Name $\qquad$ 2200 Due: Fri, 10/6/23 May be handed in Tue., 10/17 for full credit
Read about complex numbers and phasors in your textbook (sections 2.26 \& 2.27, starting on p. 159 (3rd Ed.)).

1. For the complex numbers $\quad \mathbf{z}_{\mathbf{1}}:=-4+5 \cdot j$ and $\quad \mathbf{z}_{\mathbf{2}}:=2+4 \cdot j$ Determine the following:
a) Does $\left|\mathbf{z}_{1} \cdot \mathbf{z}_{\mathbf{2}}\right|$ equal $\left|\mathbf{z}_{\mathbf{1}}\right| \cdot\left|\mathbf{z}_{\mathbf{2}}\right|$ ?
b) Does $\left|\frac{\mathbf{z}_{\mathbf{1}}}{\mathbf{z}_{\mathbf{2}}}\right| \quad$ equal $\quad \frac{\left|\mathbf{z}_{\mathbf{1}}\right|}{\left|\mathbf{z}_{\mathbf{2}}\right|} \quad$ ?
c) Does $\left|\mathbf{z}_{\mathbf{1}}+\mathbf{z}_{\mathbf{2}}\right|$ equal $\left|\mathbf{z}_{\mathbf{1}}\right|+\left|\mathbf{z}_{\mathbf{2}}\right|$ ?
2. a) Find the phasor for $\mathrm{v}(\mathrm{t})=8.4 \cdot \cos (100 \cdot \mathrm{t}-90 \cdot \mathrm{deg})$ Express in both forms, polar and rectangular.
b) The phasor representation of a current is $\mathbf{I}:=(5+\mathrm{j} \cdot 12) \cdot \mu \mathrm{A} \quad$ Find the time-domain representation, $\mathrm{i}(\mathrm{t}) . \mathrm{f}:=600 \cdot \mathrm{~Hz}$
3. Add or subtract the sinusoidal voltages using phasors. Draw a phasor diagram which shows all 3 phasors, and give your final answer in time domain form.
a) $\mathrm{v}_{1}(\mathrm{t})=50 \cdot \mathrm{v} \cdot \cos (\omega \cdot \mathrm{t}-60 \cdot \operatorname{deg}) \quad \mathrm{v}_{2}(\mathrm{t})=24 \cdot \mathrm{~V} \cdot \cos (\omega \cdot \mathrm{t}+15 \cdot \operatorname{deg}) \quad$ Find $\quad \mathrm{v}_{3}(\mathrm{t})=\mathrm{v}_{1}(\mathrm{t})+\mathrm{v}_{2}(\mathrm{t})$
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b) $\mathrm{v}_{1}(\mathrm{t})=0.9 \cdot \mathrm{~V} \cdot \cos (\omega \cdot \mathrm{t}+72 \cdot \mathrm{deg})$
$\mathrm{v}_{2}(\mathrm{t})=1.2 \cdot \mathrm{~V} \cdot \cos (\omega \cdot \mathrm{t}-20 \cdot \mathrm{deg})$
Find $\quad v_{3}(t)=v_{1}(t)+v_{2}(t)$

c) $\mathrm{v}_{1}(\mathrm{t})=0 \cdot 9 \cdot \mathrm{~V} \cdot \cos (\omega \cdot \mathrm{t}+72 \cdot \mathrm{deg}) \quad \mathrm{v}_{2}(\mathrm{t})=1.2 \cdot \mathrm{~V} \cdot \cos (\omega \cdot \mathrm{t}-20 \cdot \mathrm{deg})$

Find $\quad v_{4}(t)=v_{2}(t)-v_{1}(t) \quad$ you may add $\mathbf{V}_{4}$ to the phasor diagram you've already drawn for part b).
5. a) A capacitor impedance has a magnitude of $240 \Omega$ at a frequency of 1.8 kHz . What is the value of capacitor?
b) What value inductor has the same impedance magnitude at the same frequency?
c) Find the reactance (magnitude of the impedance with + or - sign) of this capacitor and this inductor at 3.6 kHz ?
d) What would be the total impedance of this inductance and this capacitance connected in series at 2.7 kHz ?
6. Find $\mathbf{Z}_{\mathrm{eq}}$ in each case.
a)

6. continued c)

d)

7. Find the current $\mathbf{I}(\mathrm{j} \omega)$ in each case above.
a)
b)
c)
d)
8. a) Find $\mathbf{Z}$. Hint: Find the total impedance (R+Z) first.

b) Which leads, current or voltage?
c) By how much? I.E. what is the phase angle between the voltage and current?

