ECE 2210   Lecture 7 notes   Nodal Analysis

General Network Analysis

In many cases you have multiple unknowns in a circuit, say the voltages across multiple resistors. Network analysis is a systematic way to generate multiple equations which can be solved to find the multiple unknowns. These equations are based on basic Kirchoff's and Ohm's laws.

Loop or Mesh Analysis   You may have used these methods in previous classes, particularly in Physics. The best thing to do now is to forget all that. Loop analysis is rarely the easiest way to analyze a circuit and is inherently confusing. Hopefully I've brought you to a stage where you have some intuitive feeling for how currents flow in circuits. I don't want to ruin that now by screwing around with loop currents that don't really exist.

Nodal analysis This is a much better method. It's just as powerful, usually easier, and helps you develop your intuitive feeling for how circuits work.

Nodal Analysis

Node = all points connected by wire, all at same voltage (potential)

Ground: One node in the circuit which will be our reference node. Ground, by definition, will be the zero voltage node. All other node voltages will be referenced to ground and may be positive or negative. Think of gage pressure in a fluid system. In that case atmospheric pressure is considered zero. If there is no ground in the circuit, define one for yourself. Try to chose a node which is hooked to one side of a voltage source.

Nodal Voltage: The voltage of a node referenced to ground. The objective of nodal analysis is to find all the nodal voltages. If you know the voltage at a node then it's a "known" node. Ground is a known node (duh, it's zero). If one end of a known voltage source hooked to ground, then the node on the other end is also known (another duh).

Method: Label all the unknown nodes as; "a", "b", "c", etc. Then the unknown nodal voltages become; V_a, V_b, V_c, etc. Write a KCL equation for each unknown node, defining currents as necessary. Replace each unknown current with an Ohm's law relationship using the nodal voltages. Now you have just as many equations as unknowns. Solve.

Nodal Analysis Steps

1) If the circuit doesn't already have a ground, label one node as ground (zero voltage).
   If the ground can be defined as one side of a voltage source, that will make the following steps easier.
   Label the remaining node, either with known voltages or with letters, a, b, ...
2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...
3) Write Kirchoff's current equations for each unknown node.
4) Replace the currents in your **KCL** equations with expressions like this. $\dfrac{V_a - V_b}{R_1}$ Ohm's law relationship using the nodal voltages.
5) Solve the multiple equations for the multiple unknown voltages.

Nodal Analysis Examples

Ex 1 Use nodal analysis to find the voltage across R_1 (V_{R1}).
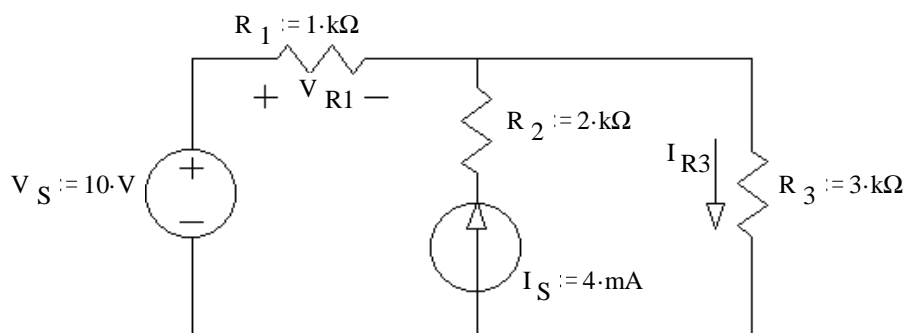
1) See next page
   Label one node as ground (zero voltage). By choosing the negative side of a voltage source as ground, the upper-left node is known ($10V$). Label the remaining nodes, either with known voltages or with letters, a, b, ...

2) Label unknown node voltages as V_a, V_b, ...
   and label the current in each resistor as I_1, I_2, ...

3) Write Kirchoff's current equations for node a.

$$I_1 \; + \; I_S \; = \; I_{R3}$$

4) Replace the currents in the **KCL** equations with Ohm's law relationships.

$$\frac{V_S - V_a}{R_1} + I_S \; = \; \frac{V_a - 0}{R_3}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} + I_S \; = \; \frac{V_a}{R_3}$$

5) Solve:

Usually it's easier to put in the numbers at this point

$$\frac{V_S}{R_1} + I_S \; = \; \frac{V_a}{R_3} + \frac{V_a}{R_1}$$

$$\frac{10 \cdot V}{1 \cdot k\Omega} + 4 \cdot mA \; = \; \frac{V_a}{3 \cdot k\Omega} + \frac{V_a}{1 \cdot k\Omega}$$

Multiply both sides by a value that will clear the denominators.

