Complex Numbers

ECE 2210 / 00

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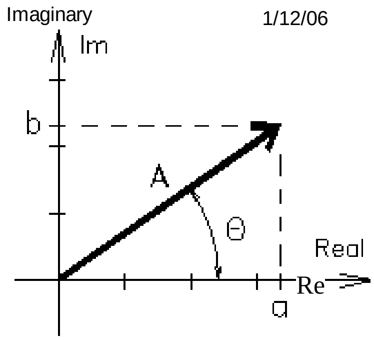
10/18/00
1/12/06



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b\cdot j$

$\mathrm{Re}(\mathbf{A}) = a$ $\qquad$ $\mathrm{Im}(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A\cdot e^{j\cdot\theta}$

$\mathrm{Re}(\mathbf{A}) = A\cdot\cos(\theta)$ $\qquad$ $\mathrm{Im}(\mathbf{A}) = A\cdot\sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2}$ $\qquad$ $\theta = \arg(\mathbf{A}) = \operatorname{atan}\left(\frac{b}{a}\right)$

$a = A\cdot\cos(\theta)$ $\qquad$ $b = A\cdot\sin(\theta)$

$\mathbf{A} = A\cdot e^{j\cdot\theta} = A\cdot\cos(\theta) + A\cdot\sin(\theta)\cdot j$ $\qquad$ $\mathbf{A} = a + b\cdot j = \left(\sqrt{a^2 + b^2}\right)\cdot e^{j\cdot\operatorname{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j\cdot 90\cdot deg}$ $\qquad$ $\frac{1}{j} = -j = e^{-j\cdot 90\cdot deg}$ $\qquad$ $e^{j\cdot 0\cdot deg} = 1$ $\qquad$ $e^{-j\cdot 180\cdot deg} = e^{-j\cdot 180\cdot deg} = -1$

$j\cdot e^{j\cdot\theta} = e^{j\cdot(\theta + 90\cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d\cdot j$ $\qquad$ polar: $\mathbf{D} = D\cdot e^{j\cdot\phi}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction $\mathbf{A} + \mathbf{D} = (a + b\cdot j) + (c + d\cdot j) = (a + c) + (b + d)\cdot j$

$\mathbf{A} - \mathbf{D} = (a + b\cdot j) - (c + d\cdot j) = (a - c) + (b - d)\cdot j$

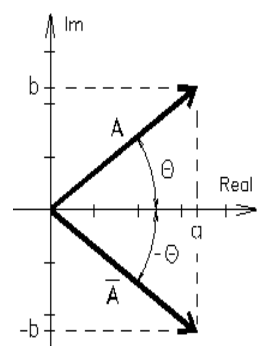
Convert polars to rectangular form first

Multiplication and Division $\mathbf{A}\cdot\mathbf{D} = (a + b\cdot j)\cdot(c + d\cdot j) = (a\cdot c - b\cdot d) + (b\cdot c + a\cdot d)\cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b\cdot j}{c + d\cdot j} = \frac{a + b\cdot j}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{a\cdot c + b\cdot d}{c^2 + d^2} + \frac{b\cdot c - a\cdot d}{c^2 + d^2}\cdot j$

Polar: $\mathbf{A}\cdot\mathbf{D} = A\cdot e^{j\cdot\theta}\cdot D\cdot e^{j\cdot\phi} = A\cdot D\cdot e^{j\cdot(\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A\cdot e^{j\cdot\theta}}{D\cdot e^{j\cdot\phi}} = \frac{A}{D}\cdot e^{j\cdot(\theta - \phi)}$



Powers $\mathbf{A}^n = A^n\cdot e^{j\cdot n\cdot\theta} = A^n\cdot\cos(n\cdot\theta) + A^n\cdot\sin(n\cdot\theta)\cdot j$ $\qquad$ Convert rectangulars first, usually

Conjugates <u>complex number</u> $\qquad$ <u>Conjugate</u>

$\mathbf{A} = a + b\cdot j$ $\qquad$ $\overline{\mathbf{A}} = a - b\cdot j$ $\qquad$ $\overline{\overline{\mathbf{A}}} = \mathbf{A}$

$\mathbf{A} = A\cdot e^{j\cdot\theta}$ $\qquad$ $\overline{\mathbf{A}} = A\cdot e^{-j\cdot\theta}$

$\mathbf{F} = \frac{3 + 5\cdot j}{(2 - 6\cdot j)\cdot e^{j\cdot 40\cdot deg}}$ $\qquad$ $\overline{\mathbf{F}} = \frac{3 - 5\cdot j}{(2 + 6\cdot j)\cdot e^{-j\cdot 40\cdot deg}}$

Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ $\qquad$ OR: $\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2}$ $\qquad$ $\sin(\alpha) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$e^{j\cdot(\omega\cdot t + \theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$

$\mathrm{Re}\left[e^{j\cdot(\omega\cdot t + \theta)}\right] = \cos(\omega\cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$

Calculus Remember, when we write $e^{j\cdot\theta}$, we really mean $e^{j\cdot(\omega\cdot t + \theta)}$

$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A\cdot e^{j\cdot\theta}\right) = j\cdot\omega\cdot A\cdot e^{j\cdot\theta} = \omega\cdot A\cdot e^{j\cdot(\theta + 90\cdot deg)}$

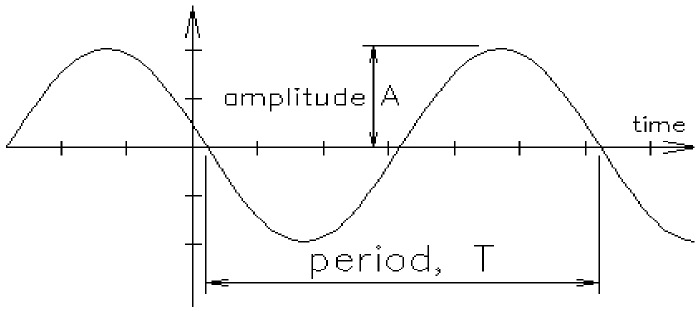
$\int \mathbf{A}\,dt = \int A\cdot e^{j\cdot\theta}\,dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot\theta} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta - 90\cdot deg)}$

ECE 2210 / 00 Lecture 12 Introduction to AC Phasors

A.Stolp 10/7/08
rev,

Phasor analysis with impedances, For steady-state sinusoidal response ONLY

Sinusoidal AC



T = Period = repeat time

f = frequency, cycles / second $\quad f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$

ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$

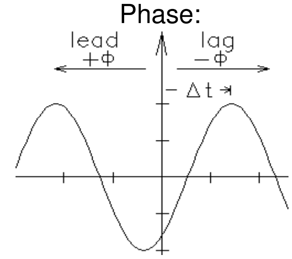
A = amplitude

Phase: $\quad \phi = -\frac{\Delta t}{T}\cdot 360\cdot deg$

or: $\quad \phi = -\frac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

$y(t) = A\cdot\cos(\omega\cdot t + \theta)$

Phase:



Phasor analysis

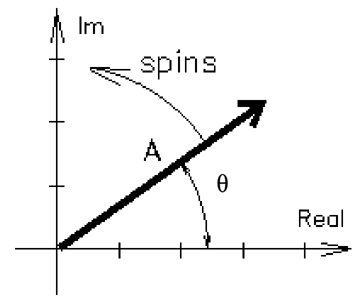
The math is all based on the Euler's equation

Euler's equation $\quad e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$

OR:

$$\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2}$$

$$\sin(\theta) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$$



$$e^{j\cdot(\omega\cdot t+\theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$$

$$Re\left[e^{j\cdot(\omega\cdot t+\theta)}\right] = \cos(\omega\cdot t + \theta)$$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$ That's the phasor

Phasor

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

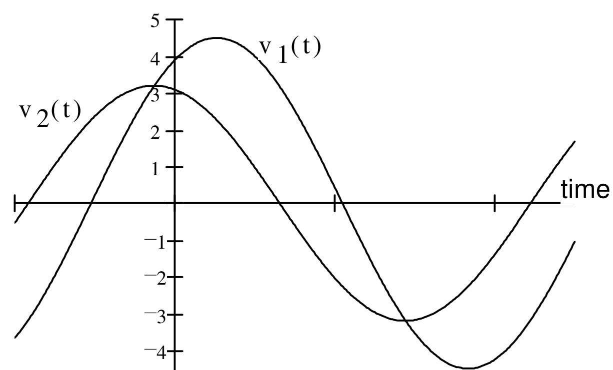
current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi) \qquad \mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Phasors are drawn on a complex plane.

Phasors are used for adding and subtracting sinusoidal waveforms.

Ex1. Add the sinusoidal voltages $v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

and $v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$



using phasor notation, draw a phasor diagram of the three phasors, then convert back to time domain form.

