback, the output voltage v_o cannot increase without bounds. It is bounded in the positive direction by V+, the op-amp positive power supply voltage, and is bounded in the negative direction by V-, the op-amp negative supply voltage. If the output voltage is within these bounds, $v_a - v_b$ must be very small. If $v_a - v_b$ were not very small, v_o would soon be forced to one of its limits. Linear circuits use negative feedback to keep this difference small. Without negative feedback you can reasonably assume that the circuit is some kind of switching circuit and that the output is always at one or the other of its limits.

This only scratches the surface of what you can do with op-amps. Get a copy of *The Op-amp Cookbook* for lots more ideas presented in a no-nonsense way.

**ECE 2210 Homework OA1   Due: Mon 4/18/22**   A. Stolp, 11/28/01

Refer to the Operational Amplifier handout. Most of the problems below are design problems. The answer should be a schematic of a circuit showing the values of all the parts. Use resistor values in the 1 kΩ to 1 MΩ range. You **MUST** choose resistor values that are **DIFFERENT** than those in my answers.

1.   Design a buffer circuit which will allow a sensor with a high source resistance to be connected to fairly low resistance load. You don't need any voltage gain, but it is important that the load does not interfere with the measurement.

2.   Design an amplifier with a gain of 12. The output voltage must be in phase with the input voltage (no inversion is allowed).

3.   Design an amplifier with a gain of 25. The output voltage may be 180° out of phase with the input voltage (inversion is allowed). Its input resistance should be ≥ 10 kΩ. That is, from the input's point of view, the amplifier should look like a 10 kΩ resistor hooked to ground, or larger.

4.   Design an amplifier with two inputs where $v_o = -10v_1 - 4v_2$.

5.   Design an amplifier with two inputs where $v_o = +10v_1 + 4v_2$. You may use more than one op-amp.

6.   Design an amplifier with two inputs where $v_o = 12v_2 - 12v_1$.

7.   Design a differentiator using an op-amp, a resistor, and an inductor. You do not need to show parts values, but you need to show that the circuit differentiate by showing a derivation similar to the ones in my handout.

8.   Design a comparator whose output will be high (about 8 or 9 V) when the input is greater than 5 V and whose output will be low (about 1 V or so) when the input is less than 5 V.

<u>**Answers**</u>
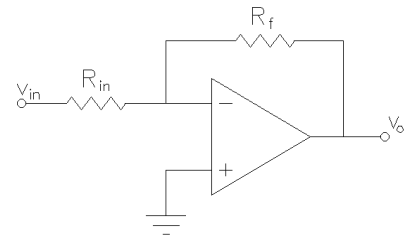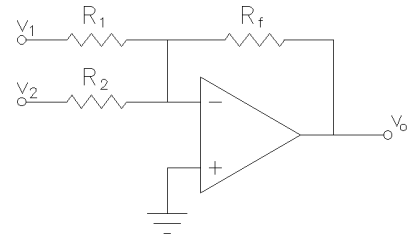
1.)   Draw a voltage follower.
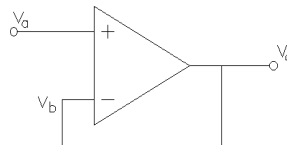
2.)   Draw a noninverting amplifier. Choose an R_1 and an R_f which is 11 times bigger than R_1. Say $R_1 = 10$ kΩ and $R_f = 110$ kΩ.
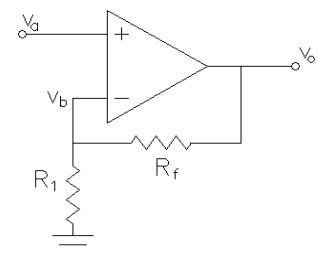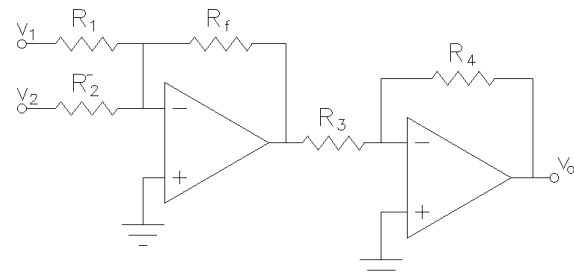
3.) Draw an inverting amplifier. R_{in} = 10 kΩ, R_f = 250 kΩ.

