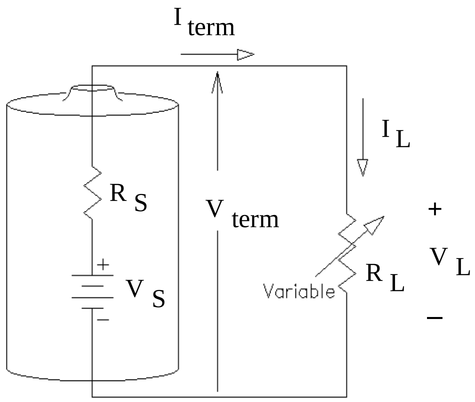
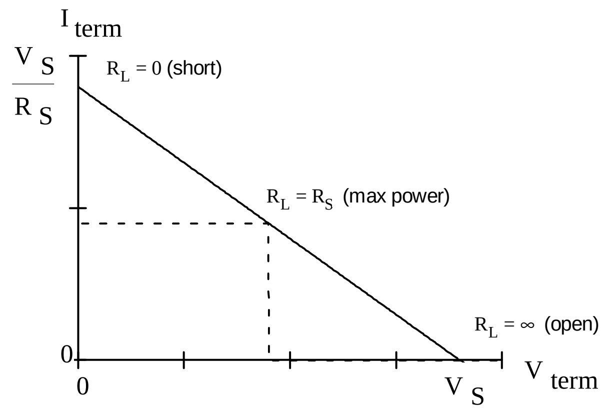
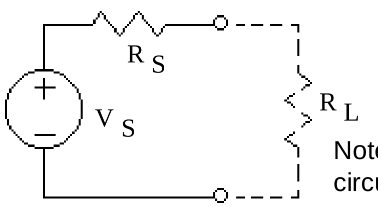
A. Stolp
1/28/06
9/2/09

Simple Model of a Real Source

Real sources are not ideal, but we will model them with two ideal components.



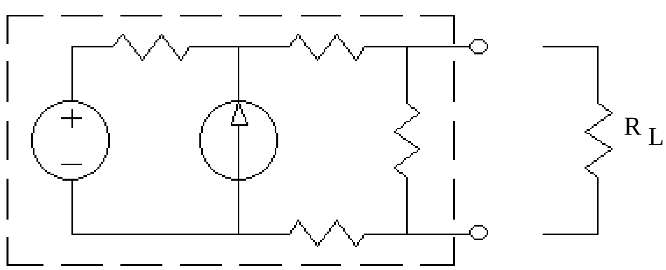


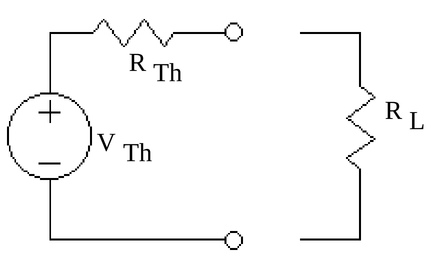


Note: R_L is NOT part of the Thévenin equivalent circuit and does not need to be shown.

Thévevin Equivalent Circuit

The same model can be used for any combination of sources and resistors.

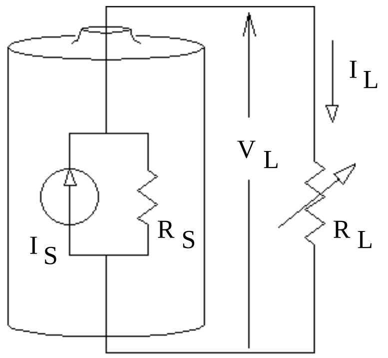


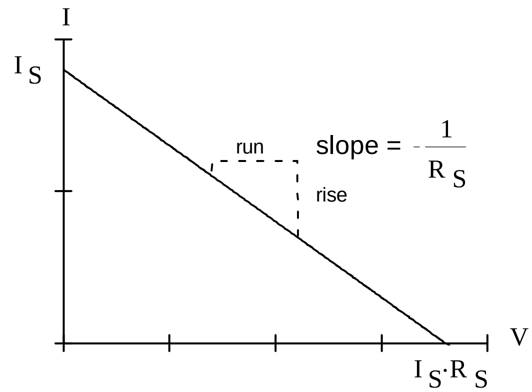


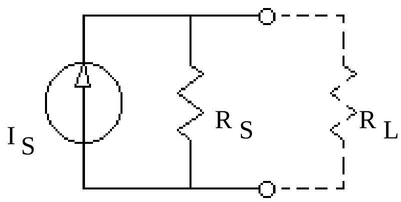
Thévenin equivalent

To calculate a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be.
   This is the Thévenin voltage (V_{Th}).
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Thévenin source resistance (R_{Th}).
4) Draw the Thévenin equivalent circuit and add your values.





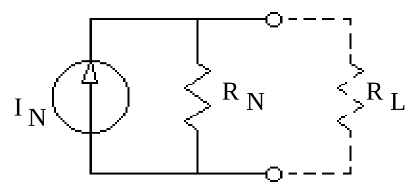


Note: R_L is not part of the Norton equivalent and does not need to be shown.