$$\frac{V_S}{R_1} + I_S \; = \; V_a \cdot \left(\frac{1}{R_1} + \frac{1}{R_3} \right)$$

$$3 \cdot k\Omega \cdot \left(\frac{10 \cdot V}{1 \cdot k\Omega} + 4 \cdot mA \right) \; = \; \left(\frac{V_a}{3 \cdot k\Omega} + \frac{V_a}{1 \cdot k\Omega} \right) \cdot 3 \cdot k\Omega$$

$$V_a := \frac{\dfrac{V_S}{R_1} + I_S}{\left(\dfrac{1}{R_1} + \dfrac{1}{R_3} \right)} \qquad V_a = 10.5 \bullet V$$

$$30 \cdot V + 3 \cdot k\Omega \cdot 4 \cdot mA \; = \; V_a + 3 \cdot V_a$$

$$30 \cdot V + 12 \cdot V \; = \; 4 \cdot V_a$$

$$V_a = \frac{42 \cdot V}{4} = 10.5 \bullet V$$
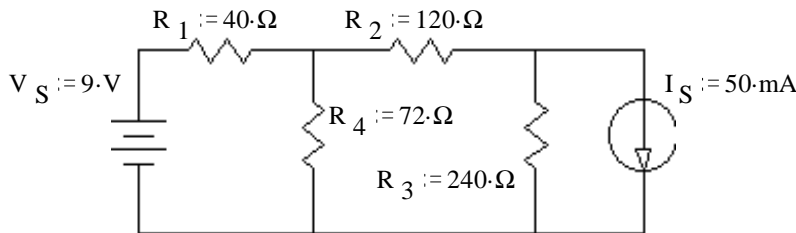
Either way, you still have to find V_{R1} from V_a.

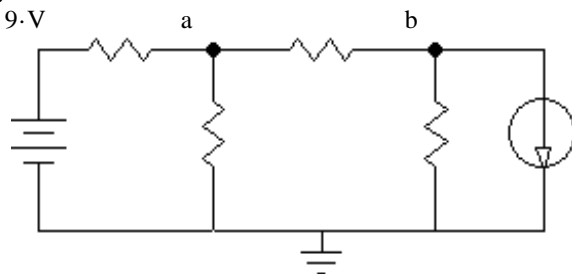$$V_{R1} := V_S - V_a \qquad\qquad V_{R1} = {-0.5} \bullet V \qquad\qquad V_b \text{ doesn't matter in this case}$$

b) Find the current through R_3 (I_{R3}).

$$I_{R3} \; = \; \frac{V_a}{R_3} = 3.5 \bullet mA$$

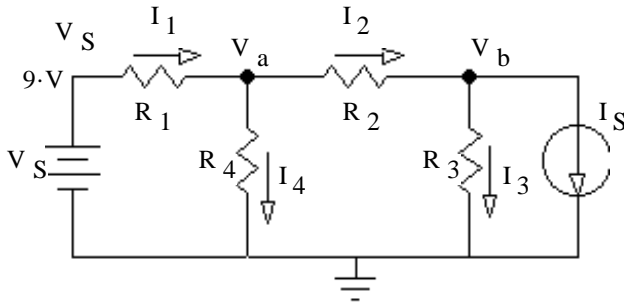**Ex 2** Same circuit used in Thévenin example, where R_4 was R_L.

1) Define ground and nodes:

2 unknown nodes ---> will need 2 equations

2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...

   It doesn't matter if these currents are in the correct directions.

3) Write Kirchoff's current equations for each unknown node.

node a $I_1 = I_2 + I_4$

node b $I_2 = I_3 + I_S$

4) Replace the currents in your **KCL** equations with expressions like this. $\dfrac{V_a - V_b}{R_1}$

node a $I_1 = I_2 + I_4$

$$\frac{V_S - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a - 0 \cdot V}{R_4}$$

node b $I_2 = I_3 + I_S$

$$\frac{V_a - V_b}{R_2} = \frac{V_b - 0 \cdot V}{R_3} + I_S$$

Now you have just as many equations as unknowns.

5) Solve the multiple equations for the multiple unknown voltages. Solve by any method you like:

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{V_b}{R_2} + \frac{V_a}{R_4}$$

$$\frac{V_a}{R_2} - \frac{V_b}{R_2} = \frac{V_b}{R_3} + I_S \qquad V_b = \frac{\dfrac{V_a}{R_2} - I_S}{\dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{\dfrac{V_a}{R_2} - I_S}{R_2 \cdot \left(\dfrac{1}{R_2} + \dfrac{1}{R_3} \right)} + \frac{V_a}{R_4}$$

$$V_a := \frac{\left[\dfrac{V_S}{R_1} - \dfrac{1}{\left[R_2 \cdot \left(\dfrac{1}{R_2} + \dfrac{1}{R_3} \right) \right]} \cdot I_S \right]}{\left[\dfrac{1}{R_1} + \dfrac{1}{R_2} - \dfrac{1}{R_2{}^2 \cdot \left(\dfrac{1}{R_2} + \dfrac{1}{R_3} \right)} + \dfrac{1}{R_4} \right]}$$

$$V_a = 4.6 \cdot V$$

$$V_b := \frac{\dfrac{V_a}{R_2} - I_S}{\dfrac{1}{R_2} + \dfrac{1}{R_3}} \qquad V_b = -0.933 \cdot V$$

