2200 Due: Fri, 10/6/23 May be handed in Tue., 10/17 for full credit

Read about complex numbers and phasors in your textbook (sections 2.26 & 2.27, starting on p.159 (3rd Ed.)).

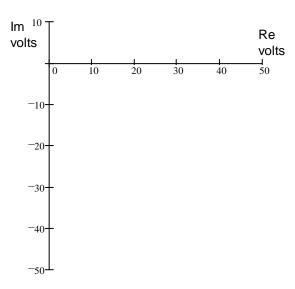
- 1. For the complex numbers  $\mathbf{z_1} := -4 + 5 \cdot \mathbf{j}$  and  $\mathbf{z_2} := 2 + 4 \cdot \mathbf{j}$  Determine the following:
  - a) Does  $|\mathbf{z}_1 \cdot \mathbf{z}_2|$  equal  $|\mathbf{z}_1| \cdot |\mathbf{z}_2|$  ?
  - b) Does  $\begin{vmatrix} z_1 \\ z_2 \end{vmatrix}$  equal  $\frac{|z_1|}{|z_2|}$  ?
  - c) Does  $\left|\mathbf{z_1} + \mathbf{z_2}\right|$  equal  $\left|\mathbf{z_1}\right| + \left|\mathbf{z_2}\right|$  ?

- 2. a) Find the phasor for  $v(t) = 8.4 \cdot \cos(100 \cdot t 90 \cdot \deg)$  Express in both forms, polar and rectangular.
  - b) The phasor representation of a current is  $I = (5 + j \cdot 12) \cdot \mu A$  Find the time-domain representation, i(t).  $f = 600 \cdot Hz$

- 3. Add or subtract the sinusoidal voltages using phasors. Draw a phasor diagram which shows all 3 phasors, and give your final answer in time domain form.
- a)  $v_1(t) = 50 \cdot V \cdot \cos(\omega \cdot t 60 \cdot \text{deg})$   $v_2(t) = 24 \cdot V \cdot \cos(\omega \cdot t + 15 \cdot \text{deg})$  Find  $v_3(t) = v_1(t) + v_2(t)$

3. continued

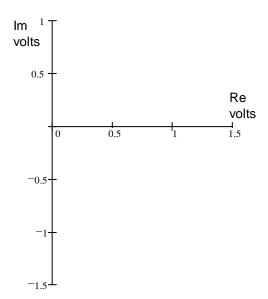
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b) 
$$v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72 \cdot \deg)$$
  $v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20 \cdot \deg)$ 

$$v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20 \cdot \deg)$$

Find 
$$v_3(t) = v_1(t) + v_2(t)$$



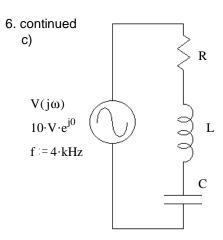
c) 
$$v_1(t) = 0.9 \cdot V \cdot \cos(\omega \cdot t + 72 \cdot \text{deg})$$
  $v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20 \cdot \text{deg})$ 

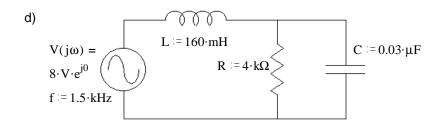
$$v_2(t) = 1.2 \cdot V \cdot \cos(\omega \cdot t - 20 \cdot \deg)$$

 $\text{Find} \quad v_4(t) \ = \ v_2(t) - v_1(t) \quad \text{ you may add } \mathbf{V_4} \text{ to the phasor diagram you've already drawn for part b)}.$ 

- 5. a) A capacitor impedance has a magnitude of  $240\Omega$  at a frequency of  $1.8 \mathrm{kHz}$ . What is the value of capacitor?
  - b) What value inductor has the same impedance magnitude at the same frequency?
  - c) Find the reactance (magnitude of the impedance with + or sign) of this capacitor and this inductor at 3.6kHz?
  - d) What would be the total impedance of this inductance and this capacitance connected in series at 2.7kHz?
- 6. Find  $\mathbf{Z}_{eq}$  in each case. a)  $R := 330 \cdot \Omega$  $L = 100 \cdot mH$

b) 
$$V(j\omega) = R := 330 \cdot \Omega$$
 
$$f := 2 \cdot kHz$$
 
$$C := 0.22 \cdot \mu F$$





7. Find the current  $I(j\omega)$  in each case above.

a)

b)

c)

d)

8. a) Find Z. Hint: Find the total impedance (R+Z) first.

 $R := 180 \cdot \Omega$  $i(t) = 30 \cdot mA \cdot cos(\omega \cdot t + 36 \cdot deg)$ 

 $v_{S}(t) = 14 \cdot V \cdot \cos(\omega \cdot t + 16 \cdot \deg)$  $\omega := 377$ 

Z = ?

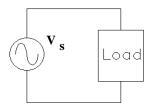
- b) Which leads, current or voltage?
- c) By how much? I.E. what is the phase angle between the voltage and current?