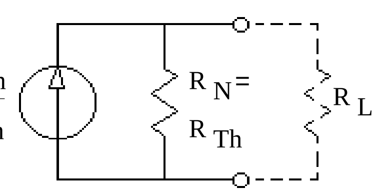
Norton equivalent

To calculate a circuit's Norton equivalent:

1) Replace the load with a short (a wire) and calculate the short-circuit current in this wire.
   This is the Norton current (I_N). Remove the short.
2) Zero all the sources.
   (To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)
3) Compute the total resistance between the load terminals.
   (DO NOT include the load in this resistance.) This is the Norton source resistance (R_N).
   (Exactly the same as the Thévenin source resistance (R_{Th})).
4) Draw the Norton equivalent circuit and add your values.



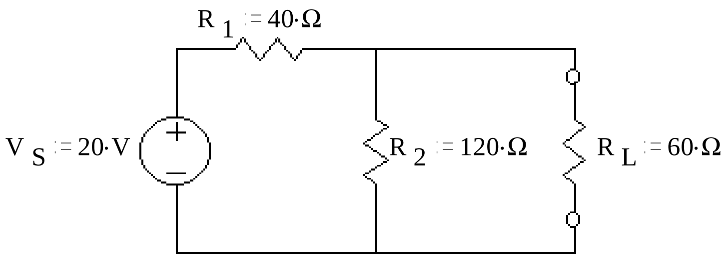
OR (the more common way)...
1) Find the Thévenin equivalent circuit.
2) Convert to Norton circuit, then >>> $R_N = R_{Th}$ and $I_N = \frac{V_{Th}}{R_{Th}}$



Thévevin & Norton Examples

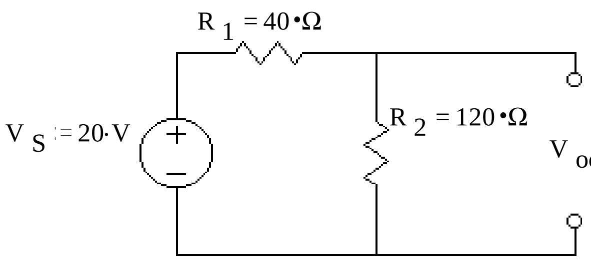
A.Stolp
1/23/03,
1/6/13

Ex 1 Find the Thévenin equivalent:



To calculate a circuit's Thévenin equivalent:

1) Remove the load and calculate the open-circuit voltage where the load used to be.
This is the Thévenin voltage (V_{Th}).



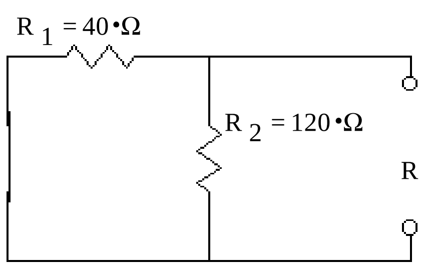
Find the open circuit voltage:

$$V_{oc} = V_{Th} := V_S \cdot \frac{R_2}{R_1 + R_2} \qquad V_{Th} = 15 \cdot V$$

2) Zero all the sources.
(To zero a voltage source, replace it with a short. To zero a current source, replace it with an open.)

3) Compute the total resistance between the load terminals.
(DO NOT include the load in this resistance.)
This is the Thévenin source resistance (R_{Th}).

Zero the source

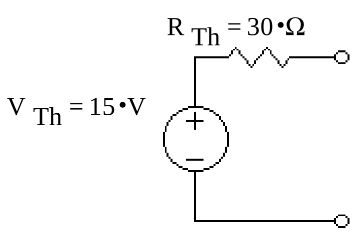


Find the Thevenin resistance:

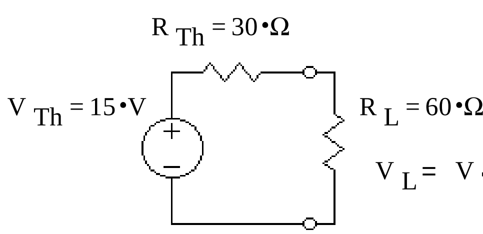
$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \qquad R_{Th} = 30 \cdot \Omega$$

4) Draw the Thévenin equivalent circuit and add your values.

Thevenin equivalent circuit:



If the load were reconnected:

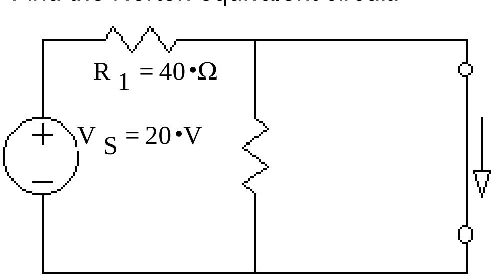


$$V_L = V_{Th} \cdot \frac{R_L}{R_{Th} + R_L} = 10 \cdot V$$

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = 166.7 \cdot mA$$

$$P_L = 10 \cdot V \cdot 166.7 \cdot mA = 1.667 \cdot W$$

b) Find the Norton equivalent circuit:



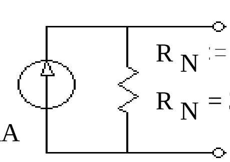
$$I_N := \frac{V_S}{R_1}$$

$$I_N = 500 \cdot mA$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 500 \cdot mA$$



$$R_N := R_{Th}$$

$$R_N = 30 \cdot \Omega$$

c) Show that the Thévenin circuit is indeed equivalent to the original at several values of R_L.

