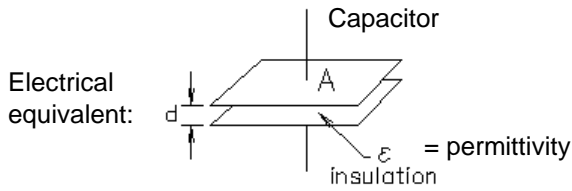


ECE 2210 / 00 Capacitor Lecture Notes

Now that we have voltages and currents which can be functions of time, it's time to introduce the capacitor and the inductor.

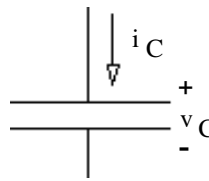
A. Stolp
2/17/03
rev 9/16/09
12/15 & 9/19
9/28/23



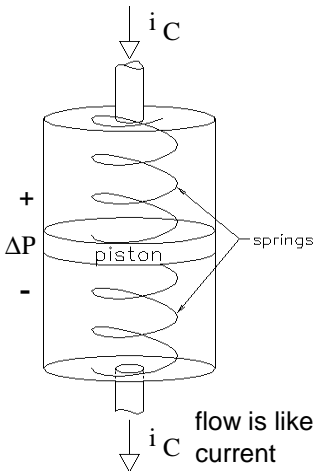
Electrical equivalent:

$$C = \epsilon \cdot \frac{A}{d} = \frac{Q}{V} = \frac{dq}{dv}$$

Units: farad = $\frac{\text{coul}}{\text{volt}} = \frac{\text{amp}\cdot\text{sec}}{\text{volt}}$



Fluid Model:



Basic equations & concepts you should know:

$$\mu\text{F} = 1 \cdot 10^{-6} \cdot \text{farad}$$

$$\text{pF} = 1 \cdot 10^{-12} \cdot \text{farad}$$

$$i_C = C \cdot \frac{d}{dt} v_C$$

$$v_C = \frac{1}{C} \int_{-\infty}^t i_C dt$$

/ initial voltage

Or...

$$v_C = \frac{1}{C} \int_0^t i_C dt + v_C(0)$$

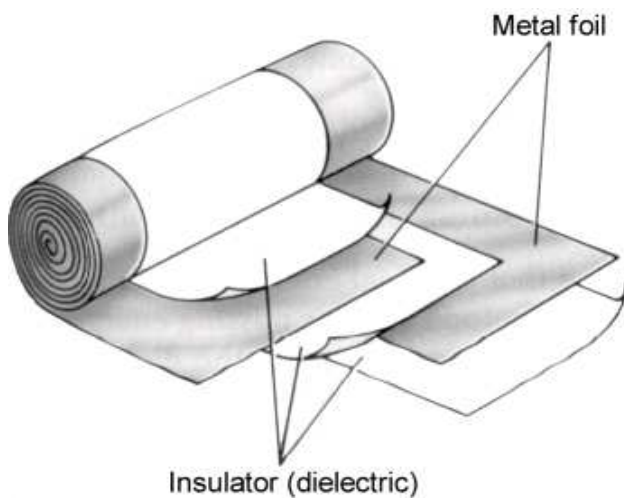
Or...

$$\Delta v_C = \frac{1}{C} \int_{t_1}^{t_2} i_C dt$$

Energy stored in electric field: $W_C = \frac{1}{2} \cdot C \cdot v_C^2$

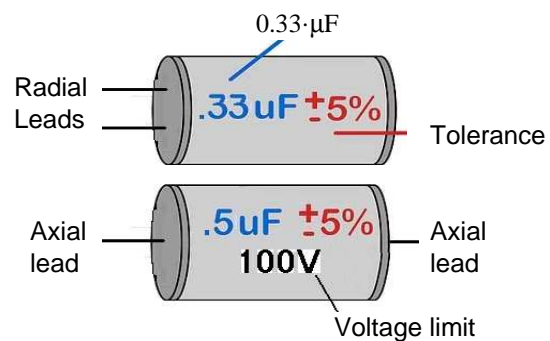
Capacitor voltage **cannot** change instantaneously

Capacitor Construction

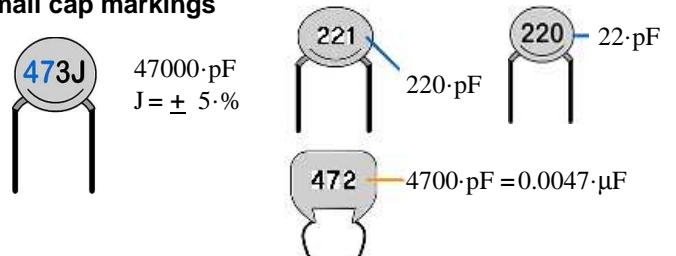


Capacitors are typically classified by the material used for insulation. The insulation determines some of the non-ideal characteristics. See Table 3.7 in text

