1. Analysis of the circuit shown yields the characteristic equation below. The switch has been in the top position for a long time and is switched down at time $t = 0$. Find the initial conditions and write the full expression for $i_L(t)$, including all the constants that you find.

\[
s^2 + \left( \frac{1}{C \cdot R_2} + \frac{R_1}{L} \right) s + \left( \frac{R_1}{L \cdot C \cdot R_2} + \frac{1}{L \cdot C} \right) = 0
\]

\[
\left( \frac{1}{C \cdot R_2} + \frac{R_1}{L} \right) = 1000 \cdot \text{sec}^{-1}
\]

\[
\left( \frac{R_1}{L \cdot C \cdot R_2} + \frac{1}{L \cdot C} \right) = 2.222 \cdot 10^6 \cdot \text{sec}^{-2}
\]

\[
s^2 + 1000 \cdot \frac{1}{\text{sec}} s + 2.222 \cdot 10^6 \cdot \frac{1}{\text{sec}^2} = 0
\]

2. What value of $R_1$ would make the above circuit critically damped?

3. Look at the circuit in HW 17, problem 2. Change $R_1$ and $R_2$ to 50Ω and consider the voltage across $R_1$ to be the output voltage. The transfer function would be:

\[
H(s) = \frac{V_{R1}(s)}{V_{in}(s)} = \frac{s^2 + \frac{R_2}{L} s + \frac{1}{L \cdot C}}{s^2 + \frac{R_1 \cdot R_2 \cdot C + L}{R_1 \cdot L \cdot C} s + \frac{R_1 + R_2}{R_1 \cdot L \cdot C}} = \frac{s^2 + 2500 \cdot s + 1.25 \cdot 10^6}{s^2 + 3000 \cdot s + 2.5 \cdot 10^6}
\]

a) What are the poles and zeros of this transfer function?

b) Plot these poles and zeros on the complex plane.

4. A feedback system is shown in the figure. a) What is the transfer function of the whole system, with feedback.

\[
H(s) = \frac{X_{out}(s)}{X_{in}(s)} = ?
\]

Simplify your expression for $H(s)$ so that the denominator is a simple polynomial.

b) $G := 5$ Find the poles and zeroes of the system.

c) What type of damping response does this system have?

d) Find the value of G to make the transfer function critically damped.

e) If $G$ is double the value found in part d) what will the damping response of the system will be?
5. a) A feedback system is shown in the figure. What is the transfer function of the whole system, with feedback.

\[ \frac{X_{\text{out}}(s)}{X_{\text{in}}(s)} = \ ? \]

Simplify your expression for \( H(s) \) so that the denominator is a simple polynomial.

b) Find the maximum value of \( F \) so that the system does not become underdamped.

c) Find the transfer function with \( F = 0.2 \)

d) With \( F = 0.2 \), at what value of \( s \) can the system produce an output even with no input? (That is, what value of \( s \) makes \( H(s) = \infty \)?)

e) Does the transfer function have a zero? Answer no or find the \( s \) value of that zero.

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**Answers**

1. \( i_L(0) = 75 \text{ mA} \) \( v_C(0) = 11.25 \text{ V} \)

\[ i_L(t) = 25 \text{ mA} e^{-\frac{500}{\text{sec}}} \left[ 50 \text{ mA} \cos \left( \frac{1404}{\text{sec}} t \right) - 457 \text{ mA} \sin \left( \frac{1404}{\text{sec}} t \right) \right] \]

2. \( R_1 = 36.64 \Omega \)

3. a) Zeroes: -691 & -1809  Poles: -1500 & -500 j  

b) \[ R(s) = \frac{G(s + 60)}{s^3 + 90s + 1800 + G \cdot 10} \]

4. a) \[ \frac{G(s + 60)}{s^2 + 90s + 1800 + G \cdot 10} \]

b) poles: -31.8 & -58.2  zero: -60  

c) overdamped  d) 22.5  e) underdamped

5. a) \[ 1000 \frac{s + 40}{s^2 + 65s + 1000 + 200 \cdot F} \]

b) 0.281  

c) \[ 1000 \frac{s + 40}{s^2 + 65s + 1040} \]

d) -28.5 or -36.5  

e) -40