Or, with numbers

node a

$$360 \cdot \Omega \cdot \left(\frac{9 \cdot V - V_a}{40 \cdot \Omega} \right) = \left(\frac{V_a - V_b}{120 \cdot \Omega} + \frac{V_a}{72 \cdot \Omega} \right) \cdot 360 \cdot \Omega$$

$$81 \cdot V - 9 \cdot V_a = 3 \cdot V_a - 3 \cdot V_b + 5 \cdot V_a$$

node b

$$240 \cdot \Omega \cdot \frac{V_a - V_b}{120 \cdot \Omega} = \left(\frac{V_b - 0 \cdot V}{240 \cdot \Omega} + 50 \cdot mA \right) \cdot 240 \cdot \Omega$$

$$2 \cdot V_a - 2 \cdot V_b = V_b + 48 \cdot mA \cdot 240 \cdot \Omega$$

$$V_a = \frac{2 \cdot V_b + V_b + 12 \cdot V}{2} = 1.5 \cdot V_b + 6 \cdot V$$

<-- substitute for V_a

$$81 \cdot V - 9 \cdot \left(1.5 \cdot V_b + 6 \cdot V \right) = 3 \cdot \left(1.5 \cdot V_b + 6 \cdot V \right) - 3 \cdot V_b + 5 \cdot \left(1.5 \cdot V_b + 6 \cdot V \right)$$

$$81 \cdot V - 13.5 \cdot V_b - 54 \cdot V = 4.5 \cdot V_b + 18 \cdot V - 3 \cdot V_b + 7.5 \cdot V_b + 30 \cdot V$$

$$81 \cdot V - 54 \cdot V - 18 \cdot V - 30 \cdot V = -21 \cdot V = 4.5 \cdot V_b - 3 \cdot V_b + 7.5 \cdot V_b + 13.5 \cdot V_b = 22.5 \cdot V_b$$

$$V_b = \frac{-21 \cdot V}{22.5} = -0.933 \cdot V \qquad V_a = 1.5 \cdot V_b + 6 \cdot V = 4.6 \cdot V$$

Same as V_L of Ex 4 of Thévenin examples:

Ex 3 Like Superposition Ex.2

a) Use nodal analysis to find the voltage across R_2 (V_{R2}).

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.

1) Define ground and nodes:

2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...

$V_S := 12 \cdot V$

$R_1 := 1 \cdot k\Omega$

$R_2 := 2 \cdot k\Omega$

V_{R2}

$R_3 := 3 \cdot k\Omega$

I_{R3}

$I_S := 2 \cdot mA$
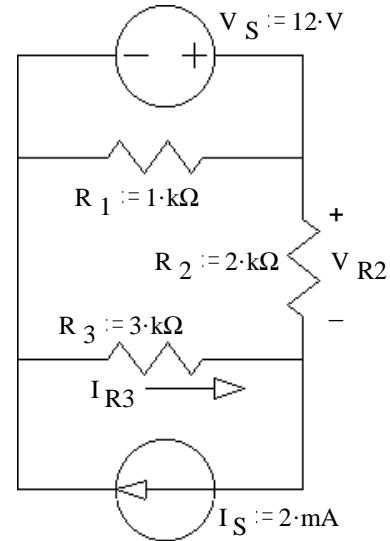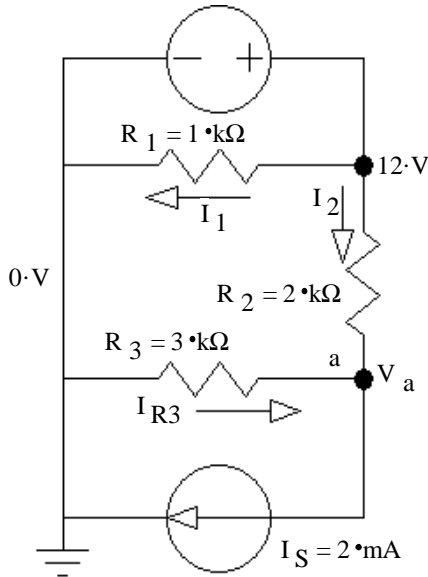
3) Write Kirchoff's current equations for each unknown node.

node a: $\quad I_2 \;+\; I_{R3} \;=\; I_S$

4) Replace the currents in the **KCL** equations with Ohm's law relationships.

$$\frac{V_S - V_a}{R_2} + \frac{0 - V_a}{R_3} = I_S$$

5) Solve the equation for the unknown voltage.

$$\frac{V_S}{R_2} - \frac{V_a}{R_2} - \frac{V_a}{R_3} = I_S$$

$$\frac{V_S}{R_2} = \frac{V_a}{R_2} + \frac{V_a}{R_3} + I_S$$

$$\frac{V_S}{R_2} - I_S = V_a \cdot \left(\frac{1}{R_2} + \frac{1}{R_3} \right)$$

$$V_a := \frac{\dfrac{V_S}{R_2} - I_S}{\left(\dfrac{1}{R_2} + \dfrac{1}{R_3} \right)} \qquad V_a = 4.8 \cdot V$$

Usually it's easier to put in the numbers at this point

$$\frac{12 \cdot V - V_a}{2 \cdot k\Omega} + \frac{0 - V_a}{3 \cdot k\Omega} = 2 \cdot mA$$

Multiply both sides by a value that will clear the denominators.