$v_1(t) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

$\mathbf{V_1}(\omega) = 4.5V\ \underline{/-30^o}$ or: $\mathbf{V_1}(\omega) = 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

and

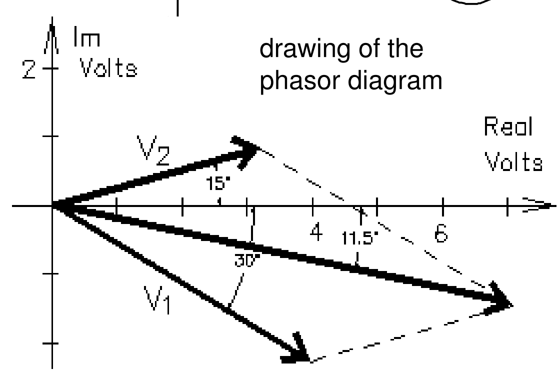
$v_2(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

$\mathbf{V_2}(\omega) = 3.2V\ \underline{/15^o}$ or: $\mathbf{V_2}(\omega) = 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

$\mathbf{V_1} = 4.5V\ \underline{/-30^o}$ or: $\mathbf{V_1} := 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

$\mathbf{V_2} = 3.2V\ \underline{/15^o}$ or: $\mathbf{V_2} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$



drawing of the phasor diagram

Add like vectors, first change to the rectangular form

$\mathbf{V_1} = 4.5V\ \underline{/-30^o}$	$4.5\cdot V\cdot\cos(-30\cdot deg) = 3.897\cdot V$	$4.5\cdot V\cdot\sin(-30\cdot deg) = -2.25\cdot V$	$\mathbf{V_1} = 3.897 - 2.25j\ \cdot V$ \
			} add
$\mathbf{V_2} = 3.2V\ \underline{/15^o}$	$3.2\cdot V\cdot\cos(15\cdot deg) = 3.091\cdot V$	$3.2\cdot V\cdot\sin(15\cdot deg) = 0.828\cdot V$	$\mathbf{V_2} = 3.091 + 0.828j\ \cdot V$ /
	Add real parts:	$3.897 + 3.091 = 6.988$	$\mathbf{V_3} := \mathbf{V_1} + \mathbf{V_2}$
	Add imaginary parts:	$-2.25 + 0.828 = -1.422$	$\mathbf{V_3} = 6.988 - 1.422j\ \cdot V$ sum

Change $\mathbf{V_3}$ back to polar coordinates:

$$\sqrt{6.988^2 + 1.422^2} = 7.131 \qquad \operatorname{atan}\left(\frac{-1.422}{6.988}\right) = -11.502\cdot deg$$

OR, in Mathcad notation (you' ll see these in future solutions):

$$\left|\mathbf{V_3}\right| = 7.131\cdot V \qquad \arg\left(\mathbf{V_3}\right) = -11.5\cdot deg$$

Change $\mathbf{V_3}$ back to the time domain:

$$v_3(t) = v_1(t) + v_2(t) = 7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$$

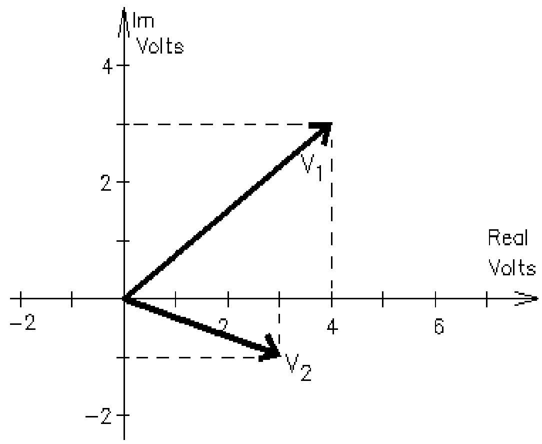
Ex 2. Two sinusoidal voltages: $v_1(t) = 5\cdot V\cdot\cos(\omega\cdot t + 36.87\cdot deg)$ and $v_2(t) = 3.162\cdot V\cdot\cos(\omega\cdot t - 18.44\cdot deg)$

a) using phasor notation, find $v_3 = v_1 - v_2$

$\mathbf{V_1} := 5\cdot V\cdot e^{j\cdot(36.87\cdot deg)}$ $\qquad 5\cdot V\cdot\cos(36.87\cdot deg) = 4\cdot V$

$5\cdot V\cdot\sin(36.87\cdot deg) = 3\cdot V$

$\mathbf{V_1} = 4 + 3j\ \cdot V$



$\mathbf{V_2} := 3.162\cdot V\cdot e^{j\cdot(-18.44\cdot deg)}$ $\qquad 3.162\cdot V\cdot\cos(-18.44\cdot deg) = 3\cdot V$

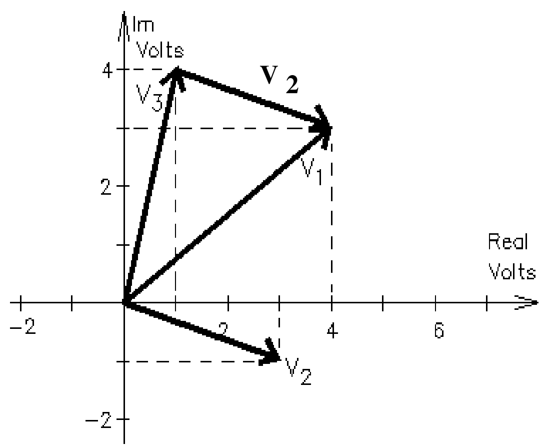
$3.162\cdot V\cdot\sin(-18.44\cdot deg) = -1\cdot V$

$\mathbf{V_2} = 3 - j\ \cdot V$

Subtract real parts: $4\cdot V - 3\cdot V = 1\cdot V$

Subtract imaginary parts: $3\cdot V - -1\cdot V = 4\cdot V$

$\mathbf{V_3} := \mathbf{V_1} - \mathbf{V_2} \qquad \mathbf{V_3} = 1 + 4j\ \cdot V$



Magnitude: $\sqrt{(1\cdot V)^2 + (4\cdot V)^2} = 4.123\cdot V$

OR:

$\left|\mathbf{V_3}\right| = 4.123\cdot V$

Angle: $\operatorname{atan}\left(\frac{4\cdot V}{1\cdot V}\right) = 75.96\cdot deg$ $\qquad \arg\left(\mathbf{V_3}\right) = 75.96\cdot deg$

So: $v_3(t) = v_1(t) - v_2(t) = 4.123\cdot V\cdot\cos(\omega\cdot t + 75.96\cdot deg)\cdot V$

What about Capacitors and Inductors?

Capacitors and Inductors in AC circuits cause 90^o phase shifts between voltages and currents because they integrate and differentiate. But... integration and differentiation is a piece-of-cake in phasors.

Calculus

$$\frac{d}{dt}\left[A\cdot e^{j\cdot(\omega\cdot t+\theta)}\right] = j\cdot\omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \omega\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta+90\cdot deg)} = \omega\cdot A\cdot e^{j\cdot(\theta+90\cdot deg)}$$

Drop the ωt (t=0) to get:

$$\int A\cdot e^{j\cdot(\omega\cdot t+\theta)}\,dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\omega\cdot t+\theta-90\cdot deg)} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta-90\cdot deg)}$$

Impedance (like resistance)

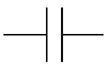
AC impedance

Inductor

$$v_L = L\cdot\frac{d}{dt}i_L = L\cdot\frac{d}{dt}I_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot L\cdot\left[I_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phasor notation ----> $\mathbf{V}_L(\omega) = j\cdot\omega\cdot L\cdot\mathbf{I}(\omega)$ $\qquad \mathbf{Z}_L = j\cdot\omega\cdot L$

Capacitor



$$i_C = C\cdot\frac{d}{dt}v_C = C\cdot\frac{d}{dt}V_p\cdot e^{j\cdot(\omega\cdot t+\theta)} = j\cdot\omega\cdot C\cdot\left[V_p\cdot e^{j\cdot(\omega\cdot t+\theta)}\right]$$

\ in phasor notation ----> $\mathbf{I}_C(\omega) = j\cdot\omega\cdot C\cdot\mathbf{V}(\omega)$

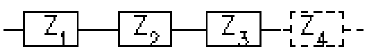
$$\mathbf{V}_C(\omega) = \frac{1}{j\cdot\omega\cdot C}\cdot\mathbf{I}(\omega) \qquad \mathbf{Z}_C = \frac{1}{j\cdot\omega\cdot C} = \frac{-j}{\omega\cdot C}$$

Resistor

$$v_R = i_R\cdot R \qquad \mathbf{V}_R(\omega) = R\cdot\mathbf{I}(\omega) \qquad \mathbf{Z}_R = R$$

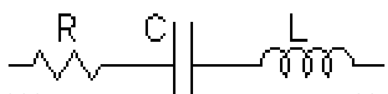
You can use impedances just like resistances as long as you deal with the complex arithmetic.
ALL the DC circuit analysis techniques will work with AC.

series:



$$\mathbf{Z}_{eq} = \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3 + \ldots$$

Example:



$f := 500\cdot Hz$

$\omega := 2\cdot\pi\cdot f \quad = \quad \omega = 3141.6\cdot\frac{rad}{sec}$

$R := 200\cdot\Omega \qquad C := 0.6\cdot\mu F \qquad L := 80\cdot mH$

$j\cdot\omega\cdot L = 251.327j\cdot\Omega$

$\frac{1}{j\cdot\omega\cdot C} = -530.516j\cdot\Omega$

$$\mathbf{Z}_{eq} := R + \frac{1}{j\cdot\omega\cdot C} + j\cdot\omega\cdot L = 200\cdot\Omega - 530.5\cdot j\cdot\Omega + 251.3\cdot j\cdot\Omega = 200 - 279.2j\cdot\Omega$$ rectangular form

$$\sqrt{(200\cdot\Omega)^2 + (279.2\cdot\Omega)^2} = 343.4\cdot\Omega \qquad \text{atan}\left(\frac{-279.2\cdot\Omega}{200\cdot\Omega}\right) = -54.38\cdot deg$$

$\mathbf{Z}_{eq} = 343.4\Omega\ \underline{/-54.4^o}$ polar form

If: $\mathbf{V} := 12\cdot V\cdot e^{j\cdot 0\cdot deg} \qquad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{12\cdot V}{343.4\cdot\Omega} = 34.945\cdot mA \quad \underline{/} \quad 0 - -54.4 = 54.4 \quad deg$