4.) Draw a two-input summer. Choose a value for R_f. Choose a value for R_1 which is $R_f/10$ and a value for R_2 which is $R_f/4$. Say 100 kΩ, 10 kΩ and 25 kΩ.

5.) Redraw the same circuit as problem 4, only now follow it with an inverting amp with a gain of 1. Say R_3 = R_4 = 10 kΩ for the second op-amp.
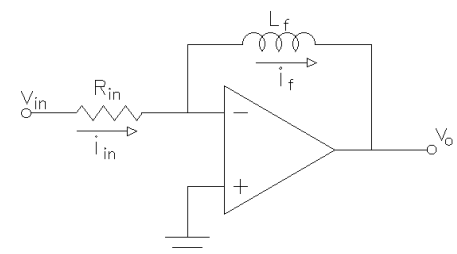
6.) Draw a differential amplifier. Choose an R_{in} value. Make R_f 12 times bigger than R_{in}. Say R_{in} = 10 kΩ and R_f = 120 kΩ.
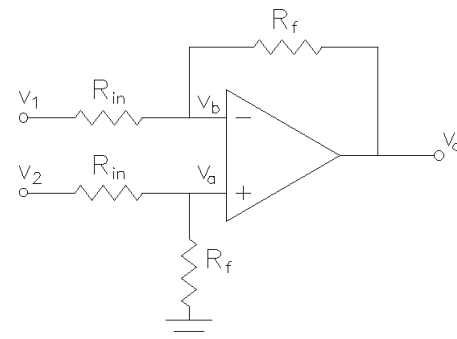
7.)

$$i_{in} = \frac{v_{in}}{R_{in}} = i_L = -\frac{1}{L}\int v_o dt$$

$$v_o = -\frac{L}{R_{in}}\frac{dv_{in}}{dt}$$

8.) Just choose the two resistor values to be equal, so the voltage at the inverting input pin will be 5 V. Now, anytime the voltage on the noninverting pin is above 5 V the output will be high (~8 V) and anytime the voltage on the noninverting pin is below 5 V the output will be low (~2 V).
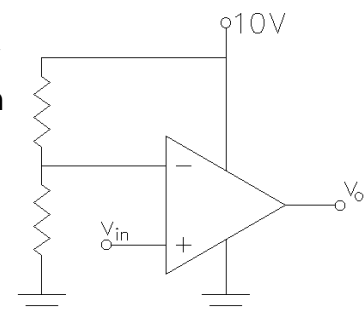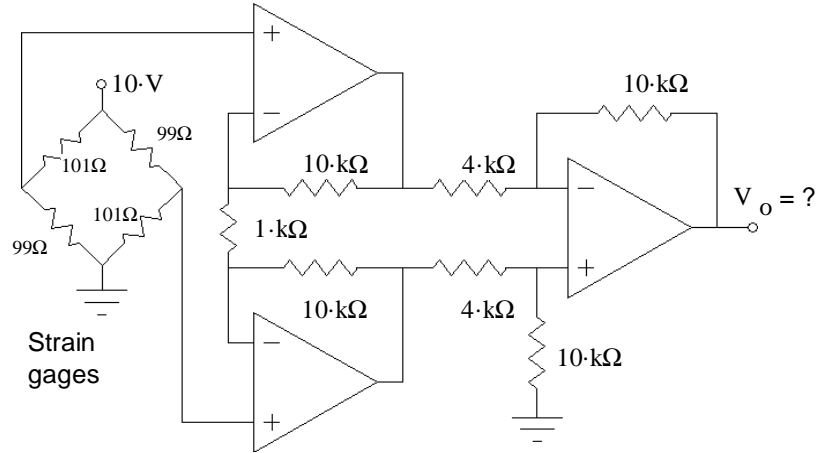
p2   ECE 2210 HW OA1