$$6 \cdot k\Omega \cdot \left(\frac{12 \cdot V - V_a}{2 \cdot k\Omega} + \frac{0 - V_a}{3 \cdot k\Omega} \right) = 2 \cdot mA \cdot 6 \cdot k\Omega$$

$$36 \cdot V - 3 \cdot V_a - 2 \cdot V_a = 12 \cdot V$$

$$-5 \cdot V_a = -24 \cdot V$$

$$V_a = \frac{-24 \cdot V}{-5} = 4.8 \cdot V$$

Remember, we needed to find the voltage across R_2 (V_{R2}).
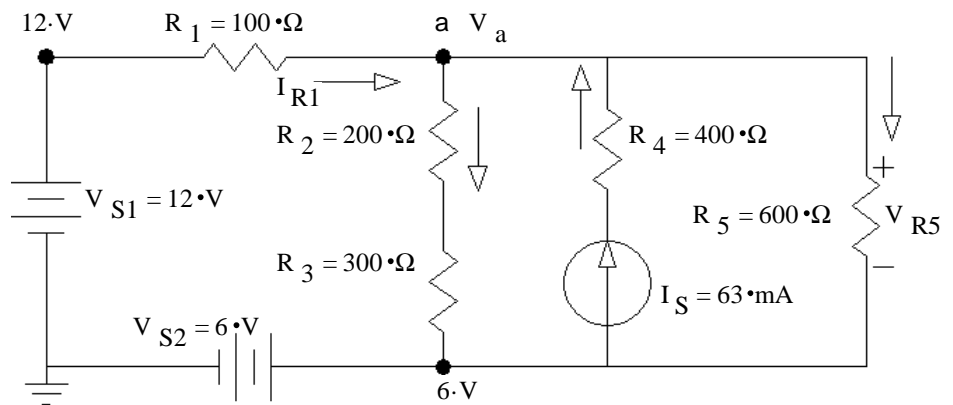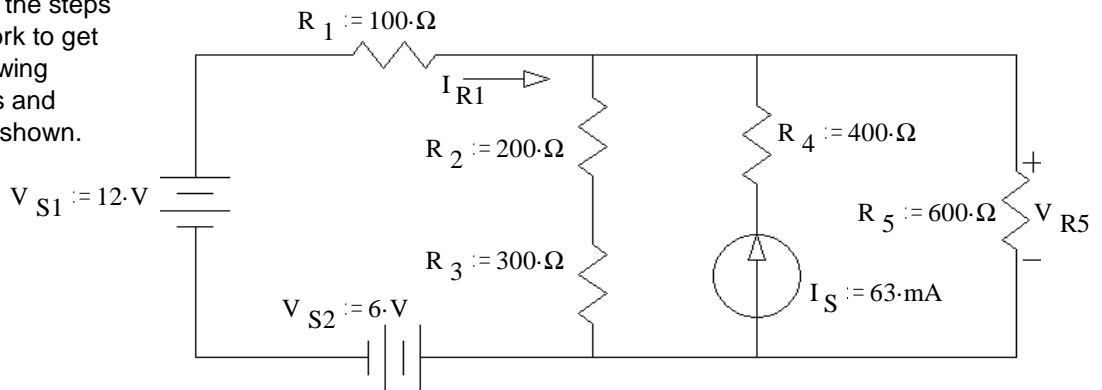
$$V_{R2} = V_S - V_a = 7.2 \cdot V$$

b) Find the current through R_3 (I_{R3}).

$$I_{R3} = \frac{0 - V_a}{R_3} = -1.6 \cdot mA \quad \text{actually flows the other way}$$

Ex 4   Use nodal analysis to find the voltage across R_5 (V_{R5}) and the current through R_1 (I_{R1}).   From exam 1, F09

You **MUST** show all the steps of nodal analysis work to get credit, including drawing appropriate symbols and labels on the circuit shown.

$R_1 := 100 \cdot \Omega$

I_{R1}

$R_2 := 200 \cdot \Omega$

$R_4 := 400 \cdot \Omega$

$V_{S1} := 12 \cdot V$

$R_5 := 600 \cdot \Omega$ V_{R5}

$R_3 := 300 \cdot \Omega$

$I_S := 63 \cdot mA$

$V_{S2} := 6 \cdot V$

$12 \cdot V$ $R_1 = 100 \cdot \Omega$ a V_a

I_{R1}

$R_2 = 200 \cdot \Omega$

$R_4 = 400 \cdot \Omega$

$V_{S1} = 12 \cdot V$

$R_5 = 600 \cdot \Omega$ V_{R5}

$R_3 = 300 \cdot \Omega$

$I_S = 63 \cdot mA$

$V_{S2} = 6 \cdot V$

$6 \cdot V$

node a:

$$I_{R1} + I_S = I_R + I_5$$

$R_3 = 300 \cdot \Omega$ $\dfrac{V_{S1} - V_a}{R_1} + I_S = \dfrac{V_a - V_{S2}}{R_2 + R_3} + \dfrac{V_a - V_{S2}}{R_5}$

$\dfrac{12 \cdot V}{100 \cdot \Omega} - \dfrac{V_a}{100 \cdot \Omega} + 63 \cdot mA = \dfrac{V_a}{500 \cdot \Omega} - \dfrac{6 \cdot V}{500} + \dfrac{V_a}{600 \cdot \Omega} - \dfrac{6 \cdot V}{600}$ multiply both sides by 3000Ω

$3000 \cdot \Omega \cdot \left(\dfrac{12 \cdot V}{100 \cdot \Omega} - \dfrac{V_a}{100 \cdot \Omega} + 63 \cdot mA \right) = \left(\dfrac{V_a}{500 \cdot \Omega} - \dfrac{6 \cdot V}{500} + \dfrac{V_a}{600 \cdot \Omega} - \dfrac{6 \cdot V}{600} \right) \cdot 3000 \cdot \Omega$

$360 \cdot V - 30 \cdot V_a + 189 \cdot V = 6 \cdot V_a - 36 \cdot V + 5 \cdot V_a - 30 \cdot V$