$\mathbf{I} = 34.95mA\ \underline{/54.4^o} = \mathbf{I} = 20.348 + 28.405j\cdot mA$

Voltage divider:

$$\mathbf{V}_{Zn} = \mathbf{V}_{total}\cdot\frac{\mathbf{Z}_n}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_3} + \ldots$$

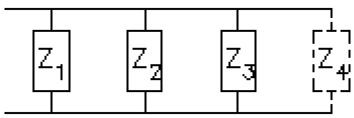
Note: $\frac{1}{j} = -j = 1\ \underline{/-90^o}$

Eg: $\mathbf{V}_C := \mathbf{V}\cdot\frac{\frac{1}{j\cdot\omega\cdot C}}{\mathbf{Z}_{eq}} = 12\cdot V\cdot e^{j\cdot 0\cdot deg}\cdot\frac{530.516\cdot e^{-j\cdot 90\cdot deg}\cdot\Omega}{343.4\cdot e^{-j\cdot 54.38\cdot deg}\cdot\Omega}$

$12\cdot V\cdot\frac{530.516\cdot\Omega}{343.4\cdot\Omega} = 18.539\cdot V \quad \underline{/} \quad 0 + -90 - -54.4 = -35.6 \quad deg$

$\mathbf{V}_C = 18.54V\ \underline{/-35.6^o} = \mathbf{V}_C = 15.069 - 10.795j\cdot V$

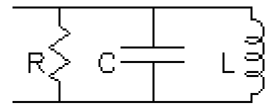
parallel:



$$\mathbf{Z}_{eq} = \frac{1}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \dots}$$

Example:

$f := 500 \cdot Hz \qquad \omega := 2 \cdot \pi \cdot f \qquad \omega = 3141.6 \cdot \frac{rad}{sec}$



$L := 80 \cdot mH$

$j \cdot \omega \cdot L = 251.327j \cdot \Omega$

$R := 200 \cdot \Omega \qquad C := 0.6 \cdot \mu F \qquad \frac{1}{\omega \cdot L} = 3.979 \cdot 10^{-3} \cdot \frac{1}{\Omega}$

$$\frac{1}{j \cdot \omega \cdot C} = -530.516j \cdot \Omega$$

$$\omega \cdot C = 1.885 \cdot 10^{-3} \cdot \frac{1}{\Omega}$$

$$\mathbf{Z}_{eq} := \frac{1}{\frac{1}{R} + \frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{j \cdot \omega \cdot L}} = \frac{1}{\frac{1}{R} + j \cdot \omega \cdot C - \frac{j}{\omega \cdot L}} = \frac{1}{\frac{1}{200 \cdot \Omega} + 1.885 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega} - 3.979 \cdot 10^{-3} \cdot j \cdot \frac{1}{\Omega}}$$

$$= \frac{1}{\left(5 \cdot 10^{-3} - 2.094 \cdot 10^{-3} \cdot j\right) \cdot \frac{1}{\Omega}} \cdot \frac{5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j}{\left(5 \cdot 10^{-3} + 2.094 \cdot 10^{-3} \cdot j\right)} = 170.156 + 71.261j \cdot \Omega$$

$2.93848 \cdot 10^{-5}$

If you want the answer in polar form, it's easier to convert the denominator first.

$$\sqrt{\left(5 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2 + \left(2.094 \cdot 10^{-3} \cdot \frac{1}{\Omega}\right)^2} = 5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega} \qquad \operatorname{atan}\left(\frac{-2.094 \cdot 10^{-3} \cdot \Omega}{5 \cdot 10^{-3} \cdot \Omega}\right) = -22.72 \cdot deg$$

$$\frac{1}{5.4 \cdot 10^{-3} \cdot \frac{1}{\Omega}} = 185.185 \cdot \Omega$$

$\mathbf{Z}_{eq} = 185.2 \angle 22.7°$

If: $\mathbf{V} := 12 \cdot V \cdot e^{j \cdot 0 \cdot deg} \qquad \mathbf{I} := \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{12 \cdot V}{185.2 \cdot \Omega} = 64.795 \cdot mA \quad \angle \quad 0 - 22.7 = -22.7 \;\; deg$

$\mathbf{I} = 60 - 25.127j \cdot mA$

Current divider:

$$\mathbf{I}_{Zn} = \mathbf{I}_{total} \cdot \frac{\frac{1}{\mathbf{Z}_n}}{\frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} + \dots}$$

Eg: $\mathbf{I}_L := \mathbf{I} \cdot \frac{\frac{1}{j \cdot \omega \cdot L}}{\frac{1}{R} + j \cdot \omega \cdot C + \frac{1}{j \cdot \omega \cdot L}} = \mathbf{I} \cdot \frac{\mathbf{Z}_{eq}}{j \cdot \omega \cdot L}$

$$= 64.795 \cdot mA \cdot e^{j \cdot 22.7 \cdot deg} \cdot \frac{185.2 \cdot e^{-j \cdot 22.7 \cdot deg} \cdot \Omega}{251.327 \cdot e^{j \cdot 90 \cdot deg} \cdot \Omega}$$

$$= 64.795 \cdot mA \cdot \frac{185.2 \cdot \Omega}{251.327 \cdot \Omega} = 47.747 \cdot mA \quad \angle \quad 22.7 + -22.7 - 90 = -90 \quad deg \qquad \mathbf{I}_L = -47.746j \cdot mA$$

Duh... $\frac{\mathbf{V}}{j \cdot \omega \cdot L} = -47.746j \cdot mA$

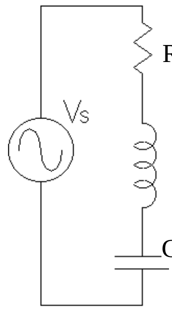
ECE 2210 / 00 Phasor Examples

Ex. 1 Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2\cdot kHz$

$\mathbf{V}(j\omega)$

$6\cdot V\cdot e^{j0}$

$f := 2\cdot kHz$

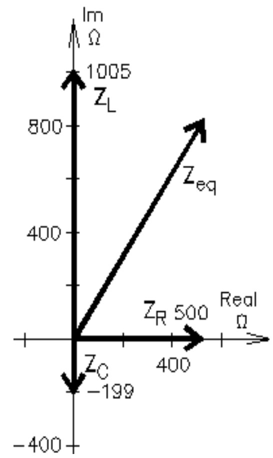


$R := 500\cdot\Omega$ $\quad \omega := 2\cdot\pi\cdot f$ $\quad \omega = 12566.371 \cdot \frac{rad}{sec}$

$L := 80\cdot mH$ $\quad \mathbf{Z}_L := j\cdot\omega\cdot L$ $\quad \mathbf{Z}_L = 1005.31i \cdot\Omega$

$C := 0.4\cdot\mu F$ $\quad \mathbf{Z}_C := \frac{1}{j\cdot\omega\cdot C}$ $\quad \mathbf{Z}_C = -198.944j \cdot\Omega$

$\mathbf{Z}_{eq} := R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}$ $\quad \mathbf{Z}_{eq} = 500 + 806.366j \cdot\Omega$



find the current: (The hard way)

$$\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z}_{eq}} = \frac{6\cdot V}{500 + 806.37\cdot j}\cdot\left(\frac{500 - 806.37\cdot j}{500 - 806.37\cdot j}\right) = \frac{6\cdot V}{(500 + 806.37\cdot j)\cdot\Omega}\cdot\left(\frac{500 - 806.37\cdot j}{500 - 806.37\cdot j}\right)$$

$$= \frac{6\cdot V\cdot(500 - 806.37\cdot j)}{\left(500^2 + 806.37^2\right)\cdot\Omega} = \frac{3000 - 4838.2\cdot j}{900232.6}\cdot A = \left(\frac{3000}{900232.6} - \frac{4838.2\cdot j}{900232.6}\right)\cdot A = 3.332 - 5.374j \cdot mA$$

It's much easier to convert the impedance to polar form first $\sqrt{500^2 + 806.37^2} = 948.806$ $\quad \operatorname{atan}\left(\frac{806.37}{500}\right) = 58.198 \cdot deg$

$\mathbf{Z_{eq}} = 948.8\Omega \,\underline{/\,58.2^o}$

find the current: $\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z}_{eq}}$ magnitude: $\frac{6\cdot V}{948.8\cdot\Omega} = 6.324 \cdot mA$

angle: $0\cdot deg - 58.2\cdot deg = -58.2 \cdot deg$ $\quad \mathbf{I} = 6.324mA \,\underline{/-58.2^o}$

	<u>find the magnitude</u>	<u>find the angle</u>	
$\mathbf{V}_R := \mathbf{I}\cdot R$	$6.324\cdot mA\cdot 500\cdot\Omega = 3.162 \cdot V$	$-58.2\cdot deg + 0\cdot deg = -58.2 \cdot deg$	$\mathbf{V_R} = 3.162V \,\underline{/-58.2^o}$
$\mathbf{V}_L := \mathbf{I}\cdot\mathbf{Z}_L$	$6.324\cdot mA\cdot 1005\cdot\Omega = 6.356 \cdot V$	$-58.2\cdot deg + 90\cdot deg = 31.8 \cdot deg$	$\mathbf{V_L} = 6.356V \,\underline{/\,31.8^o}$
$\mathbf{V}_C := \mathbf{I}\cdot\mathbf{Z}_C$	$6.324\cdot mA\cdot(-199)\cdot\Omega = -1.258 \cdot V$	$-58.2\cdot deg + (90)\cdot deg = 31.8 \cdot deg$	$\mathbf{V_C} = -1.258V \,\underline{/\,31.8^o}$
OR:	$6.324\cdot mA\cdot(199)\cdot\Omega = 1.258 \cdot V$	$-58.2\cdot deg + (-90)\cdot deg = -148.2 \cdot deg$	$\mathbf{V_C} = 1.258V \,\underline{/-148.2^o}$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V}_C := \frac{\frac{1}{j\cdot\omega\cdot C}}{R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) + j\cdot\omega\cdot L\cdot(j\cdot\omega\cdot C) + 1}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) - \omega^2\cdot L\cdot C + 1}\cdot 6\cdot V$$

$$= \frac{1}{\left(1 - \omega^2\cdot L\cdot C\right) + j\cdot\omega\cdot R\cdot C}\cdot 6\cdot V$$