R_L	Original Circuit: V_L	Original Circuit: I_L	Thévenin Circuit: I_L	Thévenin Circuit: V_L
$R_L := 0\cdot\Omega$	$0\cdot V$	$\frac{V_S}{R_1} = 500\cdot mA$	$\frac{V_{Th}}{R_{Th}+R_L} = 500\cdot mA$	$500\cdot mA\cdot 0\cdot\Omega = 0\cdot V$

Using either numbers: $P_L = V_L \cdot I_L = 0\cdot W$

$R_L := 10\cdot\Omega$

$$R_o := \frac{1}{\frac{1}{R_2}+\frac{1}{R_L}} \qquad R_o = 9.231\cdot\Omega$$

$$V_L = V_S\cdot\frac{R_o}{R_1+R_o} = 3.75\cdot V$$

$$I_L = \frac{V_L}{R_L} = 375\cdot mA$$

$$I_L := \frac{V_{Th}}{R_{Th}+R_L} \qquad I_L = 375\cdot mA$$

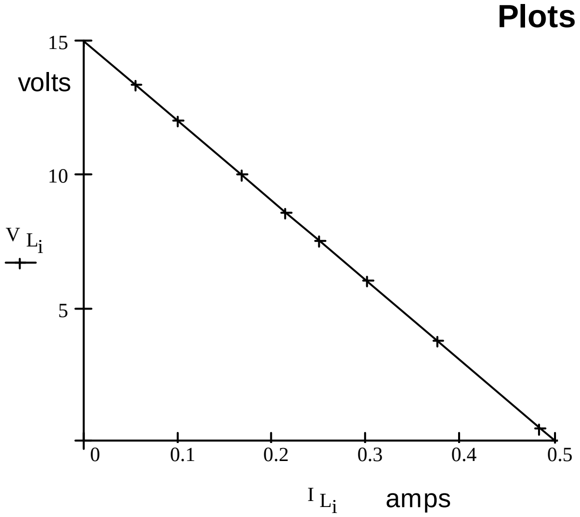
$$V_L := I_L\cdot R_L \qquad V_L = 3.75\cdot V$$

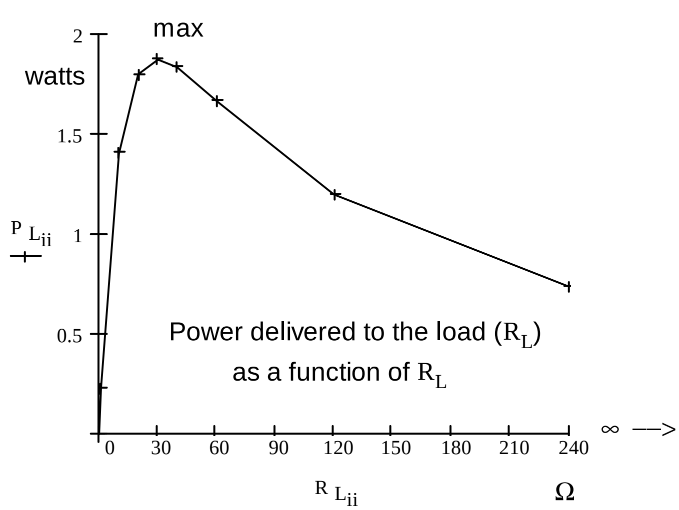
Using either numbers: $P_L = V_L \cdot I_L = 1.406\cdot W$

Repeat these calculations for a number of load resistors

$R_{L_i} :=$	R_{o_i} (Ω)	$V_L = V_S\cdot\frac{R_{o_i}}{R_1+R_{o_i}}$ (V)	$I_L = \frac{V_{L_i}}{R_{L_i}}$ (mA)	$I_L = \frac{V_{Th}}{R_{Th}+R_{L_i}}$ (mA)	$V_L = I_{L_i}\cdot R_{L_i}$ (V)	P_{L_i} (W)	
$0\cdot\Omega$	0	0	0	500	0	0	
$1\cdot\Omega$	0.992	0.484	483.871	483.871	0.484	0.234	
$10\cdot\Omega$	9.231	3.75	375	375	3.75	1.406	
$20\cdot\Omega$	17.143	6	300	300	6	1.8	
$30\cdot\Omega$	24	7.5	250	250	7.5	1.875	max
$40\cdot\Omega$	30	8.571	214.286	214.286	8.571	1.837	
$60\cdot\Omega$	40	10	166.667	166.667	10	1.667	
$120\cdot\Omega$	60	12	100	100	12	1.2	
$240\cdot\Omega$	80	13.333	55.556	55.556	13.333	0.741	
$\infty\cdot\Omega$	120	15	0	0	15	0	