$360 \cdot V + 189 \cdot V + 36 \cdot V + 30 \cdot V = 6 \cdot V_a + 5 \cdot V_a + 30 \cdot V_a$

$615 \cdot V = 41 \cdot V_a$

$V_a := \dfrac{615 \cdot V}{41}$ $V_a = 15 \cdot V$

$V_{R5} = V_a - V_{S2} = 9 \cdot V$

$I_{R1} = \dfrac{V_{S1} - V_a}{R_1} = -30 \cdot mA$

What if one side of a voltage source isn't ground?

$$I_1 + I_{VS2} = I_3$$

$$\frac{V_{S1} - V_a}{R_1} + \quad ? \quad = I_S$$
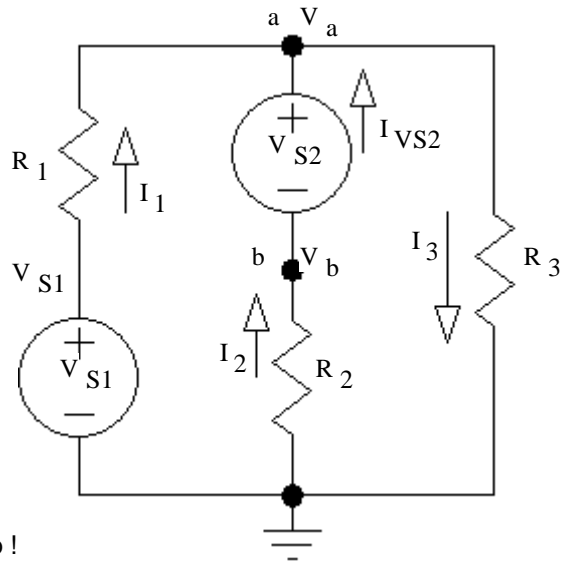
What do you put in for I_{VS2}??

Go to the other side of V_{S2}.

$$\frac{V_{S1} - V_a}{R_1} + \frac{0 - V_b}{R_2} = I_S$$

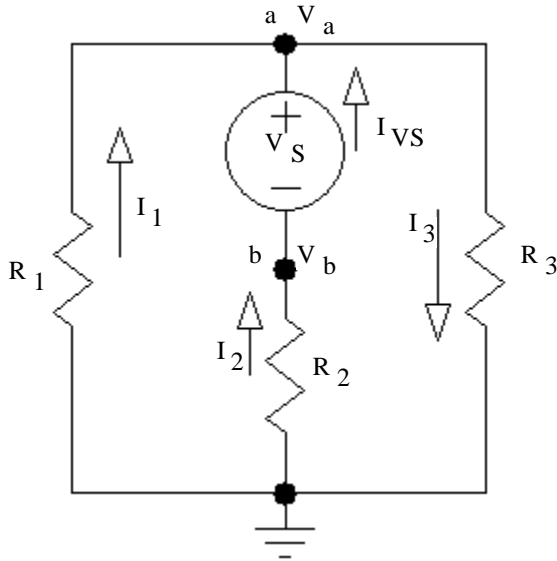Only problem is that you get the same equation at node b !

Where does the second equation come from?

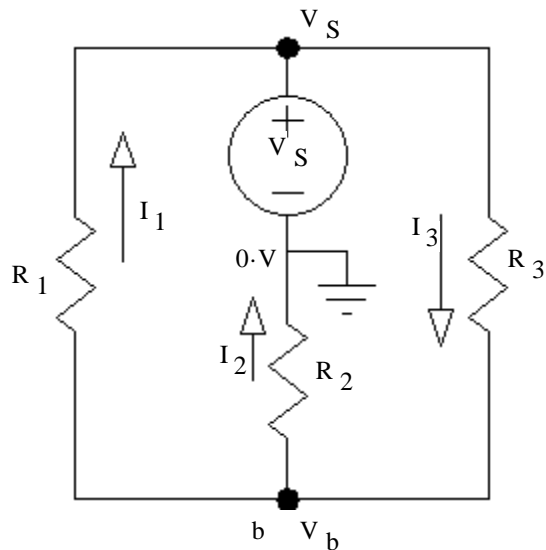Use something like this: $V_a = V_b + V_{S2}$

Similar Circuit, but no V_{S1}.

If the ground is already at the bottom, use the same method as above.

If you can chose your ground, you can make life a little simpler.

DC Notes

Basic electrical quantities

|		Unit
Charge, actually moves	Q	Coulomb (C)
Current, like fluid flow	$I = \dfrac{Q}{s}$	Amp (A, mA, µA,...)
Voltage, like pressure	V	volt (V, mV, kV,...)
Resistance	$R = \dfrac{V}{I}$	Ohm (Ω, kΩ, MΩ,...)
Conductance	$G = \dfrac{1}{R}$	Siemens (S, old unit mho)
Power energy/time	$P = V \cdot I$	Watt (W, mW, kW, MW,...)