$\left(1 - \omega^2\cdot L\cdot C\right) = -4.053$

$j\cdot\omega\cdot R\cdot C = 2.513j$

$$= \frac{6\cdot V}{-4.053 + 2.513\cdot j}$$ Let's convert denominator to polar first

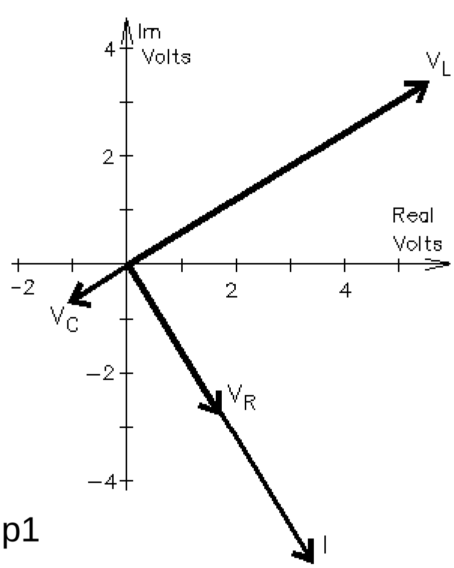
$\sqrt{(-4.053)^2 + 2.513^2} = 4.769$ $\quad \operatorname{atan}\left(\frac{2.513}{-4.053}\right) = -31.8 \cdot deg$ NO!!

but this is actually in the third quadrant, so modify your calculator's results:

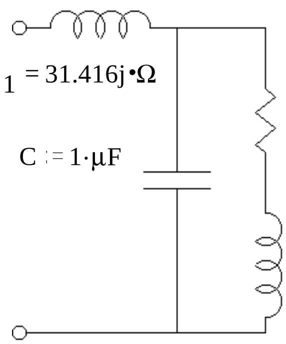
add 180º
$148.2\cdot deg$

magnitude: $\frac{6\cdot V}{4.769} = 1.258 \cdot V$ $\quad$ angle: $0\cdot deg - 148.2\cdot deg = -148.2 \cdot deg$

$= 1.258V \,\underline{/-148.2^o}$



Ex. 2   a) Find $\mathbf{Z_{eq}}$. $f := 2.5\cdot kHz$ $\omega := 2\cdot\pi\cdot f$ $\omega = 15708\cdot\frac{rad}{sec}$



$$\mathbf{Z_{eq}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} + \frac{1}{R + j\cdot\omega\cdot L_2}}$$

But it's easier to split the problem up

Left branch

$\mathbf{Z_l} := \frac{1}{j\cdot\omega\cdot C}$ $\mathbf{Z_l} = -63.662j\cdot\Omega$

$\frac{1}{\left(\frac{1}{j\cdot\omega\cdot C}\right)} = j\cdot\omega\cdot C = 0.01571i\cdot\frac{1}{\Omega}$

Right branch

$\mathbf{Z_r} := j\cdot\omega\cdot L_2 + R$ $\mathbf{Z_r} = 200 + 125.664j\ \cdot\Omega$

$\frac{1}{200 + 125.664\cdot j} = 3.585\cdot 10^{-3} - 2.252\cdot 10^{-3}j$

denominator: $j\cdot\omega\cdot C + \frac{1}{R + j\cdot\omega\cdot L_2} = 0.01571\cdot j + \left(3.585\cdot 10^{-3} - 2.252\cdot 10^{-3}\cdot j\right) = 3.585\cdot 10^{-3} + 1.346\cdot 10^{-2}i \quad \frac{1}{\Omega}$

rectangular division:

$$\frac{1}{\left(3.585\cdot 10^{-3} + 1.346\cdot 10^{-2}\cdot j\right)}\cdot\frac{\left(3.585\cdot 10^{-3} - 1.346\cdot 10^{-2}\cdot j\right)}{\left(3.585\cdot 10^{-3} - 1.346\cdot 10^{-2}\cdot j\right)} = \frac{3.585\cdot 10^{-3} - 1.346\cdot 10^{-2}\cdot j}{1.94\cdot 10^{-4}} = 18.479 - 69.381j \quad \Omega$$

$$\left(3.585\cdot 10^{-3}\right)^2 + \left(1.346\cdot 10^{-2}\right)^2 = 1.94\cdot 10^{-4}$$

add: $j\cdot\omega\cdot L_1 = 31.416j\cdot\Omega$ $31.416\cdot j + (18.479 - 69.381\cdot j) = 18.479 - 37.965j \quad \Omega$

convert to polar (if needed): $\sqrt{18.48^2 + 37.97^2} = 42.228$ $\mathrm{atan}\left(\frac{-37.97}{18.48}\right) = -64.048\cdot deg$ $\mathbf{Z_{eq}} = 42.23\Omega\ \underline{/-64.05^{o}}$

Another Way

Sometimes you might simplify a little before putting in numbers.

$$\mathbf{Z_{eq}} := j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + \frac{1}{\frac{1}{j\cdot\omega\cdot C}}} = j\cdot\omega\cdot L_1 + \frac{1}{\frac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C} = j\cdot\omega\cdot L_1 + \frac{R + j\cdot\omega\cdot L_2}{1 + j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right)}$$

$$= j\cdot\omega\cdot L_1 + \frac{R + j\cdot\omega\cdot L_2}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

$$\mathbf{Z_{eq}} = 31.416\cdot j\ \cdot\Omega + \frac{(200 + 125.664\cdot j)\cdot\Omega}{-0.974 + 3.142\cdot j}\cdot\left(\frac{-0.974 - 3.142\cdot j}{-0.974 - 3.142\cdot j}\right) = 31.416\cdot j\ \cdot\Omega + \frac{(200 + 125.664\cdot j)\cdot(-0.974 - 3.142\cdot j)}{0.974^2 + 3.142^2}$$

$$= 31.416\cdot j\ \cdot\Omega + \frac{((200\cdot(-0.974) - 125.664\cdot(-3.142)) + (125.664\cdot(-0.974) - 200\cdot 3.142)\cdot j)\cdot\Omega}{0.974^2 + 3.142^2}$$

$$= 31.416\cdot j\ \cdot\Omega + \frac{(200.036288 - 750.796736\cdot j)\cdot\Omega}{10.82084} = 31.416\cdot j\ \cdot\Omega + 18.486\cdot\Omega - 69.384\cdot j\cdot\Omega = 18.486 - 37.968j\ \cdot\Omega$$

$\sqrt{18.49^2 + 37.97^2} = 42.233$ $\mathrm{atan}\left(\frac{-37.97}{18.49}\right) = -64.036\cdot deg$ $\mathbf{Z_{eq}} = 42.23\Omega\ \underline{/-64.04^{o}}$

a little roundoff difference

b) $\mathbf{V_{in}} := 12 \cdot V \cdot e^{j \cdot 20 \cdot deg}$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$

$$\mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}} \qquad \frac{12 \cdot V}{42.23 \cdot \Omega} = 284.16 \cdot mA \qquad 20 \cdot deg - (-64.04) \cdot deg = 84.04 \cdot deg$$

$\mathbf{I_{L1}} = 284mA \ \underline{/\ 84.04^o}$

$$\mathbf{V_C} := \mathbf{I_{L1}} \cdot (18.479 - 69.381 \cdot j) \cdot \Omega \qquad 284 \cdot mA \cdot \sqrt{18.479^2 + 69.381^2} \cdot \Omega = 20.391 \cdot V$$

$$84.04 \cdot deg + atan\left(\frac{-69.381}{18.479}\right) = 8.954 \cdot deg$$

$\mathbf{V_C} = 20.4V \ \underline{/\ 8.95^o}$

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

convert to rectangular (if needed):

$$20.391 \cdot V \cdot \cos(8.954 \cdot deg) = 20.143 \cdot V$$

$$20.391 \cdot V \cdot \sin(8.954 \cdot deg) = 3.174 \cdot V$$

$\mathbf{V_C} = 20.14 + 3.174 \cdot j \quad V$

Another Way

To find V_C directly:

$$\mathbf{V_C} := \frac{\dfrac{1}{\dfrac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}}{j \cdot \omega \cdot L_1 + \dfrac{1}{\dfrac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C}} \cdot \mathbf{V_{in}}$$

--> math --> $\mathbf{V_C} = 20.153 + 3.178j \cdot V$ Same but for a little roundoff difference

c) Let's find $\mathbf{I_{L2}}$.

$$\mathbf{Z_r} = 200 + 125.664j \cdot \Omega \qquad \sqrt{200^2 + 125.664^2} = 236.202 \qquad atan\left(\frac{125.664}{200}\right) = 32.142 \cdot deg$$

$$\mathbf{I_{L2}} := \frac{\mathbf{V_C}}{\mathbf{Z_r}} = \frac{20.4 \cdot V \cdot e^{j \cdot 8.95 \cdot deg}}{236.202 \cdot \Omega \cdot e^{j \cdot 32.142 \cdot deg}} = \frac{20.4 \cdot V}{236.202 \cdot \Omega} \ \underline{/\ 8.95 - 32.142^o} \ = 86.4mA \ \underline{/-23.19}$$