Large cap markings



Small cap markings

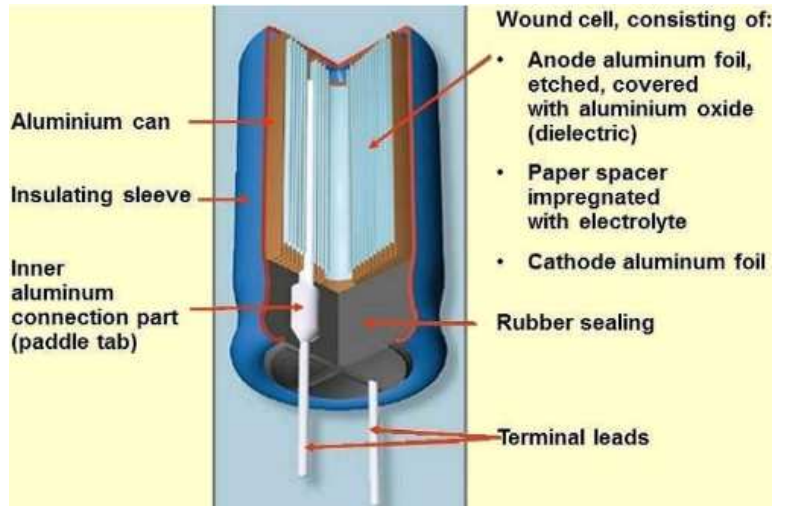
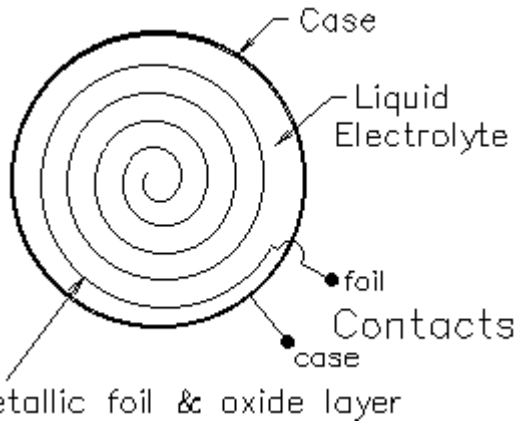


For way more about capacitors, see Section 3.6 of textbook. Especially, see Figure 3.66 for more information about markings.

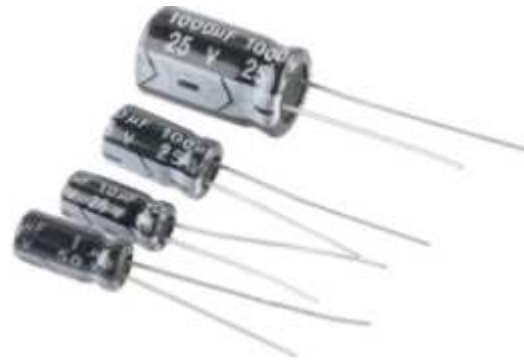
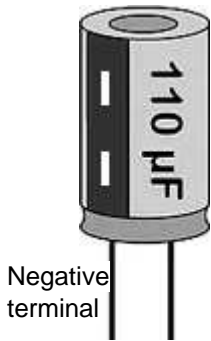
ECE 2210 / 00 Capacitor Notes p2

Electrolytic Capacitors Typically $1\mu\text{F}$ and larger

Construction (top view)

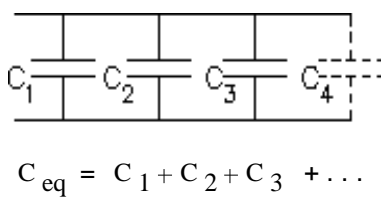


Almost all electrolytic capacitors are **polarized**. The negative terminal must always be negative with respect to the positive terminal, other wise it may be ruined.

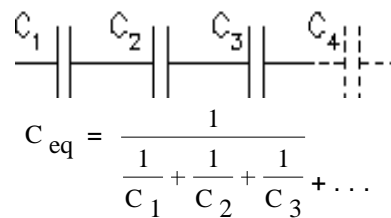


Equivalent Capacitance in series and parallel

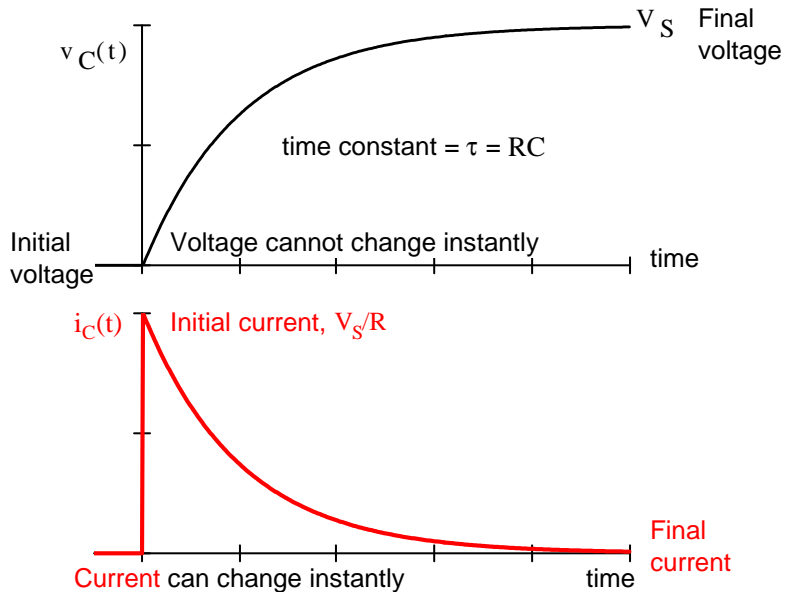
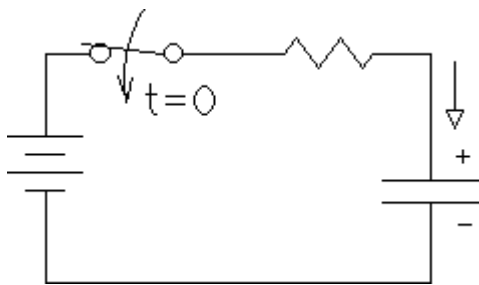
Parallel:



Series:



Capacitors are the only "backwards" components.