Name:_____   ECE 2210 Homework OA2   Due: Thur, 4/21/22   a

1. The op-amps are powered by ± 12 V power supplies. What output do you expect?
   SHOW WORK, possibly on another page.

$10 \cdot V$

99Ω

101Ω

101Ω

99Ω
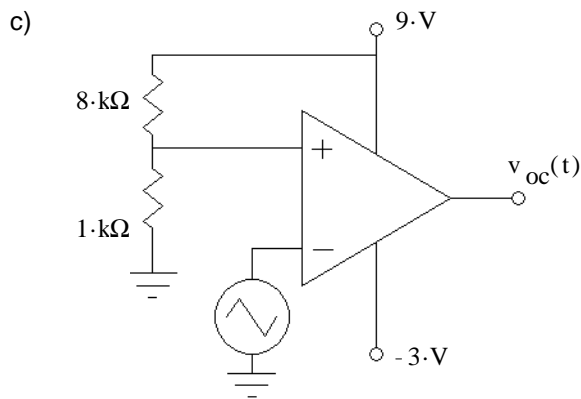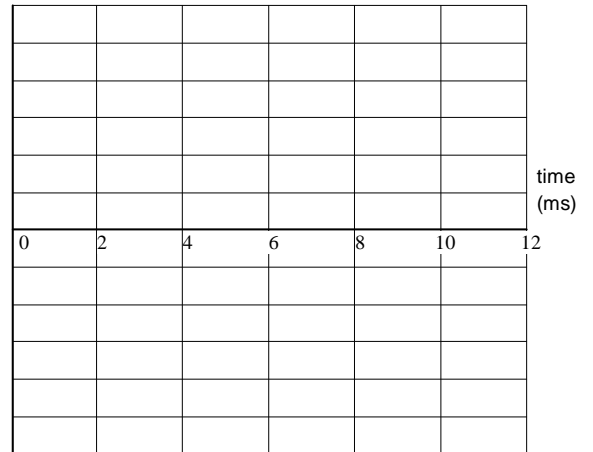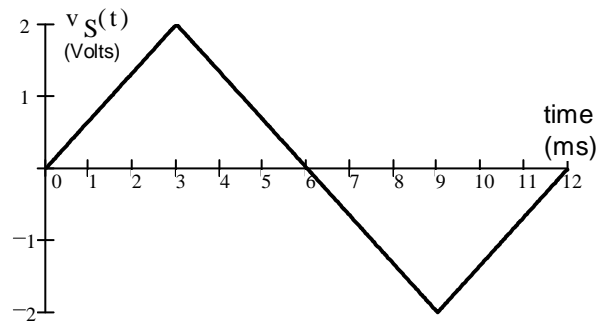
Strain
gages

$10 \cdot k\Omega$

$1 \cdot k\Omega$

$4 \cdot k\Omega$

$10 \cdot k\Omega$

$4 \cdot k\Omega$

$10 \cdot k\Omega$

$V_o = ?$

$10 \cdot k\Omega$

2. The same input signal (at right) is connected to several op-amp
   circuits. Sketch the output waveforms for the circuits shown.
   a) Op amp powered by ± 12 V power supplies.

$C := 6 \cdot \mu F$

$R := 1 \cdot k\Omega$

$v_{oa}(t)$

$v_S(t)$
(Volts)

time
(ms)

time
(ms)

2.b) $R_1 := 6 \cdot k\Omega$ $R_2 := 27 \cdot k\Omega$

$R_3 := 90 \cdot k\Omega$ $10 \cdot V$

v_S

$v_{ob}(t)$

$- 10 \cdot V$

$v_S(t)$
(Volts)

time
(ms)

time
(ms)

c) $9 \cdot V$

$8 \cdot k\Omega$

$+$

$v_{oc}(t)$

$1 \cdot k\Omega$

$-$

$- 3 \cdot V$

time
(ms)