Schematic symbols

battery, voltage sources, current source, Unknown part, ideal wire assume R=0, Node = All points connected by wire, connected, not connected, ground V=0, Volt Amp Ohm meters, Variable Resistors, potentiometer, capacitor, inductor or coil, light bulb, fuse, diode, LED, speaker, transformer, closed R=0, open R=∞ Switch, transistor, op amp
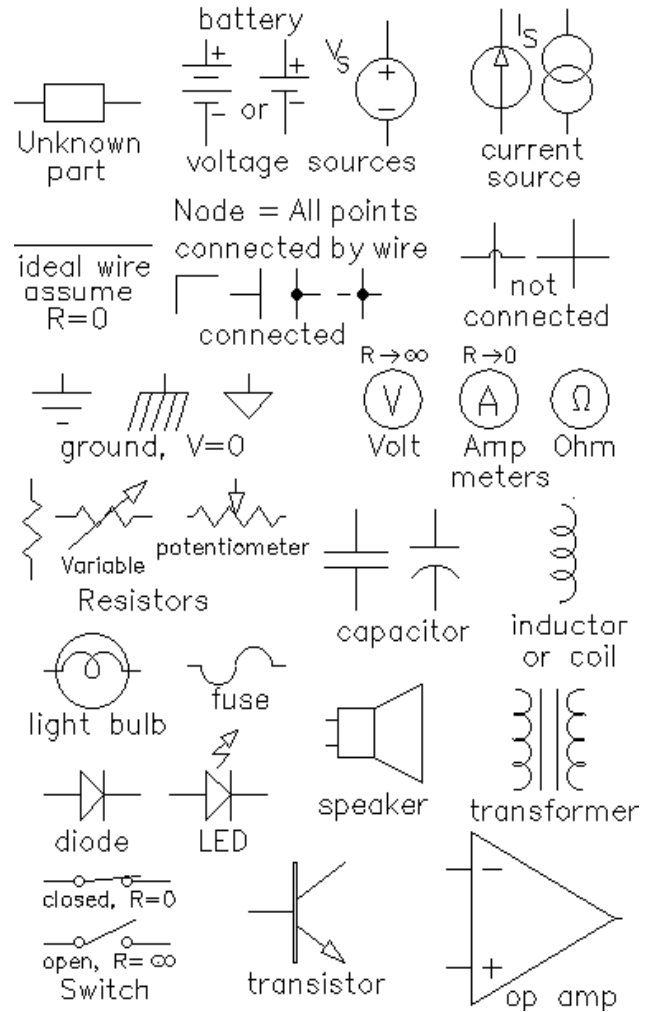
KCL, **K**irchhoff's **C**urrent **L**aw
  $I_{in} = I_{out}$ of any point, part, or section
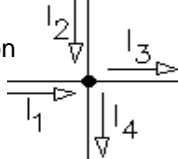
KVL, **K**irchhoff's **V**oltage **L**aw
  $V_{gains} = V_{drops}$ around any loop
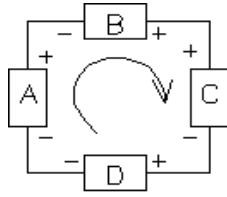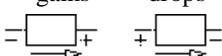
Node = all points connected by wire, all at same voltage (potential)

Ohm's law (resistors)

$$V = I \cdot R \qquad I = \frac{V}{R} \qquad R = \frac{V}{I}$$

Power $P_{IN} = P_{OUT}$ for resistor circuits

contribute   dissipate

$P = V \cdot I$ for everything

$$= I^2 \cdot R = \frac{V^2}{R} \text{ for resistors}$$

Maximum power transfer: $R_L = R_{Th}$
Load = Thevenin's

Resistors

  series: $R_{eq} = R_1 + R_2 + R_3 + \ldots$

  Exactly the **same current** through each resistor

  parallel: $R_{eq} = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots}$

  Exactly the **same voltage** across each resistor

Voltage divider:

$$V_{Rn} = V_{total} \cdot \frac{R_n}{R_1 + R_2 + R_3 + \ldots}$$

current divider:

$$I_{Rn} = I_{total} \cdot \frac{\dfrac{1}{R_n}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \ldots}$$

Multiple unknowns:
1. Combine resistors into equivalents where possible.
2. Use superposition if there are multiple sources and you know all the resistors.
3. Use KCL, KVL, & Ohm's laws to write multiple equations and solve.

DC Notes   ECE 2210 / 00

DC Notes

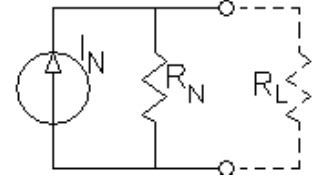## Thévenin equivalent

To calculate a circuit's Thévenin equivalent:
1) Remove the load and calculate the open-circuit voltage where the load used to be. This is the Thévenin voltage (V_{Th}).
2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
4) Draw the Thévenin equivalent circuit and add your values.
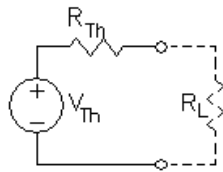
Norton equivalent

To calculate a circuit's Norton equivalent:
1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire. This is the Norton current (I_N). Remove the short.
2) Zero all the sources. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals. (DO NOT include the load in this resistance.) This is the Norton source resistance (R_N). (Exactly the same as theThévenin source resistance (R_{Th})).
4) Draw the Norton equivalent circuit and add your values.

OR (the more common way)...
1) Find the Thévenin equivalent circuit.
2) Convert to Norton circuit, $R_N = R_{Th}$ and $I_N = V_{Th}/R_{Th}$.