Initial Condition is typically found by finding the capacitor voltage just before time $t = 0$.

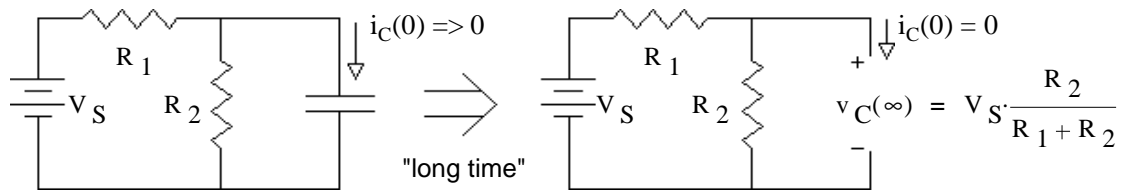
$$v_C(0^-) = v_C(0^+)$$

Voltage just before initial time = current just after initial time
 Capacitor voltage cannot change instantly

Final Condition (Steady-state)

If a circuit has been connected for "a long time", then it has reached a steady state condition. That means the currents and voltages are no longer changing.

$$\text{So... } \frac{d}{dt}v_C = 0 \quad \text{and... } i_C = C \cdot \frac{d}{dt}v_C = 0$$



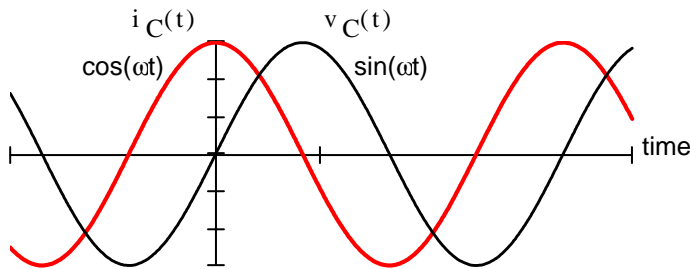
Replace the capacitor with an open and find the voltage across the open (just like finding v_{Th}).
 Applies when sources are constant (DC)

Sinusoids

$$i_C(t) = I_p \cdot \cos(\omega t) \quad v_C(t) = \frac{1}{C} \int i_C dt = \frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p \cdot \sin(\omega t) = \frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p \cdot \cos(\omega t - 90 \cdot \text{deg})$$

indefinite integral $\underbrace{\hspace{1cm}}_{V_p}$ $\underbrace{\hspace{1cm}}_{V_p}$

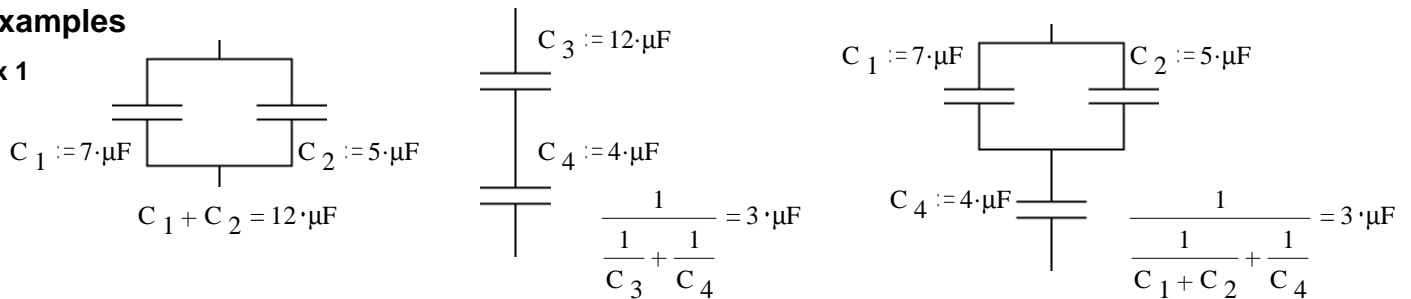
$$V_p = \frac{1}{\omega C} \cdot I_p$$



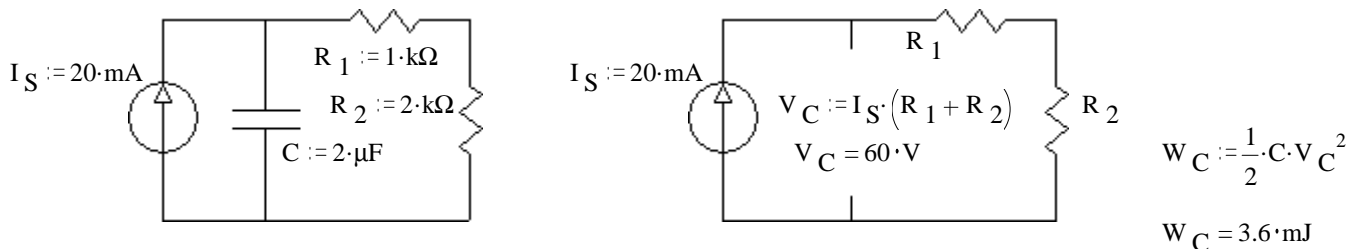
Voltage "lags" current.
 This should make sense to you,
 since current has to flow in first to charge capacitor.

