Capacitor Lecture Notes

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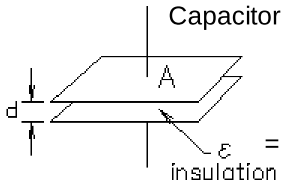
2/17/03
rev 9/16/09
12/15 & 9/19

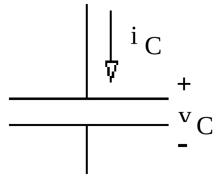
Now that we have voltages and currents which can be functions of time, it's time to introduce the capacitor and the inductor.

Electrical equivalent:



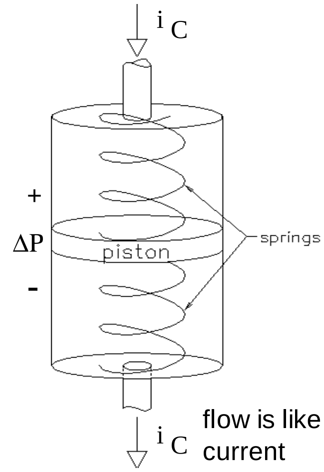
ε = permittivity

$$C = \varepsilon \cdot \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv}$$



like pressure

Fluid Model:



flow is like current

Units: $\text{farad} = \frac{\text{coul}}{\text{volt}} = \frac{\text{amp}\cdot\text{sec}}{\text{volt}}$ $\qquad \mu F = 1 \cdot 10^{-6} \cdot \text{farad} \qquad pF = 1 \cdot 10^{-12} \cdot \text{farad}$

For drawings of capacitors and info about tolerances, see Ch.3 of textbook.

Basic equations you should know:

$$C = \frac{Q}{V}$$

$$v_C = \frac{1}{C} \cdot \int_{-\infty}^{t} i_C \, dt$$

$$i_C = C \cdot \frac{d}{dt} v_C$$

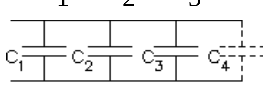
Or... $$v_C = \frac{1}{C} \cdot \int_{0}^{t} i_C \, dt + v_C(0)$$ ← initial voltage

Or... $$\Delta v_C = \frac{1}{C} \cdot \int_{t_1}^{t_2} i_C \, dt$$

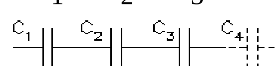
Energy stored in electric field: $W_C = \frac{1}{2} \cdot C \cdot V_C^{\,2}$

Capacitor voltage **cannot** change instantaneously

parallel: $C_{eq} = C_1 + C_2 + C_3 + \ldots$



series: $$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \ldots}$$



Capacitors are the only "backwards" components.

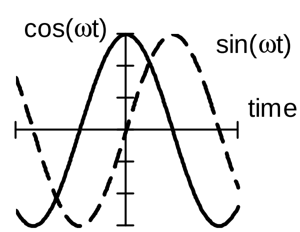
Sinusoids

$$i_C(t) = I_p \cdot \cos(\omega \cdot t)$$

$$v_C(t) = \frac{1}{C} \cdot \int i_C \, dt = \underbrace{\frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p}_{V_p} \cdot \sin(\omega \cdot t) = \underbrace{\frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t - 90 \cdot \text{deg})$$

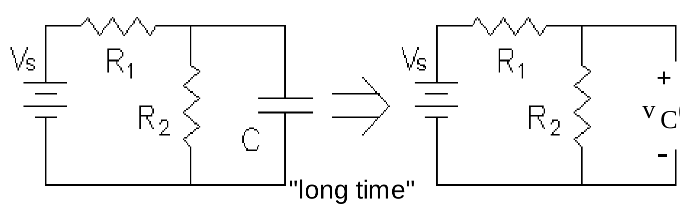
indefinite integral

Voltage "lags" current, makes sense, current has to flow in first to charge capacitor.