Another Way

Directly by Current divider:

$$\mathbf{I_{L2}} := \frac{\dfrac{1}{R + j \cdot \omega \cdot L_2}}{j \cdot \omega \cdot C + \dfrac{1}{R + j \cdot \omega \cdot L_2}} \cdot \mathbf{I_{L1}} = \frac{1}{j \cdot \omega \cdot C \cdot (R + j \cdot \omega \cdot L_2) + 1} \cdot \mathbf{I_{L1}} = \frac{\mathbf{I_{L1}}}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

denominator:

$$\sqrt{(1 - \omega^2 \cdot C \cdot L_2)^2 + (\omega \cdot C \cdot R)^2} = 3.289 \qquad atan\left(\frac{\omega \cdot C \cdot R}{1 - \omega^2 \cdot C \cdot L_2}\right) + 180 \cdot deg = 107.224 \cdot deg$$

real part is negative /

$$\mathbf{I_{L2}} = \frac{284 \cdot mA \cdot e^{j \cdot 84.04 \cdot deg}}{3.289 \cdot e^{j \cdot 107.224 \cdot deg}} = \frac{284 \cdot mA}{3.289} \ \underline{/\ 84.04 - 107.224^o} \ = 86.4mA \ \underline{/-23.18^o}$$

d) How about $\mathbf{I_C}$?

$$\mathbf{I_C} := \frac{\mathbf{V_C}}{\left(\dfrac{1}{j \cdot \omega \cdot C}\right)} = \mathbf{V_C} \cdot j \cdot \omega \cdot C = 20.4V \ \underline{/\ 8.95^o} \ 0.015708 \ \underline{/\ 90^o} \ \frac{1}{\Omega} = 320mA \ \underline{/\ 98.95^o}$$

Another Way Could also be found directly by current divider:

$$\mathbf{I_C} := \frac{j \cdot \omega \cdot C}{j \cdot \omega \cdot C + \dfrac{1}{R + j \cdot \omega \cdot L_2}} \cdot \mathbf{I_{L1}} = 320mA \ \underline{/\ 98.95^o}$$

<u>Something Weird</u>

$\mathbf{I_C}$ is greater than the input current ($\mathbf{I_{L1}}$) . What's going on?

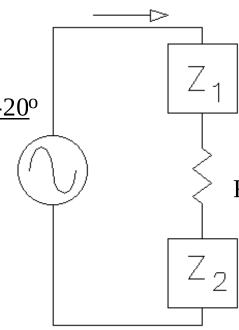
The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out (partially resonate).

Check Kirchoff's Current Law: $\mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j \cdot mA \overset{?}{=} \mathbf{I_{L1}} = 29.485 + 282.569j \cdot mA$

yes

Ex. 3 a) Find $\mathbf{Z_2}$.

$i(t) = 25\cdot mA\cdot\cos\left(377\cdot\frac{rad}{sec}\cdot t + 10\cdot deg\right)$



$\mathbf{Z_1} := (120 - 60\cdot j)\cdot\Omega$

$\mathbf{V_{in}} = 10V\ \underline{/-20^o}$

$R := 50\cdot\Omega$

$\mathbf{Z_2} = ?$

$\mathbf{I} := 25\cdot mA\cdot e^{j\cdot 10\cdot deg}$

$\mathbf{V_{in}} := 10\cdot V\cdot e^{-j\cdot 20\cdot deg}$

$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10\cdot V}{25\cdot mA}\ \underline{/ -20 - 10^o} = 400\Omega\ \underline{/-30^o}$

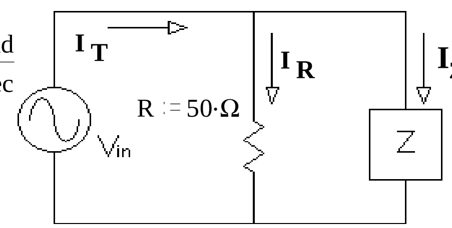
$\mathbf{Z_T} = 346.41 - 200j\ \bullet\Omega$

$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200\cdot j\ )\cdot\Omega - 50\cdot\Omega - (120 - 60\cdot j)\cdot\Omega = 176.41 - 140j\ \bullet\Omega$

b) Circle 1: i) The source current leads the source voltage <--- answer, because $10^o > -20^o$.

ii) The source voltage leads the source current

Ex 4. a) Find $\mathbf{V_{in}}$ in polar form.



$\mathbf{I_Z} = 100\ \underline{/\ 0^o}$ mA

$R := 50\cdot\Omega$

$\mathbf{Z} := (80 - 60\cdot j)\cdot\Omega$

$\mathbf{I_Z} := 100\cdot mA$ $\quad \mathbf{Z} := (80 - 60\cdot j)\cdot\Omega$ $\quad \omega := 1000\cdot\frac{rad}{sec}$

$\mathbf{V_{in}} := \mathbf{I_Z}\cdot\mathbf{Z}$ $\quad \mathbf{V_{in}} = 8 - 6j\ \bullet V$

$\sqrt{8^2 + 6^2} = 10$ $\quad \operatorname{atan}\left(\frac{-6}{8}\right) = -36.87\ \bullet deg$

$\mathbf{V_{in}} = 10V\ \underline{/-36.9^o}$

b) Find $\mathbf{I_T}$ in polar form. $\quad \mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{10\cdot V}{50\cdot\Omega}\ \underline{/-36.9^o} = \frac{10\cdot V}{50\cdot\Omega}\cdot\cos(-36.9\cdot deg) + j\cdot\frac{10\cdot V}{50\cdot\Omega}\cdot\sin(-36.9\cdot deg) = 160 - 120i\ \bullet mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (160 - 120\cdot j\ )\cdot mA + 100\cdot mA = 260 - 120j\ \bullet mA$

$\sqrt{260^2 + 120^2} = 286.356$ $\quad \operatorname{atan}\left(\frac{-120}{260}\right) = -24.78\ \bullet deg$ $\quad \mathbf{I_T} = 286mA\ \underline{/-24.8^o}$

c) Circle 1: i) The source current leads the source voltage answer i), $-24.8^o > -36.9^o$

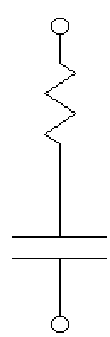
ii) The source voltage leads the source current

d) The impedance $\mathbf{Z}$ (above) is made of two components in series. What are they and what are their values?

$\mathbf{Z} = 80 - 60j\ \bullet\Omega$

Must have a resistor because there is a real part.

$R := \operatorname{Re}(\mathbf{Z})$ $\quad R = 80\ \bullet\Omega$



Must have a capacitor because the imaginary part is negative.

$\operatorname{Im}(\mathbf{Z}) = -60\ \bullet\Omega = \frac{-1}{\omega\cdot C}$ $\quad C := \frac{-1}{\omega\cdot\operatorname{Im}(\mathbf{Z})}$ $\quad C = 16.667\ \bullet\mu F$

Ex. 5   The impedance $\mathbf{Z} = 80 - 60j \cdot \Omega$ is made of two components in parallel. What are they and what are their values?

Must have a resistor because there is a real part.

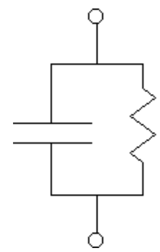
Must have an capacitor because the imaginary part is negative.

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C} \qquad \frac{1}{\mathbf{Z}} = \frac{1}{(80 - 60\cdot j)\cdot\Omega}\cdot\left(\frac{80+60\cdot j}{80+60\cdot j}\right) = \frac{80+60\cdot j}{80^2+60^2} = \frac{80+60\cdot j}{10,000}\cdot\frac{1}{\Omega}$$

$$\frac{1}{\mathbf{Z}} = 0.008 + 0.006i \cdot\Omega^{-1} = \frac{1}{R} + j\cdot\omega\cdot C$$

$$\frac{1}{R} = .008\cdot\frac{1}{\Omega} \qquad R := \frac{1}{.008\cdot\Omega^{-1}} \qquad R = 125\cdot\Omega$$

$$\omega\cdot C = .006\cdot\frac{1}{\Omega} \qquad C := \frac{.006\cdot\Omega^{-1}}{\omega} \qquad C = 6\cdot\mu F$$



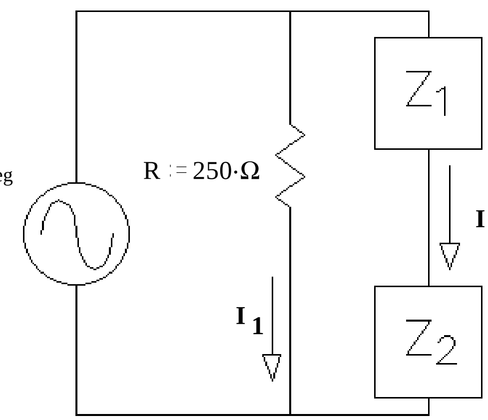
$R = 125\cdot\Omega$

Positive imaginary parts would require inductors

Ex. 6   a) Find $\mathbf{I_1}$

$$\omega := 20000\cdot\frac{rad}{sec}$$

$$\mathbf{V_{in}} := 20\cdot V\cdot e^{j\cdot 30\cdot deg}$$



$R := 250\cdot\Omega$

voltage $\mathbf{V_{Z1}} := (8 - 5\cdot j)\cdot V$

$\mathbf{I_2} := 20\cdot mA\cdot e^{j\cdot 20\cdot deg}$

$$\mathbf{I_1} := \frac{\mathbf{V_{in}}}{R} = \frac{20\cdot V}{250\cdot\Omega}\cdot e^{j\cdot 30\cdot deg} = 80\cdot mA\cdot e^{j\cdot 30\cdot deg}$$

polar division

b) Circle 1:   i) $\mathbf{V_{in}}$ leads $\mathbf{I_2}$ ii) $\mathbf{V_{in}}$ lags $\mathbf{I_2}$