Examples

Ex 1



Ex 2 This circuit has been connected as shown for a long time. Find the energy stored in the cap.

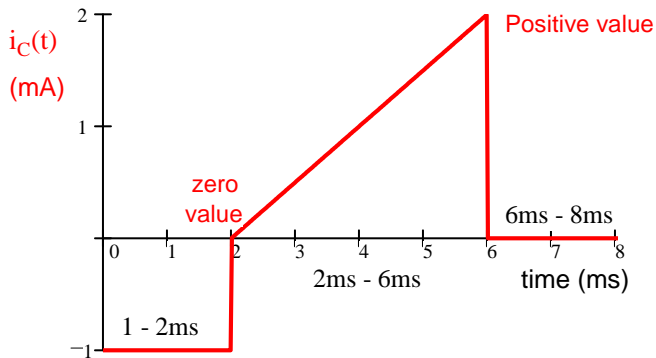
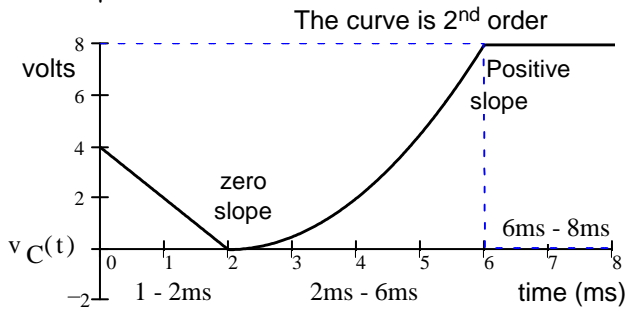


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Ex 3 The voltage across a 0.5 μF capacitor is shown below.

Make an accurate drawing of the capacitor current. Label the y-axis of your graph.

C := 0.5 · μF



1 - 2ms: $i_C = C \cdot \frac{\Delta V}{\Delta t} = 0.5 \cdot \mu F \cdot \frac{-4 \cdot V}{2 \cdot ms} = -1 \cdot mA$

2ms - 6ms: Initial slope is zero and the final slope is positive, so the current must be a triangle that starts at zero and ends at some height.

$$\Delta v_C(t) = \frac{1}{C} \int_{6ms}^{2ms} i_C(t) dt$$

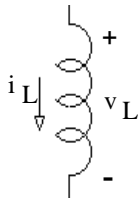
$$8 \cdot V = \frac{1}{C} \left(\frac{4 \cdot ms \cdot \text{height}}{2} \right)$$

$$\text{height} = 8 \cdot V \cdot \frac{C \cdot 2}{4 \cdot ms} = 2 \cdot mA$$

6ms - 8ms: Slope is zero, so the current must be zero.

ECE 2210 / 00 Inductor Lecture Notes

Electrical equivalent:



$$L = \mu \cdot N^2 \cdot K$$

μ is the permeability of the inductor core

K is a constant which depends on the inductor geometry

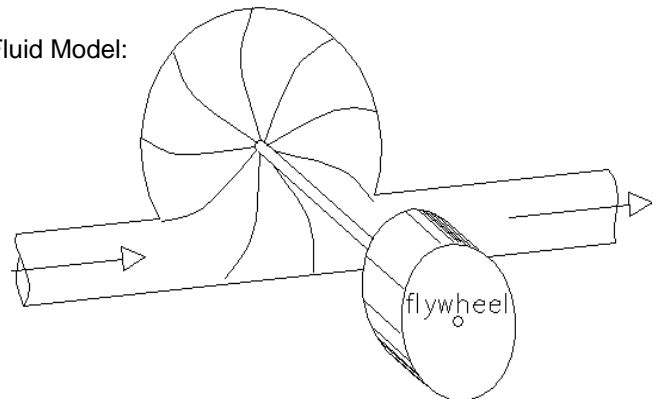
N is the number of turns of wire

$$\text{Units: henry} = \frac{\text{volt} \cdot \text{sec}}{\text{amp}} = H$$

$$mH = 10^{-3} \cdot H$$

$$\mu H = 10^{-6} \cdot H$$

Fluid Model:



Basic equations and Concepts you should know:

$$v_L = L \cdot \frac{d}{dt} i_L$$

$$i_L = \frac{1}{L} \int_{-\infty}^t v_L dt$$

Or... $i_L = \frac{1}{L} \int_0^t v_L dt + i_L(0)$ / initial current

Energy stored in electric field: $W_L = \frac{1}{2} \cdot L \cdot I_L^2$

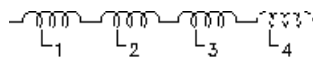
Or... $\Delta i_L = \frac{1}{L} \int_{t_1}^{t_2} v_L dt$