Steady-state or Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.



$$v_C(\infty) = V_S \cdot \frac{R_2}{R_1 + R_2}$$

$$\frac{d}{dt} v_C = 0 \qquad i_C = C \cdot \frac{d}{dt} v_C = 0$$

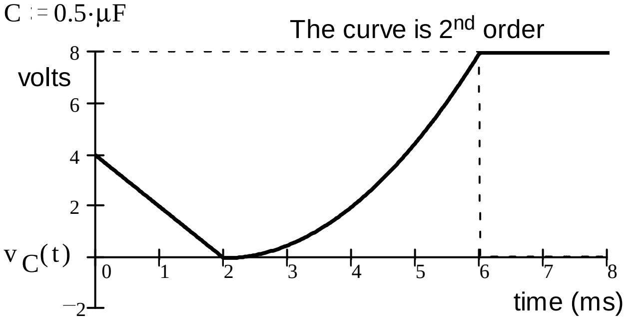
no current means it looks like an open

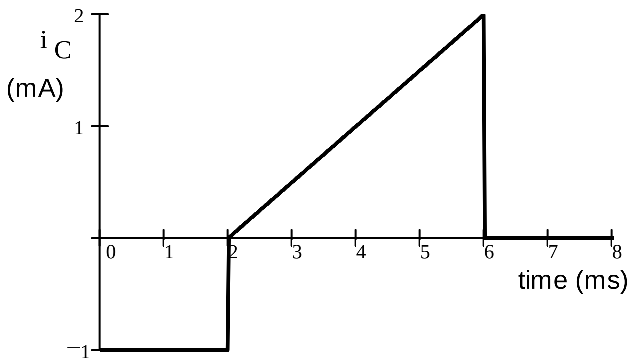
Example

The voltage across a $0.5\ \mu F$ capacitor is shown below. Make an accurate drawing of the capacitor current. Label the y-axis of your graph (I've already done the time-axis).

The accuracy of your plot at $0, 2, 6$, and 8 ms is important, so calculate those values and plot or label them carefully. Between those points your plot must simply be the correct shape.

$C := 0.5 \cdot \mu F$





1 - 2ms: $i_C = C \cdot \frac{\Delta V}{\Delta t} = 0.5 \cdot \mu F \cdot \frac{-4 \cdot V}{2 \cdot ms} = -1 \cdot mA$

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

$$\Delta v_C(t) = \frac{1}{C} \cdot \int_0^t i_C(t)\, dt$$

$$8 \cdot V = \frac{1}{C} \cdot \left(\frac{4 \cdot ms \cdot height}{2} \right)$$

$$height = 8 \cdot V \cdot \frac{C \cdot 2}{4 \cdot ms} = 2 \cdot mA$$

6ms - 8ms: Slope is zero, so the current must be zero.

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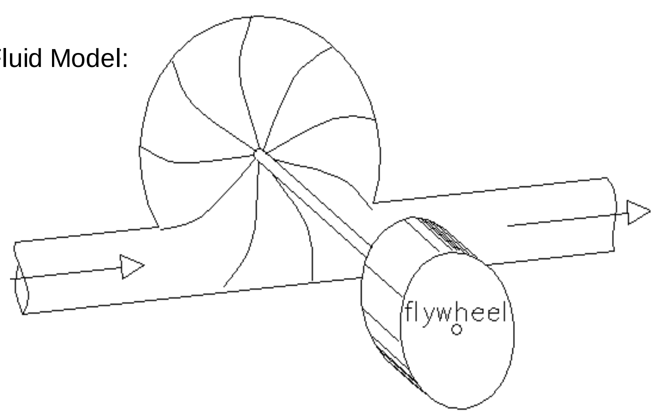
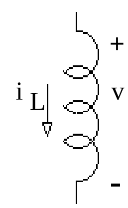
Fluid Model:



Electrical equivalent:



$L = \mu_o \cdot N^2 \cdot K$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

