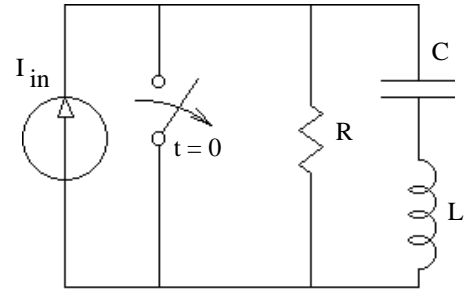4. In the circuit shown, when the switch is opened, the current
   I_{in} (current source) is forced to flow through the circuit.

   a) Write a differential equation for i_L. Hint: use LaPlace impedance method.
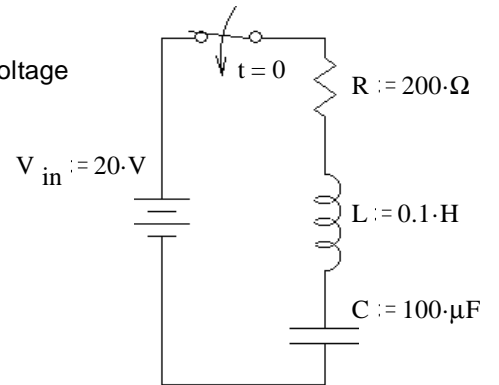

b) Write a differential equation for v_C.

c) Find the characteristic equation for the circuit shown.

1. A series RLC circuit with $R = 200\ \Omega$, $L = 0.1\ H$ and $C = 100\ \mu F$ has a constant voltage $V = 20$ volts applied at $t = 0$. The capacitor has no initial charge.

   a) Find the characteristic equation of the circuit at right.
      (hint: take $i(t)$ as the "output")

$V_{in} := 20 \cdot V$

$t = 0$

$R := 200 \cdot \Omega$

$L := 0.1 \cdot H$

$C := 100 \cdot \mu F$

   b) Find the solutions to the characteristic equation.

   c) Is this circuit over, under, or critically damped?

   d) The switch is switched down at time $t = 0$. Find the final and initial conditions:
      final: $i(\infty)$ initial: $i(0)$, $v_C(0)$ and $\dfrac{d}{dt} i(0)$

e) Write the full expression for i(t), including all the constants that you find.

2. A series RLC circuit with $R = 200\ \Omega$, $L = 0.1$ H and $C = ?\ \mu F$ is to be made critically damped by the selection of the capacitance. Find the required value of C.
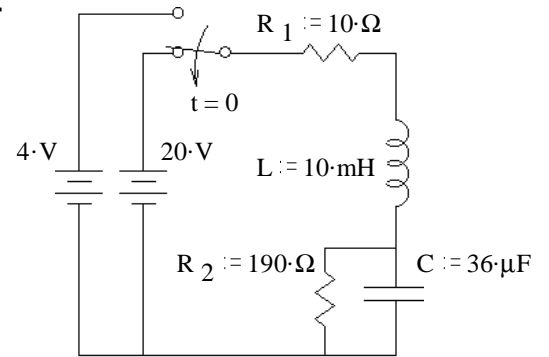
3. Find the ringing frequency of a series RLC circuit in which $R = 200\ \Omega$, $L = 0.1$ H and $C = 5\ \mu F$. (The ringing frequency is the ω part of $s_1 = \alpha + j\omega$). Express your answer in Hz.

5. The characteristic equation of the circuit shown is:

$$0 = s^2 + \left(\frac{R_1}{L} + \frac{1}{C \cdot R_2}\right) \cdot s + \left(\frac{R_1}{L \cdot C \cdot R_2} + \frac{1}{L \cdot C}\right)$$

a) Find the solutions to the characteristic equation.

$R_1 := 10 \cdot \Omega$

$t = 0$

$4 \cdot V$ $20 \cdot V$ $L := 10 \cdot mH$

$R_2 := 190 \cdot \Omega$ $C := 36 \cdot \mu F$

b) Is this circuit over, under, or critically damped?

c) The switch has been in the top position for a long time and is switched down at time $t = 0$.
   Find the final and initial conditions:

d) Write the full expression for $i_L(t)$, including all the constants that you find.

Answers

1. a) $\dfrac{R}{L}\cdot\dfrac{d}{dt}i_{in} \;=\; \dfrac{d^2}{dt^2}i_L + \dfrac{R}{L}\cdot\dfrac{d}{dt}i_L + \dfrac{1}{L\cdot C}\cdot i_L$ b) $\dfrac{R}{L\cdot C}\cdot i_{in} \;=\; \dfrac{d^2}{dt^2}v_c + \dfrac{R}{L}\cdot\dfrac{d}{dt}v_c + \dfrac{1}{L\cdot C}\cdot v_c$ c) $s^2 + \dfrac{R}{L}\cdot s + \dfrac{1}{L\cdot C} \;=\; 0$

2. a) $0 \;=\; s^2 + \dfrac{R}{L}\cdot s + \dfrac{1}{L\cdot C}$ b) $-51.3\cdot\dfrac{1}{sec} \quad -1949\cdot\dfrac{1}{sec}$ c) overdamped

   d) $i(\infty) = 0\cdot A$ $i(0) = 0$ $V_C(0) = 0$ $\dfrac{d}{dt}i(0) = 200\cdot\dfrac{A}{sec}$ e) $i(t) = 0.1054\cdot e^{-\frac{51.3}{sec}\cdot t} - 0.1054\cdot e^{-\frac{1949}{sec}\cdot t}$

3. $10\cdot\mu F$ 4. $159\cdot Hz$

5. a) $-573.1 \pm 1611j \;\; 1/sec$ b) underdamped   c) $19\cdot V$ $100\cdot mA$ $3.8\cdot V$ $20\cdot mA$ $0\cdot\dfrac{V}{sec}$ $1600\cdot\dfrac{A}{sec}$

d) $i_L(t) := 100\cdot mA + e^{\frac{-573.1}{sec}\cdot t}\cdot\left(-80\cdot\cos\left(\dfrac{1611}{sec}\cdot t\right) + 964.7\cdot\sin\left(\dfrac{1611}{sec}\cdot t\right)\right)\cdot mA$

$1600 \text{ mA/ms} = \text{initial slope, shown by dashed line}$ x-axis: $t := 0, .0001 ... .012\cdot sec$

$i_L(t)$

(mA)

Plot is **NOT REQUIRED**

This plot shows the ringing that you should expect when you get two complex solutions to the characteristic equation, as we did in this case.

$100\cdot mA + \sqrt{80^2 + 964.7^2}\cdot mA\cdot e^{\frac{-573.1}{sec}\cdot t}$ = ringing decay

final
100·mA

initial
20·mA

$T := 3.9\cdot ms$ $\dfrac{1}{T} = 256.4\cdot Hz$ $\dfrac{2\cdot\pi}{T} = 1611\cdot\dfrac{rad}{sec}$

|< - - - - - - period - - - - - - >|