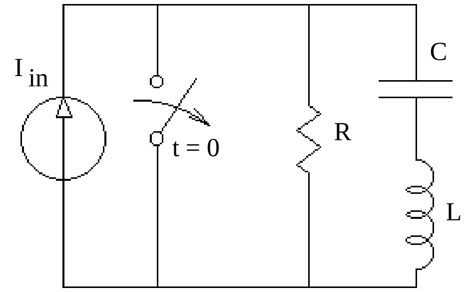
ECE 2210 Homework 2ndTr2 A.Stolp

c

2^{nd} - Order Transients Name______________________________

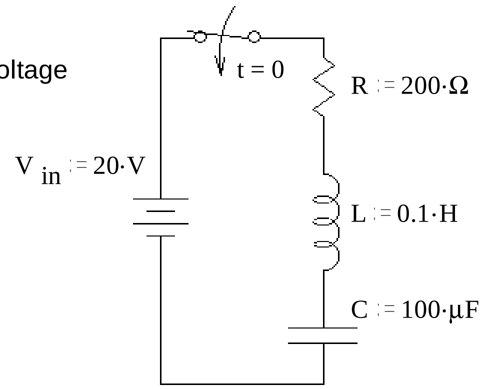
4. In the circuit shown, when the switch is opened, the current I_{in} (current source) is forced to flow through the circuit.



a) Write a differential equation for i_L. Hint: use LaPlace impedance method.

b) Write a differential equation for v_C.

c) Find the characteristic equation for the circuit shown.



1. A series RLC circuit with $R = 200\ \Omega$, $L = 0.1\ H$ and $C = 100\ \mu F$ has a constant voltage $V = 20$ volts applied at $t = 0$. The capacitor has no initial charge.
   a) Find the characteristic equation of the circuit at right.
   (hint: take $i(t)$ as the "output")

   b) Find the solutions to the characteristic equation.

   c) Is this circuit over, under, or critically damped?

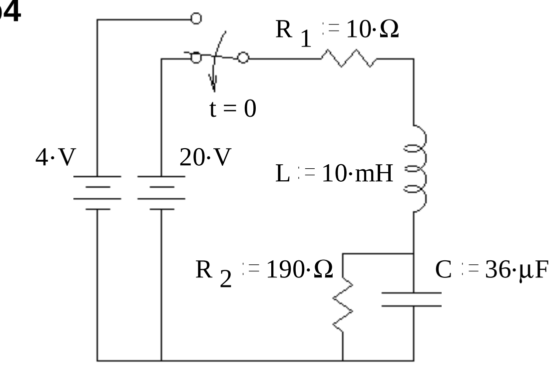
   d) The switch is switched down at time $t = 0$. Find the final and initial conditions:
   final: $i(\infty)$ initial: $i(0)$, $v_C(0)$ and $\frac{d}{dt}i(0)$

More on next page ===>

e) Write the full expression for i(t), including all the constants that you find.

2. A series RLC circuit with $R = 200\ \Omega$, $L = 0.1\ H$ and $C = ?\ \mu F$ is to be made critically damped by the selection of the capacitance. Find the required value of C.

3. Find the ringing frequency of a series RLC circuit in which $R = 200\ \Omega$, $L = 0.1\ H$ and $C = 5\ \mu F$. (The ringing frequency is the ω part of $s_1 = \alpha + j\omega$). Express your answer in Hz.



5. The characteristic equation of the circuit shown is: $0 = s^2 + \left(\frac{R_1}{L} + \frac{1}{C \cdot R_2}\right) \cdot s + \left(\frac{R_1}{L \cdot C \cdot R_2} + \frac{1}{L \cdot C}\right)$

   a) Find the solutions to the characteristic equation.

   b) Is this circuit over, under, or critically damped?

   c) The switch has been in the top position for a long time and is switched down at time $t = 0$.
   Find the final and initial conditions:

d) Write the full expression for $i_L(t)$, including all the constants that you find.

Answers

1. a) $\frac{R}{L}\cdot\frac{d}{dt} i_{in} = \frac{d^2}{dt^2} i_L + \frac{R}{L}\cdot\frac{d}{dt} i_L + \frac{1}{L\cdot C}\cdot i_L$ b) $\frac{R}{L\cdot C}\cdot i_{in} = \frac{d^2}{dt^2} v_c + \frac{R}{L}\cdot\frac{d}{dt} v_c + \frac{1}{L\cdot C}\cdot v_c$ c) $s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C} = 0$

2. a) $0 = s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}$ b) $-51.3\cdot\frac{1}{sec}$ $-1949\cdot\frac{1}{sec}$ c) overdamped

   d) $i(\infty) = 0\cdot A$ $i(0) = 0$ $V_C(0) = 0$ $\frac{d}{dt} i(0) = 200\cdot\frac{A}{sec}$ e) $i(t) = 0.1054\cdot e^{-\frac{51.3}{sec}\cdot t} - 0.1054\cdot e^{-\frac{1949}{sec}\cdot t}$

3. $10\cdot\mu F$ 4. $159\cdot Hz$

5. a) $-573.1 \pm 1611j$ 1/sec b) underdamped c) $19\cdot V$ $100\cdot mA$ $3.8\cdot V$ $20\cdot mA$ $0\cdot\frac{V}{sec}$ $1600\cdot\frac{A}{sec}$

d) $i_L(t) := 100\cdot mA + e^{\frac{-573.1}{sec}\cdot t}\cdot\left(-80\cdot\cos\left(\frac{1611}{sec}\cdot t\right) + 964.7\cdot\sin\left(\frac{1611}{sec}\cdot t\right)\right)\cdot mA$

1600 mA/ms = initial slope, shown by dashed line x-axis: $t := 0, .0001 .. .012\cdot sec$