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9. a) A resistor and a capacitor are connected in series to create an impedance of $\mathbf{Z}:=50 \cdot \Omega \cdot \mathrm{e}^{-\mathrm{j} \cdot 66 \cdot \mathrm{deg}}=50 \Omega \underline{/-66}^{\circ}$ at a frequency $f:=3 \cdot \mathrm{kHz} \quad$ Find R and C .
b) A resistor and a capacitor are connected in parallel to create an impedance of $\mathbf{Z}:=50 \cdot \Omega \cdot e^{-\mathrm{j} \cdot 66 \cdot \mathrm{deg}}=50 \Omega \underline{/-66}{ }^{\mathrm{o}}$ at a frequency $\mathrm{f}:=3 \cdot \mathrm{kHz} \quad$ Find R and C .
Hint: invert $\mathbf{Z}_{\mathbf{e q}}$, Instead of solving this: $\quad 50 \underline{/-66 \mathrm{deg}}=\frac{1}{\frac{1}{\mathrm{R}}+\mathrm{j} \cdot \omega \cdot \mathrm{C}} \quad$ solve this: $\frac{1}{\mathrm{Z}_{\mathrm{eq}}}=0.02 / \underline{66 \mathrm{deg}}=\frac{1}{\mathrm{R}}+\mathrm{j} \cdot \omega \cdot \mathrm{C}$
10. The phasor diagram at right shows the voltage and current in the circuit below


Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz .
a) What is the magnitude of the impedance?

b) What is the value of the resistor?
c) What is the reactive component (type and value)?

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11. For the circuit shown, draw a phasor diagram showing $\mathbf{V}_{\mathbf{S}}, \mathbf{I}, \mathbf{V}_{\mathbf{R}}$, and $\mathbf{V}_{\mathbf{C}}$. Draw the voltages to scale so that you can show that they obey KVL.



## Answers

1. a) $28.636=28.636 \mathrm{Yes}$
b) $1.432=1.432 \mathrm{Yes}$
c) $9.22 \mathrm{not}=10.875 \mathrm{No}$
2. a) $8.4 /-90^{\circ}=8.4 \cdot \mathrm{e}^{-\mathrm{j} \cdot 90 \cdot \mathrm{deg}}=-8.4 \mathrm{j}$
3. a) $60.8 \cdot \cos (\omega \cdot \mathrm{t}-37.6 \cdot \operatorname{deg}) \cdot V$
4. a) $0.368 \cdot \mu \mathrm{~F}$
b) $13 \cdot \mu \mathrm{~A} \cdot \cos (3770 \cdot \mathrm{t}+67.4 \cdot \mathrm{deg})$
5. $1.96 \Omega / 90^{\circ}$
b) $1.48 \cdot \cos (\omega \cdot \mathrm{t}+17.6 \cdot \mathrm{deg}) \cdot \mathrm{V}$
c) $1.525 \cdot \cos (\omega \cdot \mathrm{t}-56.15 \cdot \mathrm{deg}) \cdot \mathrm{V}$
6. a) $(330+628.3 \cdot j) \cdot \Omega=709.7 \Omega / \underline{62.29^{\circ}}$
b) $21.2 \cdot \mathrm{mH}$
c) $-120 \cdot \Omega \quad 480 \cdot \Omega$
d) $200 \cdot \mathrm{j} \cdot \Omega$
b) $(330-361.7 \cdot \mathrm{j}) \cdot \Omega=489.6 \Omega /-47.63^{\circ}$
7. a) $(6.6-12.5 \cdot \mathrm{j}) \cdot \mathrm{mA}=14.1 \mathrm{~mA} /-62.29^{\circ}$
b) $(13.8+15.1 \cdot \mathrm{j}) \cdot \mathrm{mA}=20.4 \mathrm{~mA} / 47.63^{\circ}$
d) $4.4 \mathrm{~mA} / 15.2^{\circ}$
c) $R+\left(\omega \cdot L-\frac{1}{\omega \cdot C}\right) \cdot j$
d) $1.82 \mathrm{k} \Omega /-15.2^{\circ}$
c) $\frac{10 \cdot \mathrm{~V}}{\sqrt{R^{2}+\left(\omega \cdot L-\frac{1}{\omega \cdot C}\right)^{2}}}$

8. a) $259-160 \cdot j$
b) The current leads the voltage
c) $20^{\circ}$
9. a) $20.34 \cdot \Omega$
$1.16 \cdot \mu \mathrm{~F}$
b) $123 \cdot \Omega$
$0.969 \cdot \mu \mathrm{~F}$
10. a) $19.5 \cdot \Omega$
b) $11.2 \cdot \Omega$
c) inductor
$42.3 \cdot \mathrm{mH}$