Nodal Analysis

1) If the circuit doesn't already have a ground, label one node as ground (zero voltage). If the ground can be defined as one side of a voltage source, that will make the following steps easier.
2) Label unknown node voltages as V_a, V_b, ... and label the current in each resistor as I_1, I_2, ...
3) Write Kirchoff's current equations for each unknown node.
4) Replace the currents in your KCL equations with expressions like the one below.
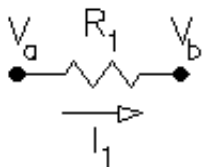
$$I_1 = \frac{V_a - V_b}{R_1}$$

5) Solve the multiple equations for the multiple unknown voltages

Superposition

For circuits with **more than 1 source**.
1) Zero all but one source. (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
2) Compute your wanted voltage or current due to the remaining source. Careful, some may be negative.
3) Repeat the first two steps for all the sources.
4) Sum all the contributions from all the sources to find the actual voltage or current. **Watch your signs!**

ECE 2210 / 00   Lecture 8 Notes   Basic AC

A. Stolp
1/27/03,
9/9/20

AC stands for **A**lternating **C**urrent as opposed to DC, **D**irect **C**urrent. AC refers to voltages and currents that change with time, usually the voltage is + sometimes and - at other times. This results in currents with go one direction when the voltage is + and the reverse direction when the voltage is -.

AC is important for two reasons.
Power is created and distributed as AC. Signals are AC.

AC Power
  Power is generated by rotating magnetic fields.
  This naturally produces sinusoidal AC waveforms.

  It is easier to make AC motors than DC motors.
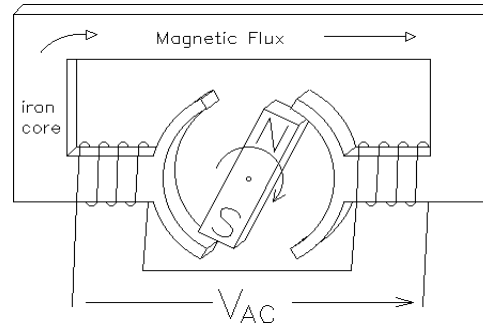
AC Power allows use of transformers to reduce line losses

Transformers work with AC, but not DC. Transformers can be used to raise or lower AC voltages (with an opposite change of current). This can be very useful in power distribution systems. Power is voltage times current. You can distribute the same amount of power with high voltage and low current as you can with low voltage and high current. However, the lower the current, the lower the I²R loses in the wires (all real wires have some resistance). So you'd like to distribute power at the highest possible voltage. Transformers allow you to do this with AC, but won't work with DC.

p Iron-core transf secondary
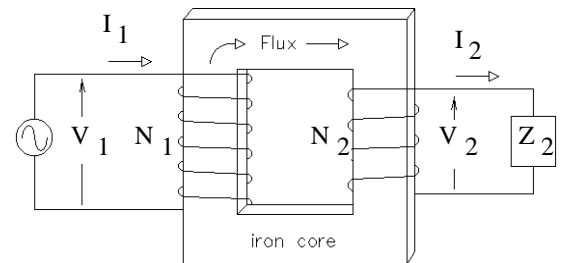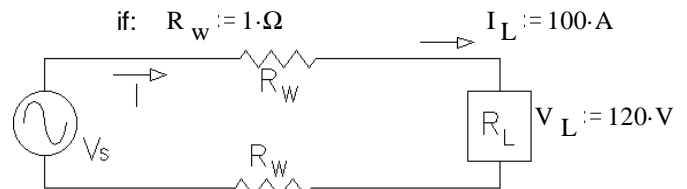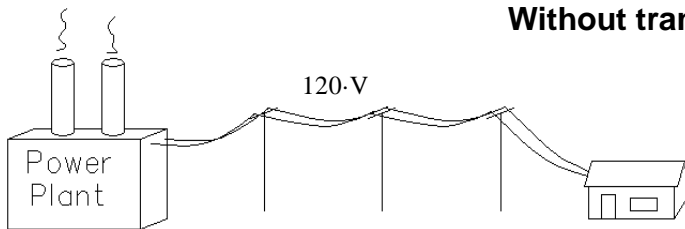
  Ideal: power in = power out   Ideal transformation of voltage and current: $\dfrac{N_1}{N_2} = \dfrac{V_1}{V_2} = \dfrac{I_2}{I_1}$
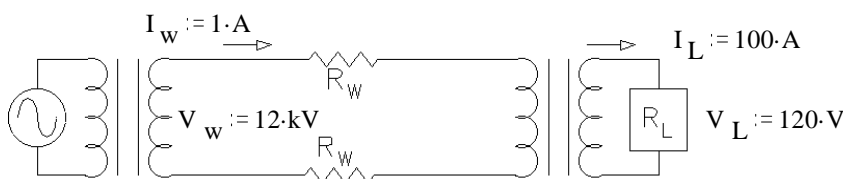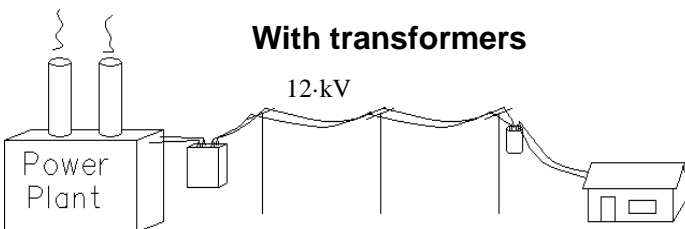
Example:

Without transformers

120·V

if: $R_w := 1 \cdot \Omega$ $I_L := 100 \cdot A$

$V_L := 120 \cdot V$

Wire loss: $P_W = I_L^2 \cdot 2 \cdot R_w = 20 \cdot kW$

With transformers

12·kV

$I_w := 1 \cdot A$ $I_L := 100 \cdot A$

$V_w := 12 \cdot kV$ $V_L := 120 \cdot V$

Wire loss: $P_W = I_w^2 \cdot 2 \cdot R_w = 2 \cdot W$

In this example, the power lost in the transmission lines is only 1/10,000th what it is without transformers.