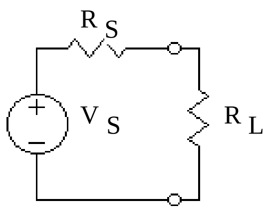
Plots





Maximum power transfer

If I wanted to maximize the power dissipated by the load, what R_L would I choose?



$$P_L = \frac{V_L^2}{R_L} = \left(\frac{R_L}{R_S + R_L}\cdot V_S\right)^2 \cdot \frac{1}{R_L} = \frac{R_L^2}{\left(R_S + R_L\right)^2}\cdot V_S^2 \cdot \frac{1}{R_L}$$

$$= \frac{R_L^2}{R_S^2 + 2\cdot R_S\cdot R_L + R_L^2}\cdot V_S^2\cdot\frac{1}{R_L} = \frac{R_L}{R_S^2 + 2\cdot R_S\cdot R_L + R_L^2}\cdot V_S^2$$

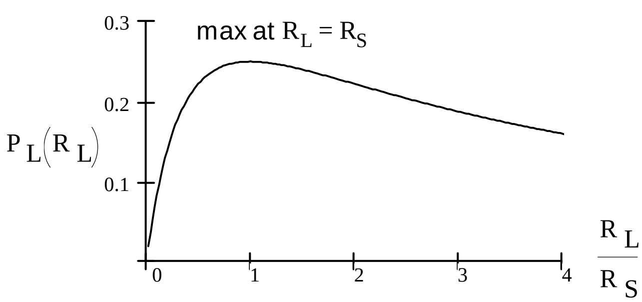
$$= \frac{1}{\dfrac{R_S^2}{R_L} + 2\cdot R_S + R_L}\cdot V_S^2$$

Next step would be to differentiate $\frac{d}{dR_L}P_L(R_L)$,

set this equal to 0 and solve for R_L to find the maximum

Unfortunately this function is a pain to differentiate. What if we just differentiate the denominator and find its minimum, wouldn't that work just as well?

$$\frac{d}{dR_L}\left(\frac{R_S^2}{R_L} + 2\cdot R_S + R_L\right) = -1\cdot\frac{R_S^2}{R_L^2} + 0 + 1 = 0$$



Maximum power transfer happens when: $R_L = R_S$

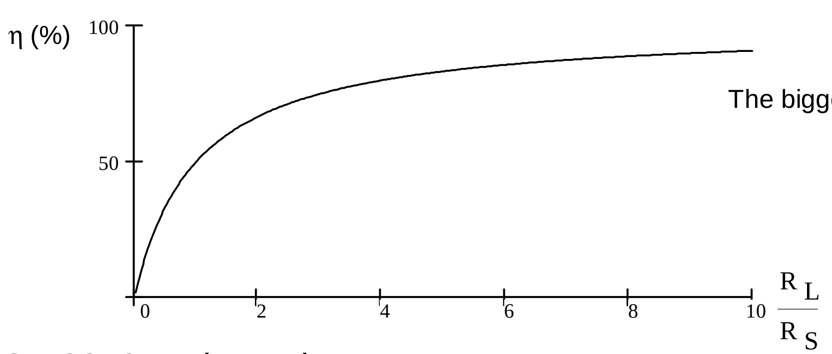
Just what we saw in Example 1

This is rarely important in power circuitry, where there should be plenty of power and R_S should be small. It is much more likely to be important in signal circuitry where the voltages can be very small and the source resistance may be significant -- say a microphone or a radio antenna.

All you need to remember is: $R_L = R_S$ to maximize the power dissipation in R_L

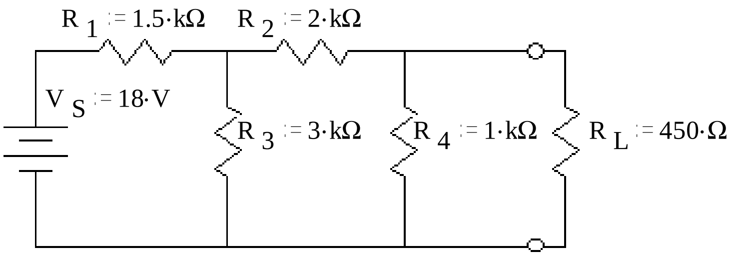
What about efficiency?

$$\frac{P_L(R_L)}{P_S(R_L)} = \frac{I^2\cdot R_L}{I^2\cdot\left(R_S + R_L\right)} = \frac{R_L}{R_S + R_L}$$

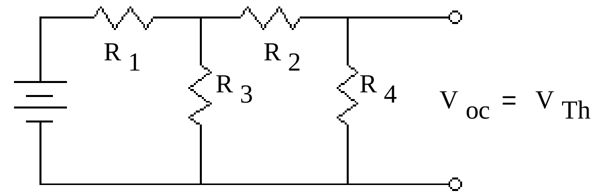


The bigger R_L is, the higher the efficiency.

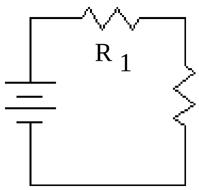
Ex 2 a) Find and draw the Thévenin equivalent circuit.