Inductor current **cannot** change instantaneously

ECE 2210 / 00 Capacitor / Inductor Notes p4

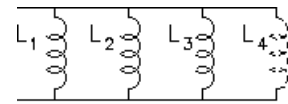
ECE 2210 / 00 Inductor Notes p5

series: $L_{eq} = L_1 + L_2 + L_3 + \dots$

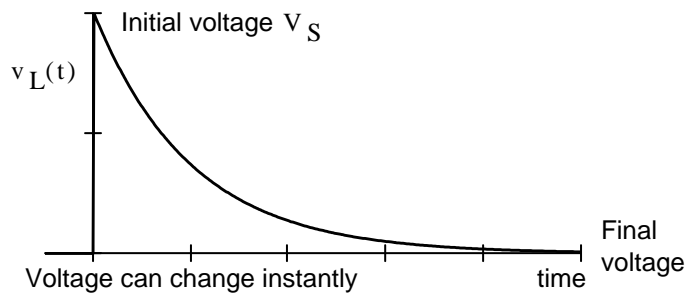
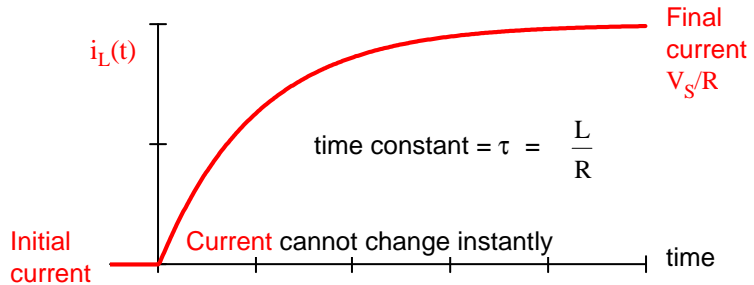
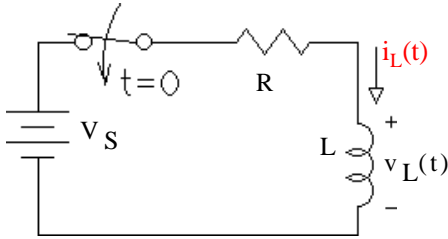


parallel:

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots}$$



"Energizing" an Inductor



Initial Condition is typically found by finding the inductor current just before time $t = 0$.

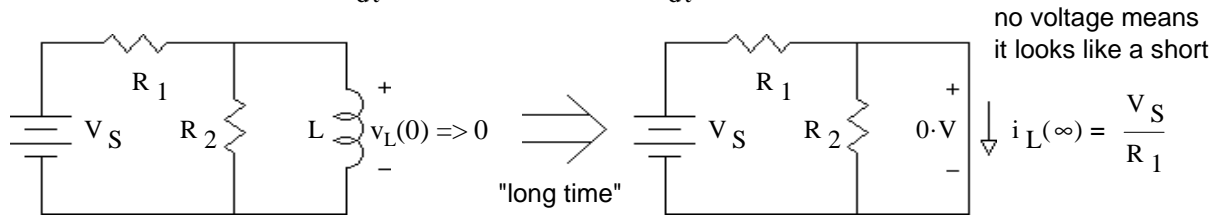
$$i_L(0^-) = i_L(0^+)$$

Current just before initial time = current just after initial time
Inductor current cannot change instantly

Final Condition (Steady-state)

If a circuit has been connected for "a long time", then it has reached a steady state condition. that means the currents and voltages are no longer changing.

$$\frac{d}{dt}i_L = 0 \quad v_L = L \cdot \frac{d}{dt}i_L = 0$$



Replace the inductor with a short and find the voltage across the.
Applies when sources are constant (DC)

Sinusoids

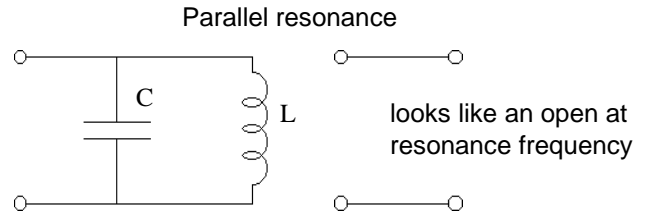
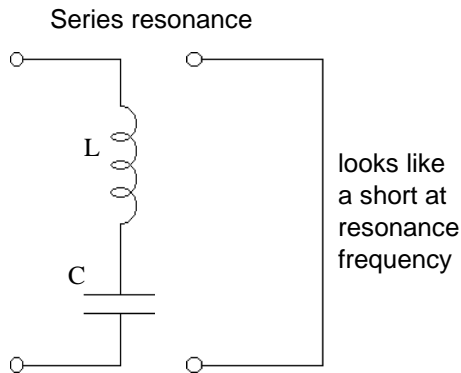
$$i_L(t) = I_p \cdot \cos(\omega t) \quad v_L(t) = L \cdot \frac{d}{dt}i_L = L \cdot \omega (-I_p \cdot \sin(\omega t)) = -L \cdot \omega I_p \cdot \sin(\omega t) = L \cdot \omega I_p \cdot \cos(\omega t + 90\text{-deg})$$



Voltage "leads" current.
This should make sense to you, since voltage has to present to make current change, so voltage comes first.

Resonance At what frequency does $-L \cdot \omega \cdot I_p \cdot \sin(\omega t) + \frac{1}{C} \cdot \frac{1}{\omega} \cdot I_p \cdot \sin(\omega t) = 0$??