That's why they raise the voltage in transmission lines to the point where they crackle and buzz. That crackle is the sound of the losses into the surrounding air and can become significant if the voltage is *too* high.

A time-varying voltage or current that carriers information. If it varies in time, then it has an AC component.

       | |

       | Audio, video, position, temperature, digital data, etc...

In some unpredictable fashion

DC is not a signal, Neither is a pure sine wave. If you can predict it, what information can it provide?
Neither DC nor pure sine wave have any "bandwidth". In fact, no periodic waveform is a signal & no periodic
waveform has bandwidth. You need bandwidth to transmit information.

Signal sources

Microphone	Audio
Camera	Video
Thermistor or other thermal sensor	Temperature
Potentiometer	Position
LVDT (Linear Variable Differential Transformer)	Position
Light sensor	
Computer	
switch	
etc...	

A transducer is a device which transforms one form of energy to another. Some sensors are transducers, many are not

Most often a signal comes from some other system.
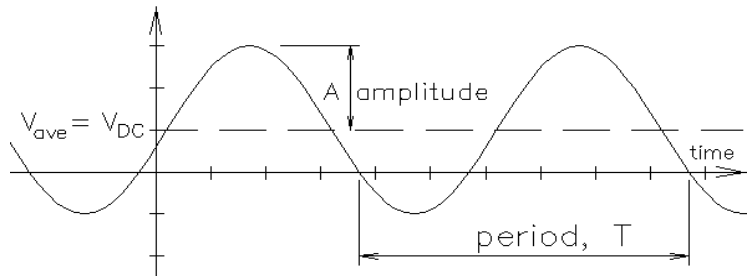
Periodic waveforms: Waveshape repeats

T = Period = repeat time

f = frequency, cycles / second $f = \dfrac{1}{T} = \dfrac{\omega}{2\cdot\pi}$

ω = radian frequency, radians/sec $\omega = 2\cdot\pi\cdot f$

A = amplitude

DC = average

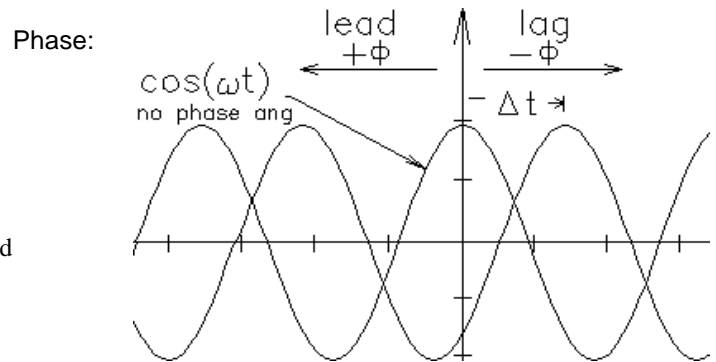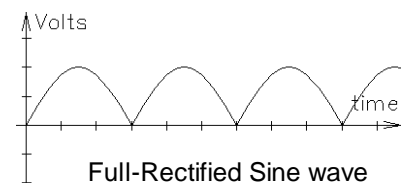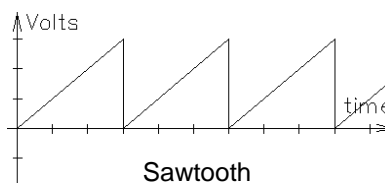## Sinusoidal AC

$y(t) = A\cdot\cos(\omega\cdot t + \phi)$
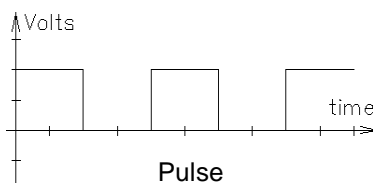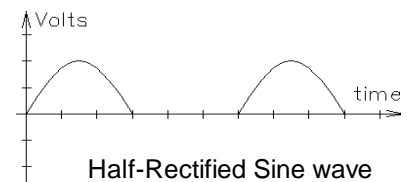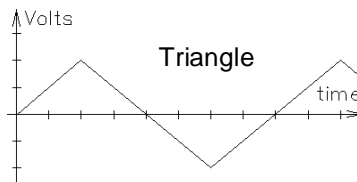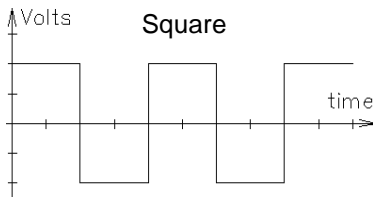
voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi)$

current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi)$

Phase: $\phi = -\dfrac{\Delta t}{T}\cdot 360\cdot deg$ or: $\phi = -\dfrac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

Phase:

Other common periodic waveforms

All but the square and triangle waves have a DC component as well as AC.