Find the open circuit voltage:



First do some simplification:

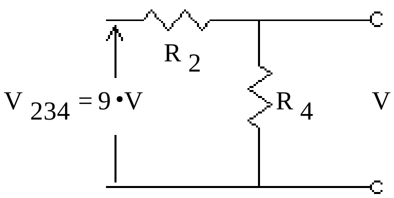


$$R_{eq234} := \frac{1}{\frac{1}{R_3} + \frac{1}{R_2 + R_4}} \qquad R_{eq234} = 1.5\cdot k\Omega$$

$$V_{234} := \frac{R_{eq234}}{R_1 + R_{eq234}}\cdot V_S$$

$$V_{234} = 9\cdot V$$

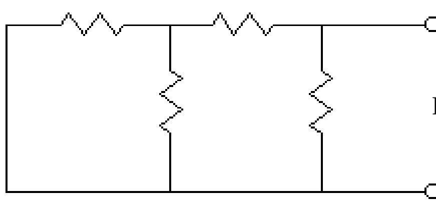
Divide this voltage between R_2 and R_4:



$$V_{Th} := \frac{R_4}{R_2 + R_4}\cdot V_{234} \qquad V_{Th} = 3\cdot V$$

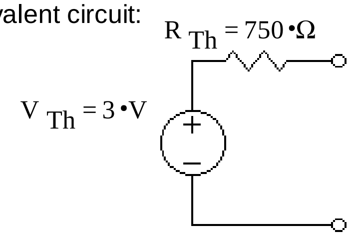
Find the Thévenin resistance:

Zero the source



$$R_{Th} := \frac{1}{\frac{1}{R_4} + \frac{1}{R_2 + \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_3}\right)}}} \qquad R_{Th} = 750\cdot\Omega$$

Thévenin equivalent circuit:



If the load were reconnected:

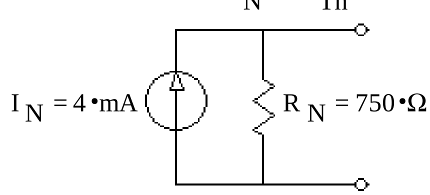
$$V_L := V_{Th}\cdot\frac{R_L}{R_{Th} + R_L} \qquad V_L = 1.125\cdot V$$

$$I_L := \frac{V_{Th}}{R_{Th} + R_L} \qquad I_L = 2.5\cdot mA$$

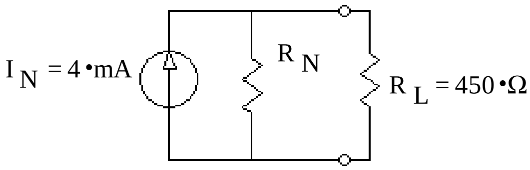
b) Find and draw the Norton equivalent circuit.

$$R_N := R_{Th}$$

$$I_N := \frac{V_{Th}}{R_{Th}}$$



c) Use your Norton equivalent circuit to find the current through the load.



$$I_L := \frac{\frac{1}{R_L}}{\left(\frac{1}{R_N} + \frac{1}{R_L}\right)} \cdot I_N \qquad I_L = 2.5 \cdot mA$$

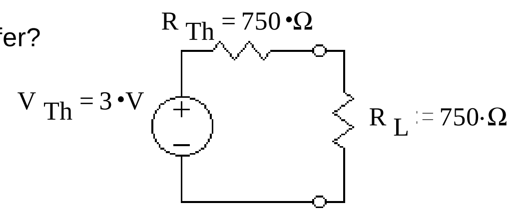
$$V_L := I_L \cdot R_L \qquad V_L = 1.125 \cdot V$$

same as above

d) What value of R_L would result in the maximum power delivery to R_L?

For maximum power transfer $R_L = R_{Th} = 750 \cdot \Omega$

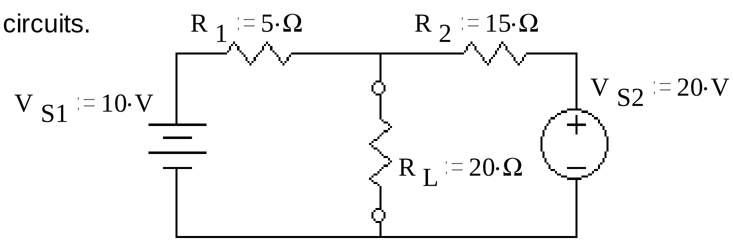
e) What is the maximum power transfer?



$$V_L := \frac{V_{Th}}{2}$$

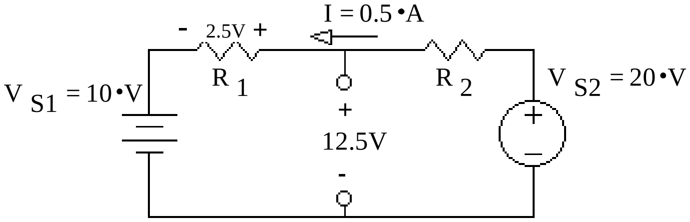
$$P_L = \frac{V_L^{\,2}}{R_L} = 3 \cdot mW$$

Ex 3 a) Find and draw the Thévenin & Norton equivalent circuits.