Why? Show numbers: __30__ > __20__   ______ < ______

c) Find $\mathbf{Z_2}$ in polar form

Convert $\mathbf{V_{in}}$ to rectangular coordinates

$20\cdot V\cdot\cos(30\cdot deg) = 17.321\cdot V$ $20\cdot V\cdot\sin(30\cdot deg) = 10\cdot V$ pol to rect

$\mathbf{V_{in}} = 17.321 + 10j \cdot V$

$\mathbf{V_{Z2}} := \mathbf{V_{in}} - \mathbf{V_{Z1}}$ $\mathbf{V_{Z2}} = 9.321 + 15j \cdot V$ subtract

rect to pol $\sqrt{9.321^2 + 15^2} = |\mathbf{V_{Z2}}| = 17.66\cdot V$

$$\text{atan}\left(\frac{15}{9.321}\right) = \arg(\mathbf{V_{Z2}}) = 58.145\cdot deg$$

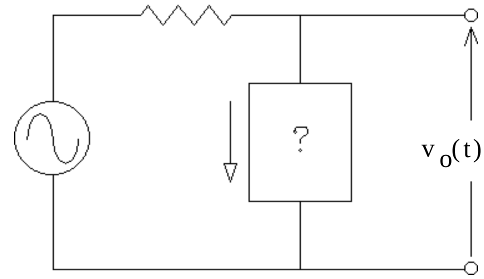
div $\mathbf{Z_2} := \frac{\mathbf{V_{Z2}}}{\mathbf{I_2}}$ $\frac{17.66\cdot V}{20\cdot mA} = 883\cdot\Omega$ $\angle$ $58.145\cdot deg - 20\cdot deg = 38.145\cdot deg$ $\mathbf{Z_2} = 883 \angle 38.15^\circ\ \Omega$

$\mathbf{Z_2} = 694.436 + 545.379j \cdot\Omega$

Ex. 7 You need to design a circuit in which the "output" voltage leads the input voltage ($v_S(t)$) by 40º of phase.

$R := 400 \cdot \Omega$

$v_S(t)$



$f := 1 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f$

$\omega = 6.283 \cdot 10^3 \cdot \frac{rad}{sec}$

a) What should go in the box: R, L, C?

$$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}} \cdot \mathbf{V_S}$$

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is 40º.

This can only happen if the angle of $\mathbf{Z_{box}}$ is positive, so $\mathbf{Z_{box}}$ is a inductor

b) Find its value. $\mathbf{V_o} =$ $\mathbf{V_o} = \frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L} \cdot \mathbf{V_S}$ angle $\frac{j \cdot \omega \cdot L}{R + j \cdot \omega \cdot L}$ is $90 - \operatorname{atan}\left(\frac{\omega \cdot L}{R}\right) = 40º$.

So: $\operatorname{atan}\left(\frac{\omega \cdot L}{R}\right) = 50º$ $\frac{\omega \cdot L}{R} = \tan(50 \cdot deg) = 1.192$ $L = \frac{R \cdot 1.192}{\omega} = 75.9 \cdot mH$

c) Repeat if the "output" voltage should lag the input voltage ($v_S(t)$) by 20º of phase.

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is -20º. This can only happen if the angle of $\mathbf{Z_{box}}$ is negative, so $\mathbf{Z_{box}}$ is a capacitor

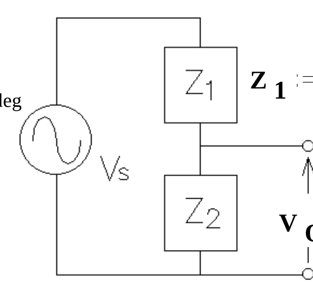
$$\mathbf{V_o} = \frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}} \cdot \mathbf{V_S}$$

angle $\frac{\frac{1}{j \cdot \omega \cdot C}}{R + \frac{1}{j \cdot \omega \cdot C}}$ is $-90 - \operatorname{atan}\left(\frac{-\frac{1}{\omega \cdot C}}{R}\right) = -90 - \operatorname{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right)$

$\operatorname{atan}\left(-\frac{1}{\omega \cdot C \cdot R}\right) = -70º$. $-\frac{1}{\omega \cdot C \cdot R} = \tan(-70 \cdot deg) = -2.747$ $C = \frac{1}{\omega \cdot R \cdot 2.747} = 0.145 \cdot \mu F$

Ex. 8 Find $\mathbf{V_O}$ in the circuit shown. Express it as a magnitude and phase angle (polar).

$\mathbf{V_S} := 6 \cdot V \cdot e^{j \cdot 18 \cdot deg}$



$\mathbf{Z_1} := 25 \cdot \Omega + 35j \cdot \Omega$

$\mathbf{Z_2} := 80 \cdot \Omega \cdot e^{-j \cdot 60 \cdot deg}$

$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}} \cdot \mathbf{V_S}$ Simple voltage divider

$\left|\mathbf{Z_2}\right| \cdot \cos(-60 \cdot deg) = 40 \cdot \Omega$ $\left|\mathbf{Z_2}\right| \cdot \sin(-60 \cdot deg) = -69.282 \cdot \Omega$ $\mathbf{Z_2} = 40 - 69.282j \cdot \Omega$

$\mathbf{Z_1} + \mathbf{Z_2} = 25 \cdot \Omega + 35j \cdot \Omega + 40 \cdot \Omega - 69.282 \cdot j \cdot \Omega = 65 - 34.282j \cdot \Omega = 73.486 \cdot \Omega \cdot e^{-j \cdot 27.81 \cdot deg}$

$$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}} \cdot \mathbf{V_S} = \frac{80 \cdot \Omega \cdot e^{-j \cdot 60 \cdot deg}}{73.486 \cdot \Omega \cdot e^{-j \cdot 27.81 \cdot deg}} \cdot \left(6 \cdot V \cdot e^{j \cdot 18 \cdot deg}\right) = \frac{80 \cdot \Omega}{73.486 \cdot \Omega} \cdot 6 \cdot V \cdot e^{j \cdot (-60 - (-27.81) + 18) \cdot deg} = 6.53 \cdot V \cdot e^{-j \cdot 14.2 \cdot deg}$$

ECE 2210/00 homework # 12 Due: Mon, 2/28/22 b

Read about complex numbers and phasors in your textbook (sections 2.26 & 2.27, starting on p.159).

1. For the complex numbers $\mathbf{z_1} := -4 + 5 \cdot j$ and $\mathbf{z_2} := 2 + 4 \cdot j$ Determine the following

   a) Does $|\mathbf{z_1} \cdot \mathbf{z_2}|$ equal $|\mathbf{z_1}| \cdot |\mathbf{z_2}|$?

   b) Does $\left|\frac{\mathbf{z_1}}{\mathbf{z_2}}\right|$ equal $\frac{|\mathbf{z_1}|}{|\mathbf{z_2}|}$?

   c) Does $|\mathbf{z_1} + \mathbf{z_2}|$ equal $|\mathbf{z_1}| + |\mathbf{z_2}|$?

2. a) Find the phasor for $v(t) = 8.4 \cdot \cos(100 \cdot t - 90 \cdot deg)$ Express in both forms, polar and rectangular.

   b) The phasor representation of a current is $\mathbf{I} := (5 + j \cdot 12) \cdot \mu A$ Find the time-domain representation, $i(t)$. $f := 600 \cdot Hz$

3. Add or subtract the sinusoidal voltages using phasors. Draw a phasor diagram which shows all 3 phasors, and give your final answer in time domain form.

   a) $v_1(t) = 1.5 \cdot V \cdot \cos(\omega \cdot t + 10 \cdot deg)$ $v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 25 \cdot deg)$ Find $v_3(t) = v_1(t) + v_2(t)$

   b) $v_1(t) = 1.5 \cdot V \cdot \cos(\omega \cdot t + 10 \cdot deg)$ $v_2(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 25 \cdot deg)$ Find $v_4(t) = v_1(t) - v_2(t)$

   you may add $\mathbf{V_4}$ to the phasor diagram you've already drawn for part a).

   c) $v_1(t) = 50 \cdot V \cdot \cos(\omega \cdot t - 60 \cdot deg)$ $v_2(t) = 24 \cdot V \cdot \cos(\omega \cdot t + 15 \cdot deg)$ Find $v_3(t) = v_1(t) + v_2(t)$

   d) $v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72 \cdot deg)$ $v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20 \cdot deg)$ Find $v_3(t) = v_1(t) + v_2(t)$

   e) $v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72 \cdot deg)$ $v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20 \cdot deg)$ Find $v_4(t) = v_2(t) - v_1(t)$

   you may add $\mathbf{V_4}$ to the phasor diagram you've already drawn for part d).

4. Express the impedance of a 5.2mH inductor at 60 Hz in polar form.

5. a) A capacitor impedance has a magnitude of 240Ω at a frequency of 1.8kHz. What is the value of capacitor?