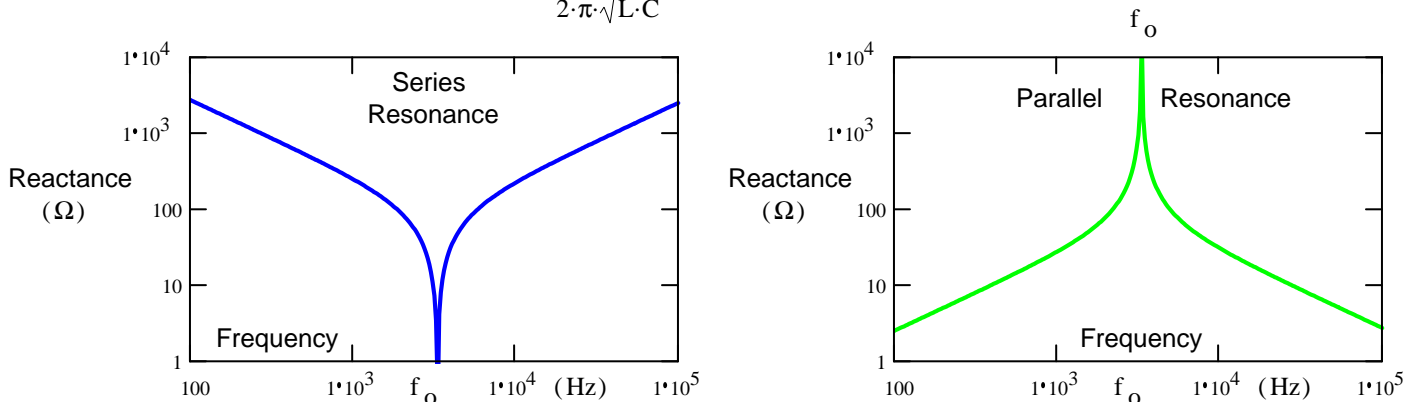
$$L \cdot \omega_o \cdot I_p = \frac{1}{C} \cdot \frac{1}{\omega_o} \cdot I_p \quad \dots \quad \omega_o = \frac{1}{\sqrt{L \cdot C}} \left(\frac{\text{rad}}{\text{sec}} \right)$$



The resonance frequency is calculated the same way for either case:

$$\omega_o = \frac{1}{\sqrt{L \cdot C}} \left(\frac{\text{rad}}{\text{sec}} \right) \quad \text{OR..} \quad \omega_o = \frac{1}{\sqrt{L_{eq} \cdot C_{eq}}} \quad \text{If you have multiple capacitors or inductors which can be combined.} \quad f_o = \frac{\omega_o}{2 \cdot \pi} \quad (\text{Hz})$$

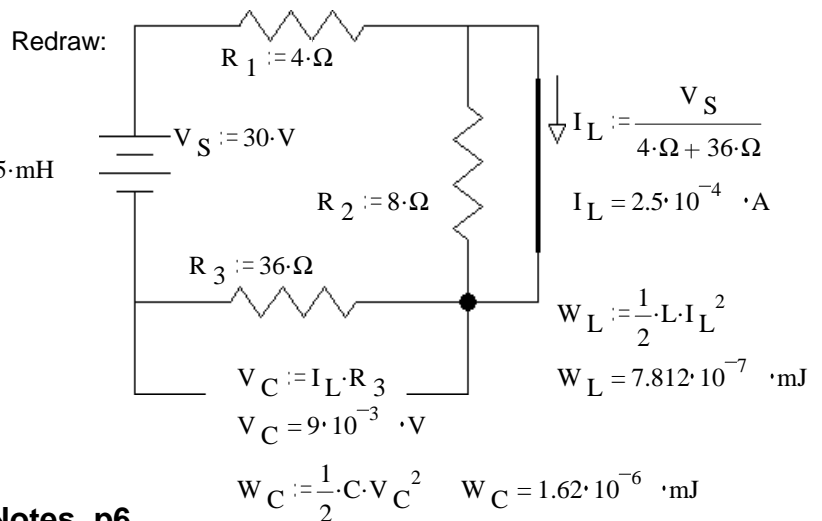
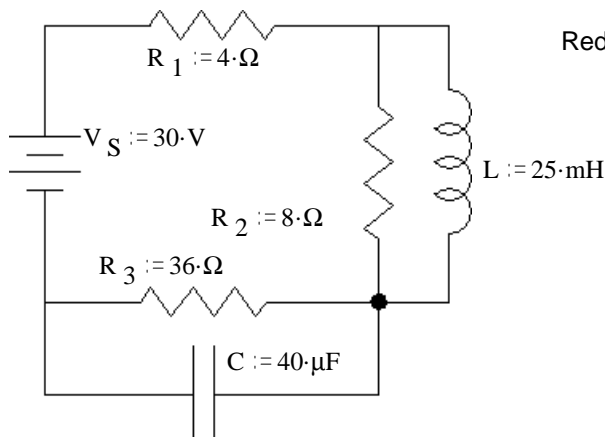
IF $C := 0.58 \cdot \mu\text{F}$ $L := 4 \cdot \text{mH}$ Then $f_o = \frac{1}{2 \cdot \pi \cdot \sqrt{L \cdot C}} = 3.3 \cdot \text{kHz}$ And you get these frequency response curves:



You will soon learn more about reactance and impedance.