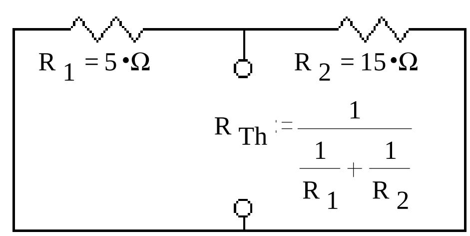


$$I := \frac{V_{S2} - V_{S1}}{R_1 + R_2}$$

$$I = 0.5 \cdot A$$



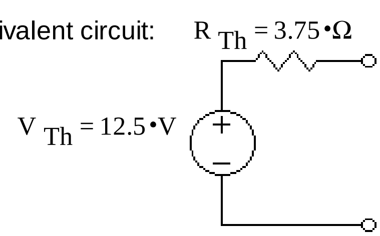
$$V_{Th} := 10 \cdot V + 2.5 \cdot V$$



$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$R_{Th} = 3.75 \cdot \Omega$$

Thévenin equivalent circuit:

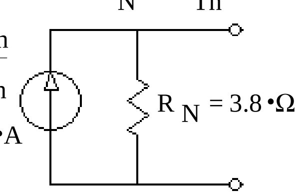


Norton equivalent circuit:

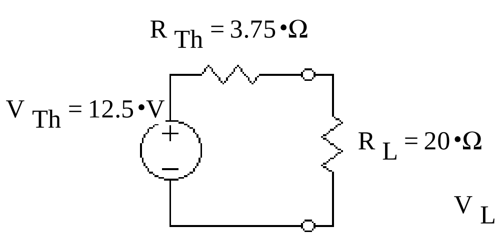
$$R_N := R_{Th}$$

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 3.333 \cdot A$$

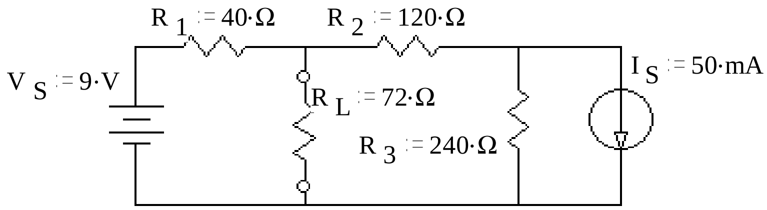


b) Use your Thévenin equivalent circuit to find the voltage across the load.

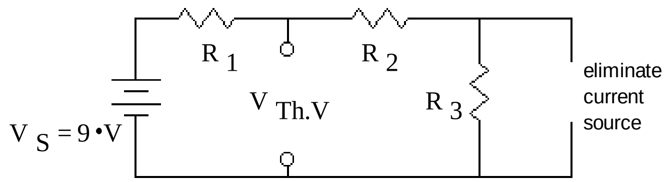


$$V_L = \frac{R_L}{R_{Th} + R_L} \cdot V_{Th} = 10.526 \cdot V$$

Ex 4 a) Find and draw the Thévenin & Norton equivalent circuits.

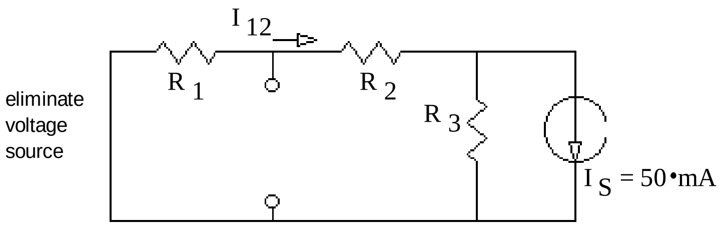


Use superposition to find V_{Th}.



$$V_{Th.V} := \frac{R_2 + R_3}{R_1 + R_2 + R_3}\cdot V_S$$

$$V_{Th.V} = 8.1\cdot V$$

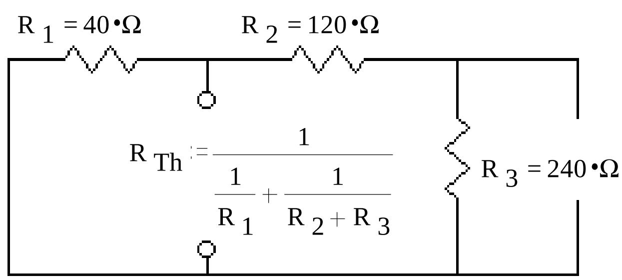


current divider: $$I_{12} := \frac{\frac{1}{R_1 + R_2}}{\frac{1}{R_1 + R_2} + \frac{1}{R_3}}\cdot I_S$$ $I_{12} = 30\cdot mA$

$V_{Th.I} := -I_{12}\cdot R_1$ $\quad V_{Th.I} = -1.2\cdot V$

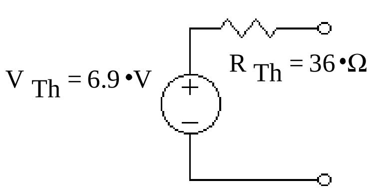
$V_{Th} := V_{Th.V} + V_{Th.I}$ $\quad V_{Th} = 6.9\cdot V$