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- 9. a) A resistor and a capacitor are connected in series to create an impedance of  $\mathbf{Z} := 50 \cdot \Omega \cdot \mathrm{e}^{-\mathrm{j} \cdot 66 \cdot \mathrm{deg}} = 50\Omega / -66^{\circ}$  at a frequency  $f := 3 \cdot \mathrm{kHz}$  Find R and C.
  - b) A resistor and a capacitor are connected in parallel to create an impedance of  $\mathbf{Z} := 50 \cdot \Omega \cdot e^{-j \cdot 66 \cdot deg} = 50\Omega / -66^{o}$  at a frequency  $f := 3 \cdot kHz$  Find R and C.

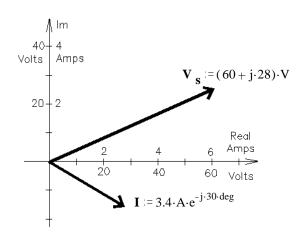
Hint: invert  $\mathbf{Z}_{eq}$ , Instead of solving this:  $50 \, \underline{/-66 deg} = \frac{1}{\frac{1}{R} + \mathbf{j} \cdot \omega \cdot C}$  solve this:  $\frac{1}{Z_{eq}} = 0.02 \underline{/66 deg} = \frac{1}{R} + \mathbf{j} \cdot \omega \cdot C$ 

10. The phasor diagram at right shows the voltage and current in the circuit below



Assume the load consists of a resistor in series with a reactive component and the frequency is 60 Hz.

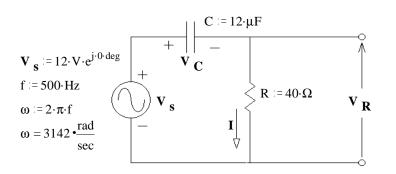
a) What is the magnitude of the impedance?

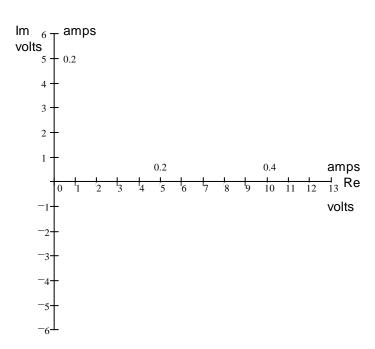


- b) What is the value of the resistor?
- c) What is the reactive component (type and value)?

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11. For the circuit shown, draw a phasor diagram showing  $V_S$ , I,  $V_R$ , and  $V_C$ . Draw the voltages to scale so that you can show that they obey KVL.





## **Answers**

1. a) 
$$28.636 = 28.636$$
 Yes

b) 
$$1.432 = 1.432$$
 Yes

c) 
$$9.22 \text{ not} = 10.875 \text{ No}$$

11.

2. a) 
$$8.4 \frac{\text{--}90^{\circ}}{\text{--}} = 8.4 \cdot e^{-j \cdot 90 \cdot \text{deg}} = -8.4j$$

b) 
$$13 \cdot \mu A \cdot \cos(3770 \cdot t + 67.4 \cdot deg)$$

3. a) 
$$60.8 \cdot \cos(\omega \cdot t - 37.6 \cdot \deg) \cdot V$$

b) 
$$1.48 \cdot \cos(\omega \cdot t + 17.6 \cdot deg) \cdot V$$

c) 
$$1.525 \cdot \cos(\omega \cdot t - 56.15 \cdot \deg) \cdot V$$

c) 
$$-120\cdot\Omega$$
  $480\cdot\Omega$ 

d) 
$$200 \cdot \mathbf{j} \cdot \Omega$$

6. a) 
$$(330 + 628.3 \cdot j) \cdot \Omega = 709.7\Omega / 62.29^{\circ}$$

c) 
$$R + \left(\omega \cdot L - \frac{1}{\omega}\right)$$

angle: - atan

b) 
$$(330 - 361.7 \cdot j) \cdot \Omega = 489.6\Omega / -47.63^{\circ}$$

c) 
$$R + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right) \cdot j$$

7. a) 
$$(6.6 - 12.5 \cdot j) \cdot mA = 14.1 mA / -62.29^{\circ}$$

$$\frac{10 \cdot V}{\sqrt{R^2 + \left(\omega \cdot L - \frac{1}{\omega}\right)^2}}$$

b) 
$$(13.8 + 15.1 \cdot j) \cdot mA = 20.4 mA / 47.63^{\circ}$$

$$\int_{0}^{10 \cdot V} R^2 + \left(\omega \cdot L - \frac{1}{\omega \cdot C}\right)^2$$

8. a)  $259 - 160 \cdot j$ 

Volts 
$$\bigwedge$$
 Im  $\bigvee_{S=2.2 \text{ Amps}}$   $\bigvee_{R=8.335+5.527j}$   $\bigvee_{R=8.335+5.527j}$ 

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