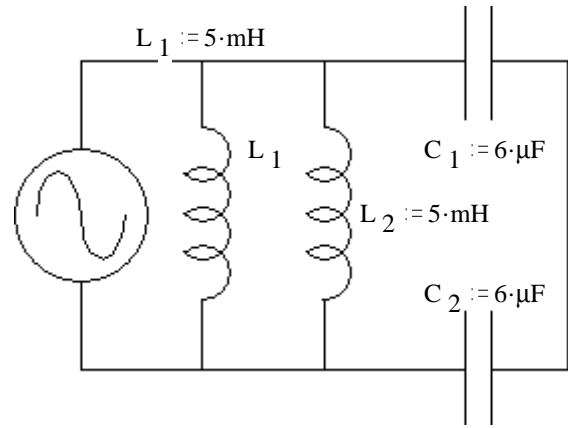
Examples

Ex 1 The circuit has been connected as shown for a long time. Find the energy stored in the capacitor and the inductor.



ECE 2210 / 00 Capacitor / Inductor Notes p7

Ex 2 Find the resonant frequency (or frequencies) of the circuit shown (in cycles/sec or Hz).

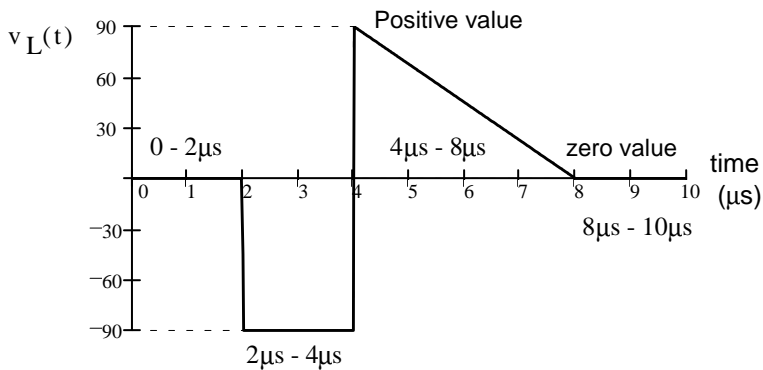
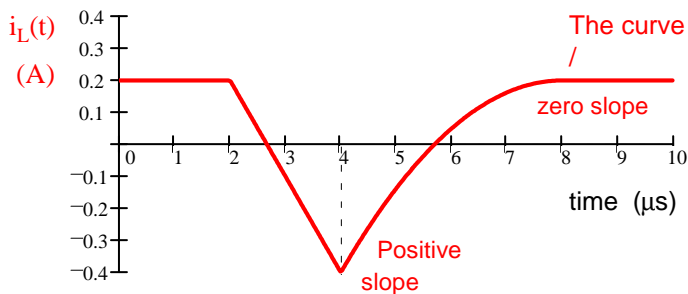


$$L_{eq} := \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} \quad L_{eq} = 2.5 \cdot \text{mH}$$

$$C_{eq} := \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}} \quad C_{eq} = 3 \cdot \mu\text{F}$$

$$\omega_o := \frac{1}{\sqrt{L_{eq} \cdot C_{eq}}} \quad \omega_o = 11547 \cdot \frac{\text{rad}}{\text{sec}} \quad f_o = \frac{\omega_o}{2 \cdot \pi} = 1838 \cdot \text{Hz}$$

Ex 3 The current through a 0.3mH inductor is shown below. Make an accurate drawing of the inductor voltage. Make reasonable assumptions where necessary. Label your graph. $L := 0.3 \cdot \text{mH}$



0 - 2 μs : No change in current, so: $v_L = 0$

$$2\mu\text{s} - 4\mu\text{s}: v_L = L \cdot \frac{\Delta I}{\Delta t} = 0.3 \cdot \text{mH} \cdot \frac{-0.6 \cdot \text{A}}{2 \cdot \mu\text{s}} = -90 \cdot \text{V}$$

4 μs - 8 μs : Initial slope is positive and the final slope is zero, so the voltage must be a triangle that starts at some height and ends at zero.

$$\Delta i_L(t) = \frac{1}{L} \int_{4 \cdot \mu\text{s}}^{8 \cdot \mu\text{s}} v_L(t) dt$$

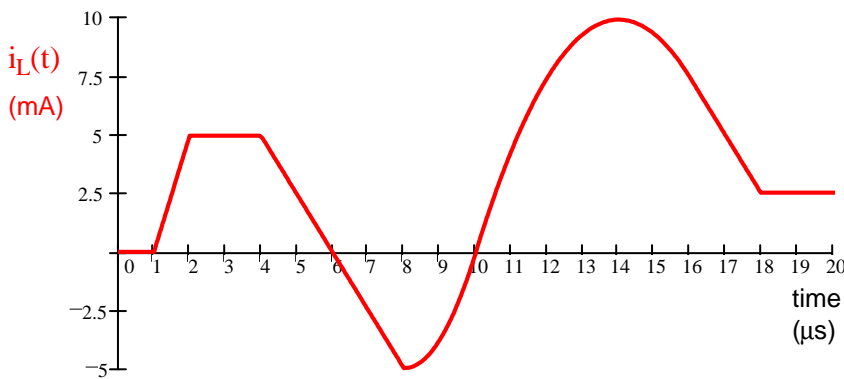
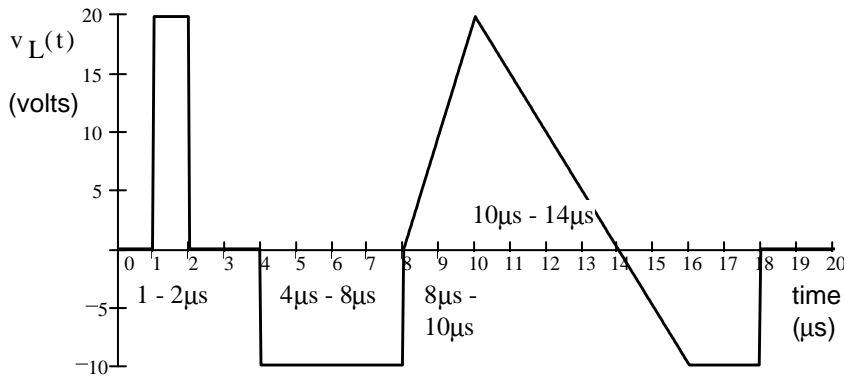
$$0.6 \cdot \text{A} = \frac{1}{0.3 \cdot \text{mH}} \cdot \left(\frac{4 \cdot \mu\text{s} \cdot \text{height}}{2} \right)$$