Find the Thévenin resistance

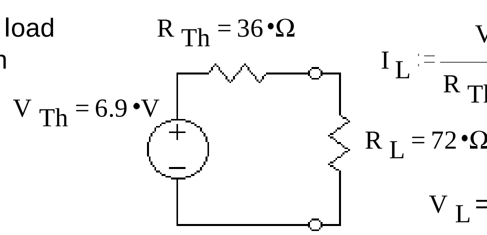


$$R_{Th} := \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}}$$

Thévenin equivalent circuit:



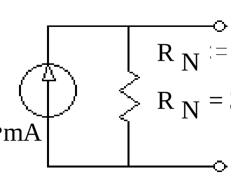
Put the load back on



$$I_L := \frac{V_{Th}}{R_{Th} + R_L}$$ $I_L = 63.889\cdot mA$

$V_L = I_L\cdot R_L = 4.6\cdot V$

Norton equivalent circuit: $I_N := \frac{V_{Th}}{R_{Th}}$ $\quad I_N = 191.7\cdot mA$



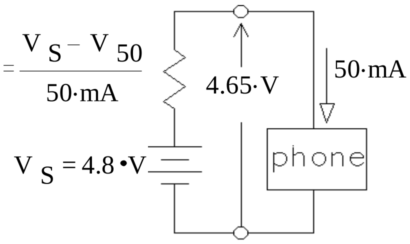
$R_N := R_{Th}$

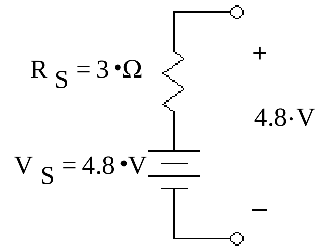
$R_N = 36\cdot\Omega$

Ex 5 A NiCad Battery pack is used to power a cell phone. When the phone is switched on the battery pack voltage drops from 4.80 V to 4.65 V and the cell phone draws 50 mA. $V_S := 4.80 \cdot V$ $\quad V_{50} := 4.65 \cdot V$

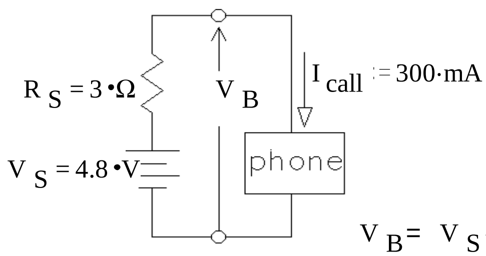
a) Draw a simple, reasonable model of the battery pack using ideal parts.
Find the value of each part.

$$R_S := \frac{V_S - V_{50}}{50 \cdot mA}$$



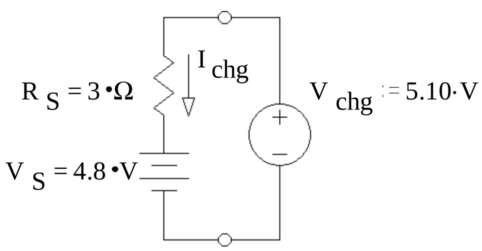


b) The cell phone is used to make a call. Now it draws 300 mA.
What is the battery pack voltage now?



$$V_B = V_S - I_{call} \cdot R_S = 3.9 \cdot V$$

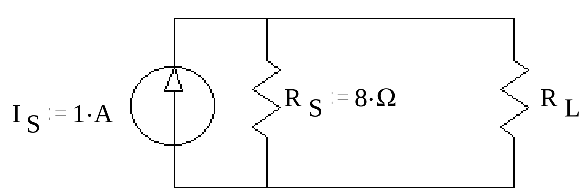
c) The battery pack is placed in a charger. The charger supplies 5.10 V. How much current flows into the battery pack?



$$I_{chg} = \frac{V_{chg} - V_S}{R_S} = 100 \cdot mA$$

Ex 6 Consider the circuit at right.

a) What value of load resistor (R_L) would you choose if you wanted to maximize the power dissipation in that load resistor.