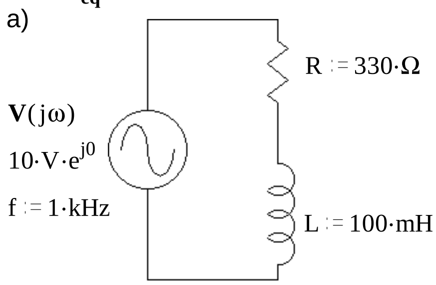
   b) What value inductor has the same impedance magnitude at the same frequency?

   c) Find the reactance (magnitude of the impedance with + or - sign) of this capacitor and this inductor at 3.6kHz?

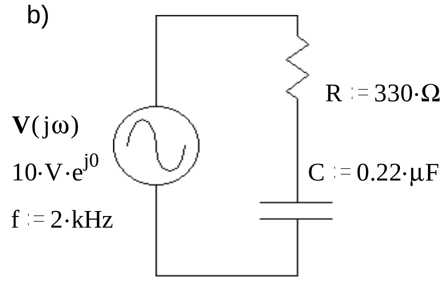
   d) What would be the total impedance of this inductance and this capacitance connected in series at 2.7kHz?

6. Find $\mathbf{Z_{eq}}$ in each case.

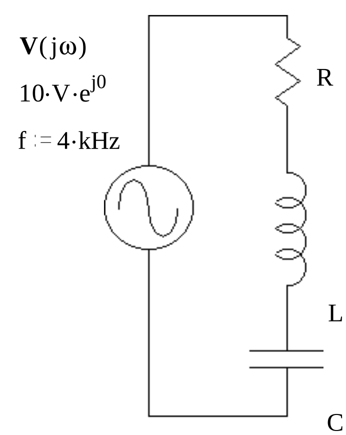
   a)



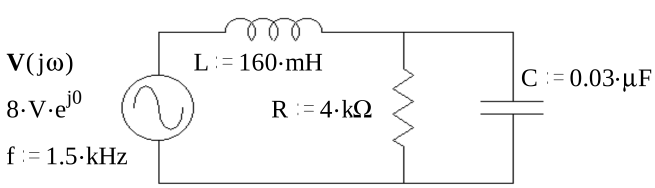
   b)



   c)



   d)



7. Find the current $\mathbf{I}(j\omega)$ in each case above.

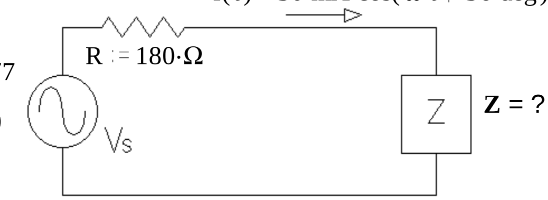
8. a) Find **Z**. Hint: Find the total impedance (R+**Z**) first.

$i(t) = 30\cdot mA\cdot\cos(\omega\cdot t + 36\cdot deg)$

b) Which leads, current or voltage?

$\omega := 377$

$v_s(t) = 14\cdot V\cdot\cos(\omega\cdot t + 16\cdot deg)$



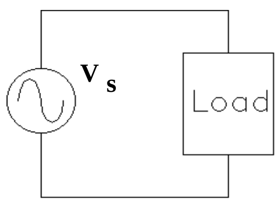
c) By how much?
I.E. what is the phase angle between the voltage and current?

9. a) A resistor and a capacitor are connected in <u>series</u> to create an impedance of $\mathbf{Z} := 50\cdot\Omega\cdot e^{-j\cdot 66\cdot deg}$ = 50Ω /-66º
at a frequency $f := 3\cdot kHz$ Find R and C.

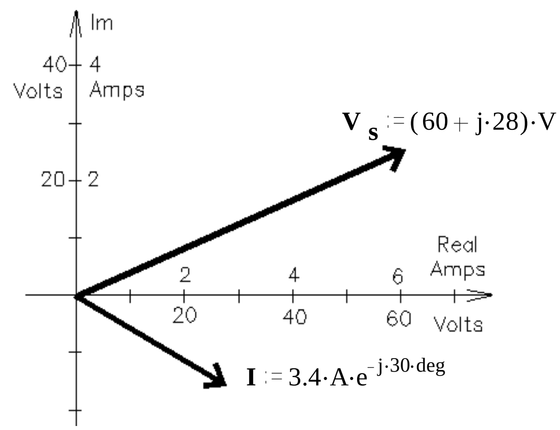
b) A resistor and a capacitor are connected in <u>parallel</u> to create an impedance of $\mathbf{Z} := 50\cdot\Omega\cdot e^{-j\cdot 66\cdot deg}$ = 50Ω /-66º
at a frequency $f := 3\cdot kHz$ Find R and C.

Hint: invert $\mathbf{Z_{eq}}$, Instead of solving this: $50\ \angle{-66deg} = \dfrac{1}{\dfrac{1}{R} + j\cdot\omega\cdot C}$ solve this: $\dfrac{1}{Z_{eq}} = 0.02\angle{66deg} = \dfrac{1}{R} + j\cdot\omega\cdot C$

10. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.



a) What is the magnitude of the impedance?

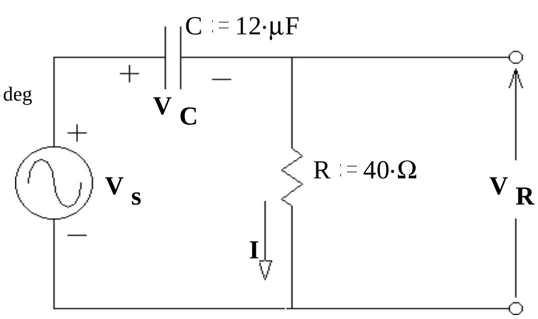
b) What is the value of the resistor?

c) What is the reactive component (type and value)?

11. For the circuit shown, draw a phasor diagram showing $\mathbf{V_S}$, **I**, $\mathbf{V_R}$, and $\mathbf{V_C}$. Draw the voltages to scale so that you can show that they obey KVL.

$\mathbf{V_s} := 12\cdot V\cdot e^{j\cdot 0\cdot deg}$

$f := 500\cdot Hz$



Answers

1. a) 28.636 = 28.636 Yes b) 1.432 = 1.432 Yes
c) 9.22 not = 10.875 No

2. a) 8.4 /-90º = $8.4\cdot e^{-j\cdot 90\cdot deg} = -8.4j$ b) $13\cdot\mu A\cdot\cos(3770\cdot t + 67.4\cdot deg)$

3. a) $v_1(t) + v_2(t) = 4.67\cdot\cos(\omega\cdot t + 20.2\cdot deg)\cdot V$

b) $v_1(t) - v_2(t) = 1.794\cdot\cos(\omega\cdot t - 142.5\cdot deg)\cdot V$ c) $v_1(t) + v_2(t) = 60.8\cdot\cos(\omega\cdot t - 37.6\cdot deg)\cdot V$

d) $v_1(t) + v_2(t) = 1.48\cdot\cos(\omega\cdot t + 17.6\cdot deg)\cdot V$ e) $v_2(t) - v_1(t) = 1.525\cdot\cos(\omega\cdot t - 56.15\cdot deg)\cdot V$

4. 1.96 Ω /90º 5. a) $0.368\cdot\mu F$ b) $21.2\cdot mH$ c) $-120\cdot\Omega$ $480\cdot\Omega$ d) $200\cdot j\cdot\Omega$

6. a) $(330 + 628.3\cdot j)\cdot\Omega$ = 709.7Ω /62.29º c) $R + \left(\omega\cdot L - \dfrac{1}{\omega\cdot C}\right)\cdot j$ d) 1.82kΩ /-15.2º

b) $(330 - 361.7\cdot j)\cdot\Omega$ = 489.6Ω /-47.63º

7. a) $(6.6 - 12.5\cdot j)\cdot mA$ = 14.1mA /-62.29º c) $\dfrac{10\cdot V}{\sqrt{R^2 + \left(\omega\cdot L - \dfrac{1}{\omega\cdot C}\right)^2}}$ angle: $-\operatorname{atan}\left(\dfrac{\omega\cdot L - \dfrac{1}{\omega\cdot C}}{R}\right)$

b) $(13.8 + 15.1\cdot j)\cdot mA$ = 20.4mA /47.63º

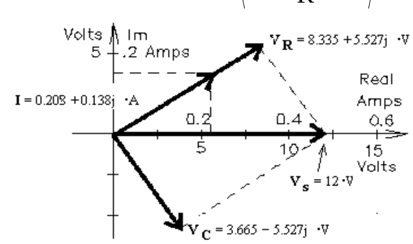
d) 4.4mA /15.2º

8. a) $259 - 160\cdot j$ b) The current leads the voltage c) 20º

9. a) $20.34\cdot\Omega$ $1.16\cdot\mu F$ b) $123\cdot\Omega$ $0.969\cdot\mu F$

10. a) $19.5\cdot\Omega$ b) $11.2\cdot\Omega$ c) inductor $42.3\cdot mH$

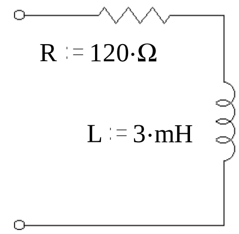
11.



The 2nd exam **will** include this material

Warning: This homework is longer than normal -- DO NOT put it off until the last minute.