0   1.5   3   4.5   6   7.5   9   10.5   12

Answers

1. $5.25 \cdot V$

ECE 2210 Hw OA2
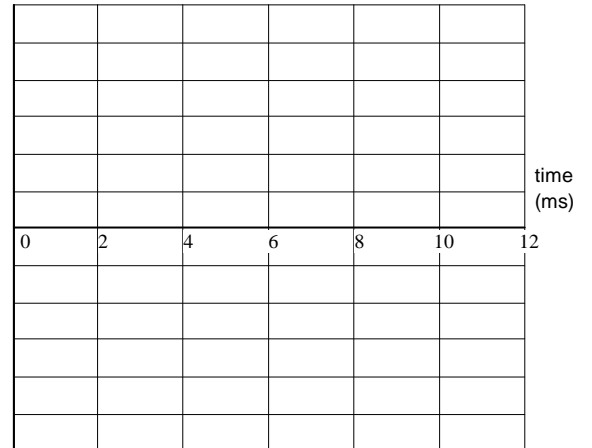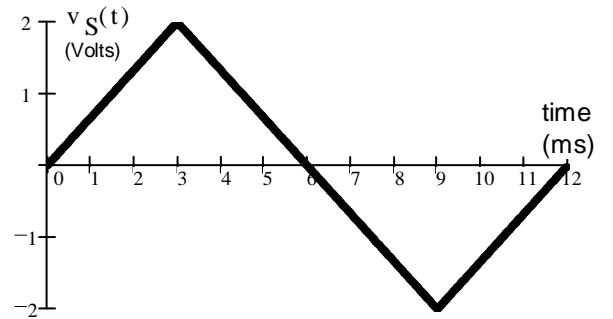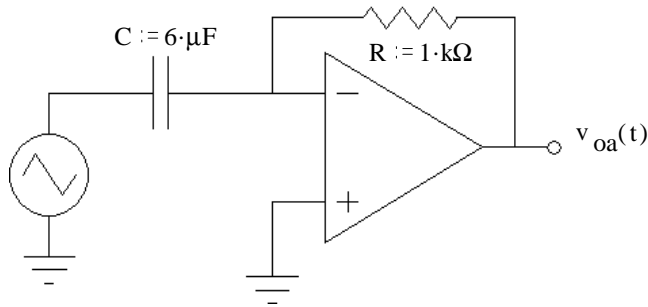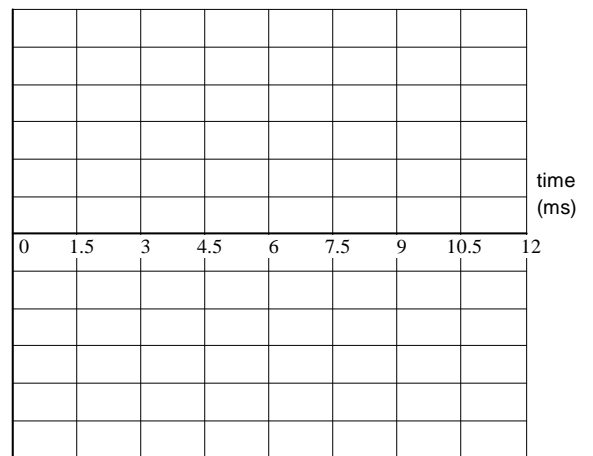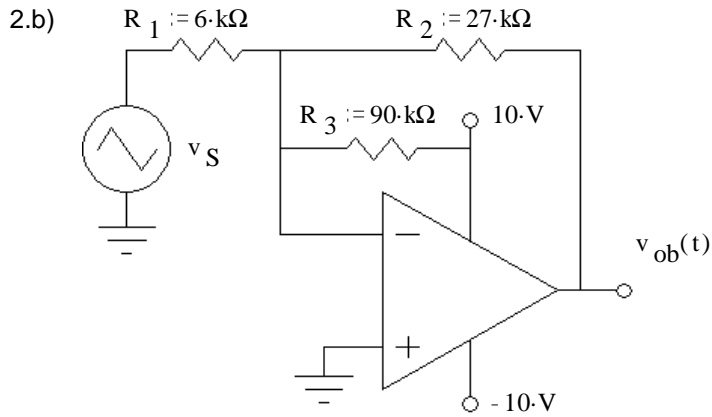
2. a) $(0ms, -4V)$ to $(3ms, -4V)$ to $(3ms, 4V)$ to $(9ms, 4V)$ to $(9ms, -4V)$ to $(12ms, -4V)$

b) $(0ms, -3V)$ to $(2ms, -9V)$ to $(4ms, -9V)$ to $(9ms, 6V)$ to $(12ms, -3V)$

c) $(0ms, 8V)$ to $(1.5ms, 8V)$ to $(1.5ms, -2V)$ to $(4.5ms, -2V)$ to $(4.5ms, 8V)$ to $(12ms, 8V)$