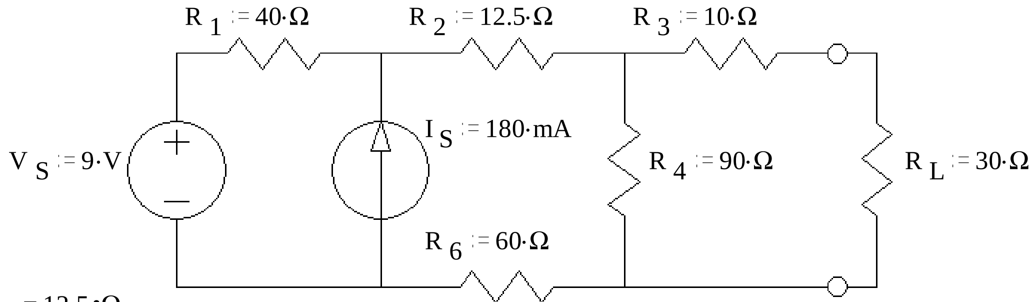


$R_L := R_S \qquad R_L = 8 \cdot \Omega$

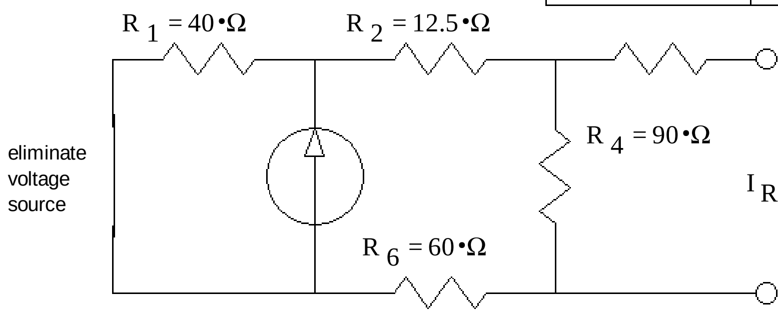
b) With that load resistor (R_L) find the power dissipation in the load.

$I_L := \dfrac{I_S}{2} \qquad P_L = I_L^2 \cdot R_L = 2 \cdot W$

Ex 7



Use superposition to find V_{Th}.



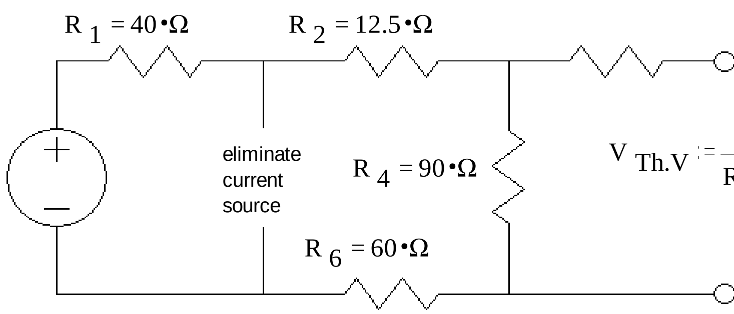
current divider:

$$I_{R4} := \frac{\dfrac{1}{R_2 + R_4 + R_6}}{\dfrac{1}{R_2 + R_4 + R_6} + \dfrac{1}{R_1}} \cdot I_S \qquad I_{R4} = 35.556\cdot mA$$

$$V_{Th.I} := I_{R4}\cdot R_4$$

$$V_{Th.I} = 3.2\cdot V$$

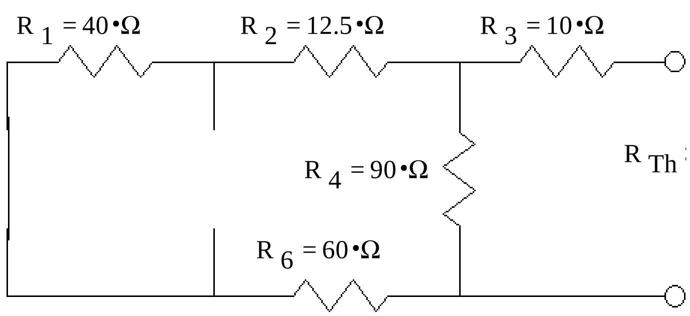
$$R_{246} := R_2 + R_4 + R_6 \qquad R_{246} = 162.5\cdot\Omega$$



$$V_{Th.V} := \frac{R_4}{R_1 + R_2 + R_4 + R_6}\cdot V_S \qquad V_{Th.V} = 4\cdot V$$

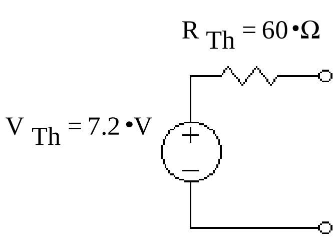
$$V_{Th} := V_{Th.V} + V_{Th.I}$$

$$V_{Th} = 7.2\cdot V$$

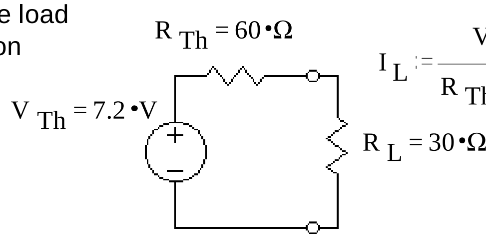


$$R_{Th} := \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_1 + R_2 + R_6}} + R_3 \qquad R_{Th} = 60\cdot\Omega$$

Thévenin equivalent circuit:



Put the load back on



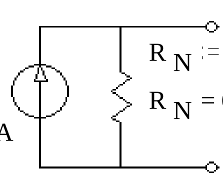
$$I_L := \frac{V_{Th}}{R_{Th} + R_L} \qquad I_L = 80\cdot mA$$

$$V_L = I_L\cdot R_L = 2.4\cdot V$$

Norton equivalent circuit:

$$I_N := \frac{V_{Th}}{R_{Th}}$$

$$I_N = 120\cdot mA$$



$$R_N := R_{Th}$$

$$R_N = 60\cdot\Omega$$