N is the number of turns of wire

Basic equations you should know: $v_L = L \cdot \frac{d}{dt} i_L$

$$i_L = \frac{1}{L} \cdot \int_{-\infty}^{t} v_L\, dt$$

Or... $i_L = \frac{1}{L} \cdot \int_0^t v_L\, dt + i_L(0)$ ← initial current

Or... $\Delta i_L = \frac{1}{L} \cdot \int_{t_1}^{t_2} v_L\, dt$

Energy stored in electric field: $W_L = \frac{1}{2} \cdot L \cdot I_L^2$

Inductor current **cannot** change instantaneously

Units: $henry = \frac{volt \cdot sec}{amp}$

$mH = 10^{-3} \cdot H$ $\quad \mu H = 10^{-6} \cdot H$

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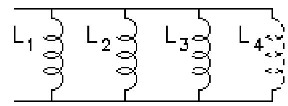
series: $L_{eq} = L_1 + L_2 + L_3 + ...$

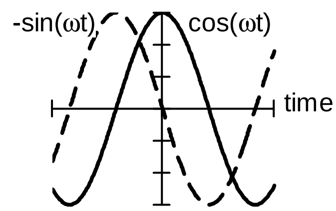
parallel: $L_{eq} = \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2} + \dfrac{1}{L_3} + ...}$



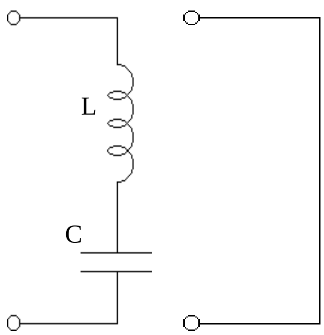
Sinusoids $i_L(t) = I_p \cdot \cos(\omega \cdot t)$

$$v_L(t) = L \cdot \frac{d}{dt} i_L = \underbrace{L \cdot \omega \cdot}_{V_p} \left(-I_p \cdot \sin(\omega \cdot t)\right) = \underbrace{L \cdot \omega \cdot I_p}_{V_p} \cdot \cos(\omega \cdot t + 90 \cdot deg)$$

Voltage "leads" current, makes sense, voltage has to present to make current change, so voltage comes first.

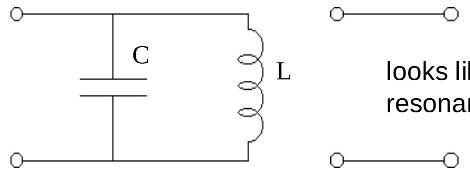


Resonance Series resonance



looks like a short at resonance frequency

Parallel resonance



looks like an open at resonance frequency

The resonance frequency is calculated the same way for either case:

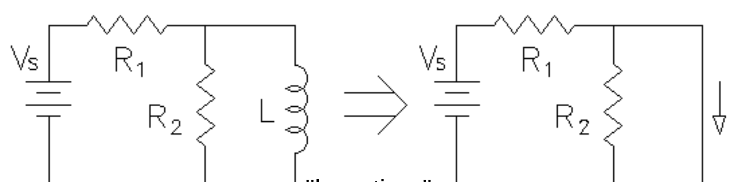
$\omega_o = \dfrac{1}{\sqrt{L \cdot C}} \left(\dfrac{rad}{sec}\right)$ OR.. $\omega_o = \dfrac{1}{\sqrt{L_{eq} \cdot C_{eq}}}$ If you have multiple capacitors or inductors which can be combined. $f_o = \dfrac{\omega_o}{2 \cdot \pi}$ (Hz)

Steady-state of Final conditions

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$\dfrac{d}{dt} i_L = 0$ $v_L = L \cdot \dfrac{d}{dt} i_L = 0$

no voltage means it looks like a short



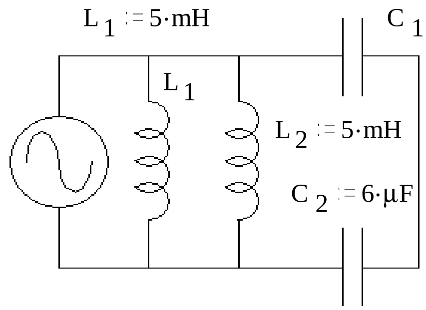
$i_L(\infty) = \dfrac{V_S}{R_1}$

Examples

Ex 1

Find the resonant frequency (or frequencies) of the circuit shown (in cycles/sec or Hz).

