Appendix, Calculations

Series RLC Circuit

For transient analysis, use the LaPlace s instead of \( \omega \) for the impedances. Remember that the LaPlace \( s = \alpha + j\omega \)

Transfer function: \( H(s) = \frac{R}{Ls + R_T + \frac{1}{C_s}} \)

\[ R_T := R + R_s + R_{\text{sub}} \quad R_s := 50\,\Omega \quad R_{\text{sub}} := 0\,\Omega \]

\[ L := 2.8\,\text{mH} \quad C := 0.001\,\mu\text{F} \]

If you take the denominator of the transfer function and set it equal to zero, you get the characteristic equation:

\[ 0 = s^2 + \frac{R_T}{L}s + \frac{1}{LC} \]

Solve the characteristic equation for \( s \) values, using the quadratic equation:

\[ s_1 := \frac{-\sqrt{\frac{R_T}{L}} - \sqrt{\frac{R_T}{L}^2 - 4\frac{1}{LC}}}{2} \quad s_2 := \frac{-\sqrt{\frac{R_T}{L}} + \sqrt{\frac{R_T}{L}^2 - 4\frac{1}{LC}}}{2} \]

\[ s = \alpha + j\omega \text{, so:} \quad \alpha := -\frac{R_T}{2L} \quad \omega := \frac{1}{\sqrt{\frac{4}{LC} - \frac{R_T^2}{L}}} \quad \frac{1}{\text{sec}} \]

\( e^{\alpha t} \) is a decaying exponential

The time constant is: \( \tau := \frac{1}{\alpha} \quad \tau = 37.3\,\mu\text{s} \)

Critical Damping happens when the part of \( s \) under the radical is 0:

\[ \left( \frac{R_T}{L} \right)^2 = \frac{4}{LC} \quad R_T = \frac{4}{\sqrt{\frac{4}{LC} - \frac{R_T^2}{L}}} = 3347\,\Omega \]

Parallel RLC Circuit

Impedance of \( C, L, \) & \( R_L \): \( Z(s) = \frac{1}{C_s + \frac{1}{Ls + R_L}} \)

Transfer function:

\[ H(s) = \frac{Z(s)}{Z(s) + R} = \frac{1}{1 + \frac{R}{Z(s)}} = \frac{1}{1 + R\left( \frac{1}{C_s + \frac{1}{Ls + R_L}} \right)} \]

\[ = \frac{1}{1 + R\left( \frac{Ls + R_L}{Ls + R_L} \right)} = \frac{Ls + R_L}{Ls + R_L + R\cdot C\cdot s + (L + R\cdot C\cdot R_L)\cdot s + (L + R)\left( \frac{1}{R\cdot C} \right)} \]

\[ = \frac{Ls + R_L}{R\cdot C\cdot L\cdot s^2 + (L + R\cdot C\cdot R_L)\cdot s + (L + R)\left( \frac{1}{R\cdot C} \right)} \]

characteristic equation: \( 0 = \left( s^2 + \frac{1}{R\cdot C + \frac{R_L}{L}} \right)s + \left( \frac{R_L}{R\cdot C + \frac{1}{L}} \right)s + \left( \frac{1}{R\cdot C + \frac{1}{L}} \right) \)

Find solutions to the characteristic eq. as above:

\[ \alpha := -\frac{1}{2} \left( \frac{1}{R\cdot C + \frac{R_L}{L}} \right) \quad \tau = \frac{1}{\alpha} \quad \omega := \frac{1}{\frac{1}{2} \sqrt{\frac{1}{R\cdot C + \frac{R_L}{L}}} - 4\left( \frac{R_L}{R\cdot C + \frac{1}{L}} \right)} \]

\[ f := \frac{\omega}{2\pi} = 6.41\,\text{kHz} \]

Compare these to what you measured.