$$\text{height} = 0.6 \cdot \text{A} \cdot \frac{0.3 \cdot \text{mH} \cdot 2}{4 \cdot \mu\text{s}} = 90 \cdot \text{V}$$

8 μs - 10 μs : No change in current, so: $v_L = 0$

Ex 4 Given an inductor voltage, find the current.

$L := 4 \cdot \text{mH}$



1 - 2μs:

$$\Delta i_L(t) = \frac{1}{L} \int_{1 \cdot \mu\text{s}}^{2 \cdot \mu\text{s}} 20 \cdot \text{V} dt = 5 \cdot \text{mA}$$
 = change from 1 - 2μs

4μs - 8μs:

$$\Delta i_L(t) = \frac{1}{L} \int_{4 \cdot \mu\text{s}}^{8 \cdot \mu\text{s}} -10 \cdot \text{V} dt = -10 \cdot \text{mA}$$
 = change from 4μs - 8μs

8μs - 10μs:

$$\Delta i_L(t) = \frac{1}{L} \int_{8 \cdot \mu\text{s}}^{10 \cdot \mu\text{s}} V(t) dt$$

$$= \frac{1}{L} \cdot \frac{20 \cdot \text{V} \cdot 2 \cdot \mu\text{s}}{2} = 5 \cdot \text{mA} = \text{change from } 8\mu\text{s} - 10\mu\text{s}$$

Voltage ramps from 0 to +20V, so **current curve** progresses from 0 slope to a positive slope

10μs - 14μs:

$$\Delta i_L(t) = \frac{1}{L} \int_{10 \cdot \mu\text{s}}^{14 \cdot \mu\text{s}} V(t) dt + 0 \cdot \text{mA}$$

$$= \frac{1}{L} \cdot \frac{20 \cdot \text{V} \cdot 4 \cdot \mu\text{s}}{2} = 10 \cdot \text{mA}$$

Current curves from a positive slope to 0 slope etc...

Ex 5 The current through and the voltage across an unknown component are shown below.

a) What type of component is it?
 Give a good reason for your choice.

inductor ? $v(t) := L \cdot \frac{d}{dt} i(t)$

Doesn't fit graphs, still have current even when voltage isn't changing.

Also Inductor current can't change instantly.

Can't be an Inductor

capacitor ? $i(t) := C \cdot \frac{d}{dt} v(t)$

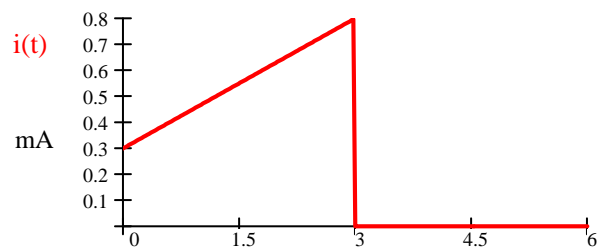
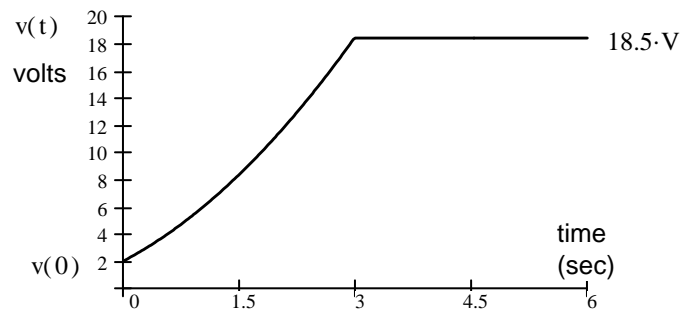
This fits the graphs, $i(t)$ corresponds to slope of $v(t)$

Must be a Capacitor

b) What is the value of the component?

$$v(3 \cdot \text{sec}) = \frac{1}{C} \int_0^{3 \cdot \text{sec}} i(t) dt + v(0)$$

OR: $\Delta v = \frac{1}{C} \cdot (\text{current_area}) \quad \Delta v := 16.5 \cdot \text{V}$



$$\text{current_area} := 0.3 \cdot \text{mA} \cdot 3 \cdot \text{sec} + \frac{1}{2} \cdot 0.5 \cdot \text{mA} \cdot 3 \cdot \text{sec}$$

$$C := \frac{\text{current_area}}{\Delta v} \quad C = 100 \cdot \mu\text{F}$$