$L_1 := 5 \cdot mH$ $C_1 := 6 \cdot \mu F$



$L_2 := 5 \cdot mH$ $C_2 := 6 \cdot \mu F$

$L_{eq} := \dfrac{1}{\dfrac{1}{L_1} + \dfrac{1}{L_2}}$ $L_{eq} = 2.5 \cdot mH$

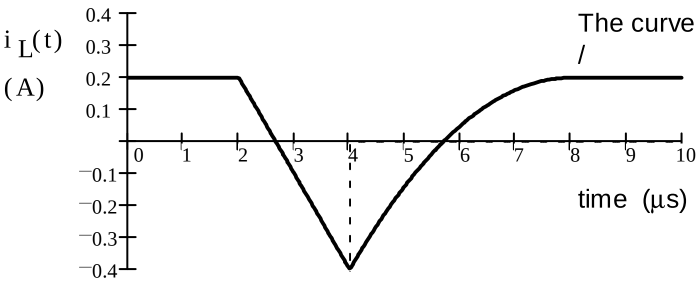
$C_{eq} := \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2}}$ $C_{eq} = 3 \cdot \mu F$

$\omega_o := \dfrac{1}{\sqrt{L_{eq} \cdot C_{eq}}}$ $\omega_o = 11547 \cdot \dfrac{rad}{sec}$ $f_o = \dfrac{\omega_o}{2 \cdot \pi} = 1838 \cdot Hz$

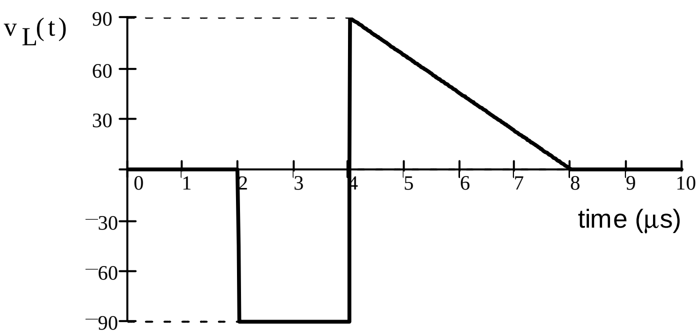
Ex 2

The current through a 0.3mH inductor is shown below. Make an accurate drawing of the inductor voltage. Make reasonable assumptions where necessary. Label your graph.

$L := 0.3 \cdot mH$



The curve is 2[nd] order and ends at 8µs /



0 - 2µs: No change in current, so: $v_L = 0$

2µs - 4µs: $v_L = L \cdot \frac{\Delta I}{\Delta t} = 0.3 \cdot mH \cdot \frac{-0.6 \cdot A}{2 \cdot \mu s} = -90 \cdot V$

4µs - 8µs: Initial slope is positive and the final slope is zero, so the voltage must be a triangle that starts at some height and ends at zero.

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_0^t v_L(t)\, dt$$

$$0.6 \cdot A = \frac{1}{0.3 \cdot mH} \cdot \left(\frac{4 \cdot \mu s \cdot height}{2} \right)$$

$$height = 0.6 \cdot A \cdot \frac{0.3 \cdot mH \cdot 2}{4 \cdot \mu s} = 90 \cdot V$$