1. For the circuit shown, find the following:

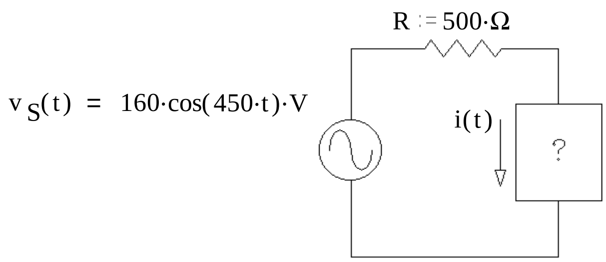


   a) At what frequency would the magnitude of the total impedance be 240Ω?

   b) At this frequency, what is the phase angle of the impedance?

   c) At this frequency, you want to add a capacitor in series to make the circuit appear purely resistive (the impedance has no imaginary component). Find the value of the capacitor.

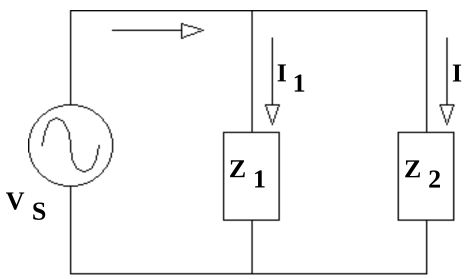
2. You need to design a circuit in which the current (i(t)) leads the voltage ($v_S(t)$) by 36º of phase.

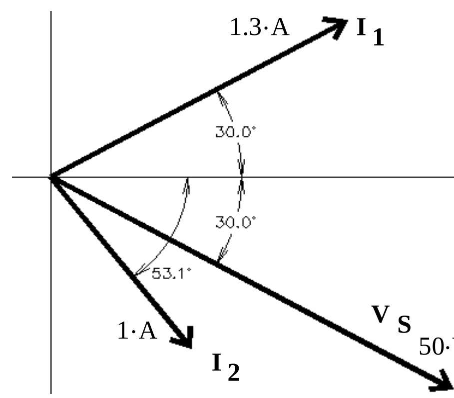


   a) What should go in the box: R, L, C?

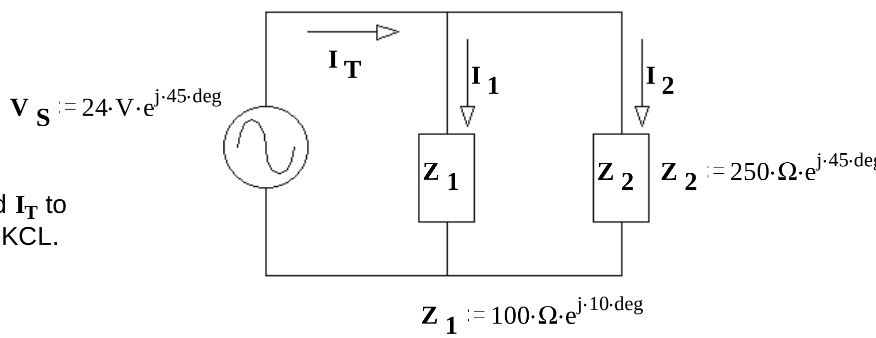
   b) Find its value.

3. The phasor diagram at right shows the source voltage and two branch currents of a parallel circuit. Find the inpedance of each of the two branches.



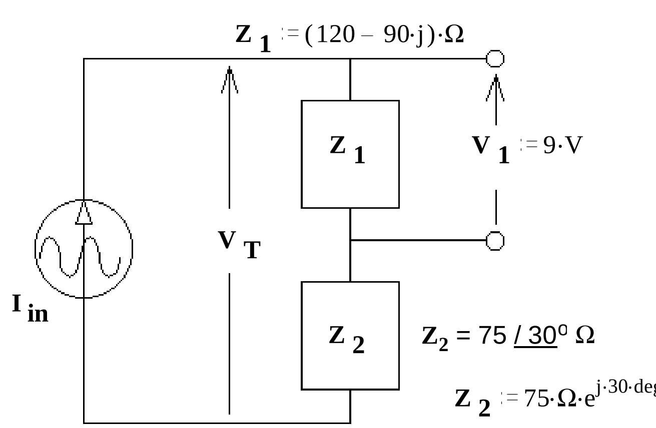


4. a) Find all the currents, $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_T}$.



   b) Draw a phasor diagram showing $\mathbf{I_1}$, $\mathbf{I_2}$, and $\mathbf{I_T}$ to scale so that you can show that they obey KCL.

5. a) Find the AC current source, $\mathbf{I_{in}}$ in polar form.



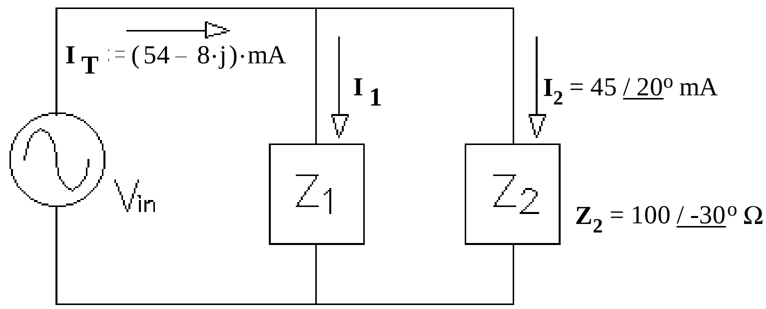
   b) Find $\mathbf{V_T}$.

   c) Choose one:

      i) The source current leads the source voltage.

      ii) The source current lags the source voltage.

6. a) Find $\mathbf{Z_1}$.



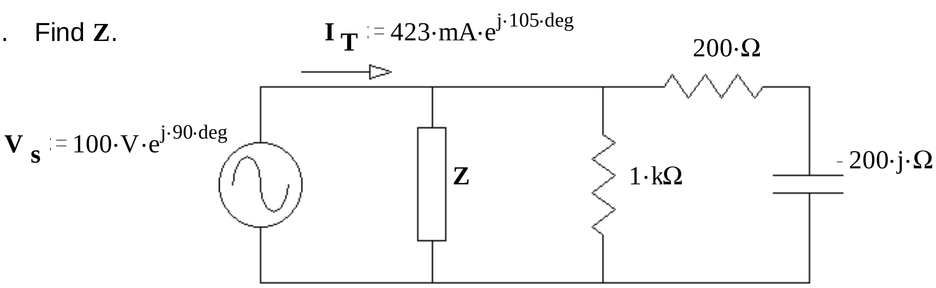
b) To make $\mathbf{Z_1}$ in the simplest way, what part(s) would you need? Just determine the needed part(s) from the list below and state why you made that choice, don't find the values.

resistor capacitor inductor power supply current source
Thevenin resistor Ideal transformer voltmeter ammeter scope

c) Choose one: i) $\mathbf{I_2}$ leads the source voltage ($\mathbf{V_{in}}$) ii) $\mathbf{I_2}$ lags the source voltage ($\mathbf{V_{in}}$)

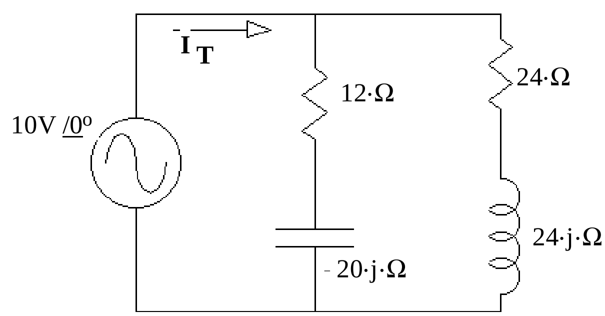
d) Choose one: i) $\mathbf{I_1}$ leads $\mathbf{I_2}$ ii) $\mathbf{I_1}$ lags $\mathbf{I_2}$

7. Find $\mathbf{Z}$.



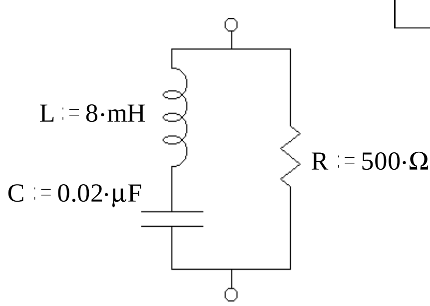
8. a) Find the total impedance of the circuit.

b) Find $\mathbf{I_T}$.



9. Find $\mathbf{Z_{eq}}$ in simple polar form.

$f := 8000\cdot Hz$



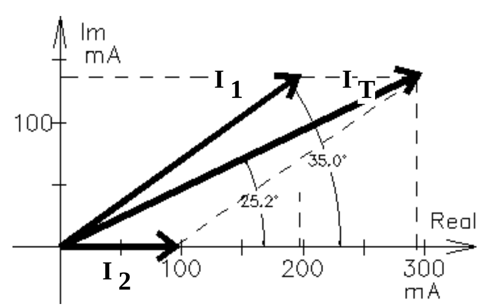
Answers

1. a) $11\cdot kHz$ b) 60^o c) $0.0694\cdot\mu F$

2. a) C b) $6.12\cdot\mu F$

3. $Z_1 = (19.2 - 33.3\cdot j)\cdot\Omega$ $Z_2 = (46.0 + 19.6\cdot j)\cdot\Omega$

4. a) $(0.197 + 0.138\cdot j)\cdot A + 0.096\cdot A = 0.293 + 0.138j\ \cdot A$ b)



5. a) $60 \angle 36.87^o$ mA b) $11.54 \angle 21^o$ V c) i)

6. a) $172 \angle 53.4^o\ \Omega$ b) phase angle > 0, resistor and inductor

c) i) d) ii)

7. $657\ \Omega \angle 67.4^o$ 8. a) $21.86\Omega \angle -20.38^o$ b) $0.457A \angle 20.38^o$

9. $382\Omega \angle -40.2^o$