8µs - 10µs: No change in current, so: $v_L = 0$

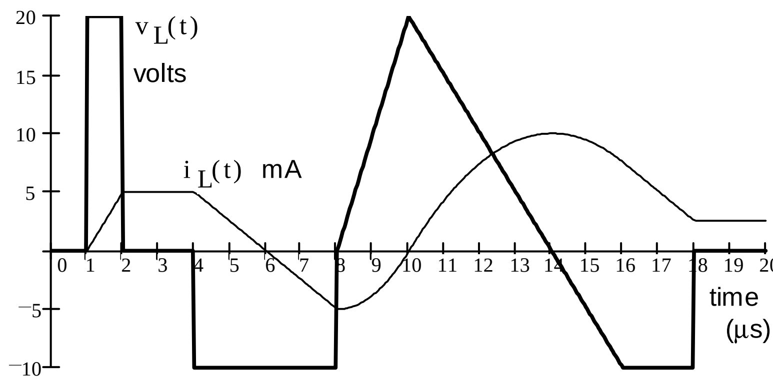
Ex 3 Given a voltage, find the current, $L := 4 \cdot mH$

$$\Delta i_L(t) = \frac{1}{L} \cdot \int_{1 \cdot \mu s}^{2 \cdot \mu s} 20 \cdot V\, dt = 5 \cdot mA$$

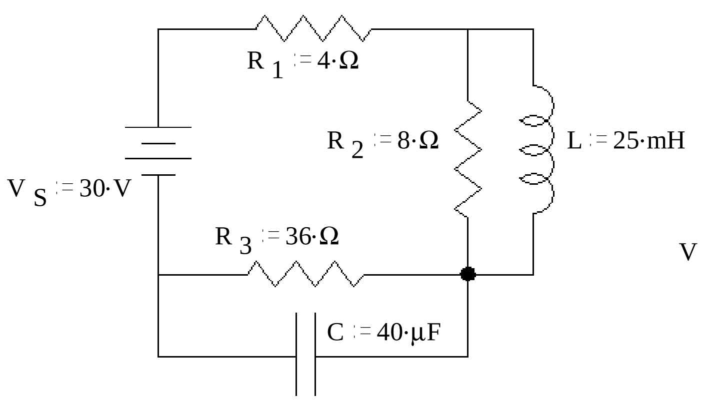
$$\frac{1}{L} \cdot \int_{4 \cdot \mu s}^{8 \cdot \mu s} -10 \cdot V\, dt + 5 \cdot mA = -5 \cdot mA$$

$$\frac{1}{L} \cdot \int_{8 \cdot \mu s}^{10 \cdot \mu s} V(t)\, dt + -5 \cdot mA$$

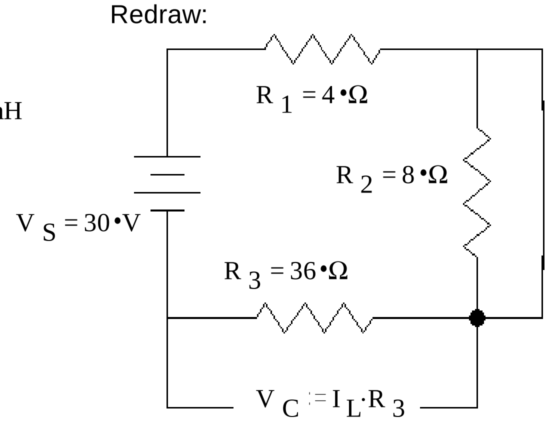
$$= \frac{1}{L} \cdot \frac{20 \cdot V \cdot 2 \cdot \mu s}{2} - 5 \cdot mA = 0 \cdot mA \quad \text{etc...}$$



Ex 4 The following circuit has been connected as shown for a long time. Find the energy stored in the capacitor and the inductor.



Redraw:



$V_C = 27 \cdot V$

$W_C := \frac{1}{2} \cdot C \cdot V_C^2$ $\qquad W_C = 14.58 \cdot mJ$

$I_L := \frac{V_S}{R_1 + R_3}$

$I_L = 0.75 \cdot A$

$W_L := \frac{1}{2} \cdot L \cdot I_L^2$

$W_L = 7.031 \cdot mJ$