AC Power Examples

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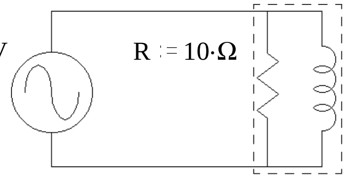
Ex. 1 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

$\mathbf{V_{in}} := 110 \cdot V$ $\quad R := 10 \cdot \Omega$ $\quad L := 25 \cdot mH$

$\omega := 377 \cdot \frac{rad}{sec}$



$$\mathbf{Z} := \frac{1}{\left(\frac{1}{R} + \frac{1}{j \cdot \omega \cdot L}\right)} = \frac{1}{0.1458 \cdot \frac{1}{\Omega} \cdot e^{-j \cdot 46.7 \cdot deg}}$$

$\mathbf{Z} = 4.704 + 4.991j \;\cdot\Omega$ $\quad |\mathbf{Z}| = 6.859 \cdot \Omega$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 46.7 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.686$

$\mathbf{I} := \frac{\mathbf{V_{in}}}{\mathbf{Z}}$ $\quad \mathbf{I} = 11 - 11.671j \;\cdot A$ $\quad |\mathbf{I}| = 16.038 \cdot A$ $\quad \arg(\mathbf{I}) = -46.7 \cdot deg$

$P := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 1.21 \cdot kW$

$Q := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 1.284 \cdot kVAR$ OR... $Q := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot \sqrt{1^2 - pf^2}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := \mathbf{V_{in}} \cdot \overline{\mathbf{I}}$ OR.. $\mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 1.21 + 1.284j \;\cdot kVA = 1.764kVA \;\underline{/46.7^o}$ $\quad \operatorname{atan}\left(\frac{\operatorname{Im}(\mathbf{S})}{\operatorname{Re}(\mathbf{S})}\right) = 46.696 \cdot deg$ same angle as **Z**

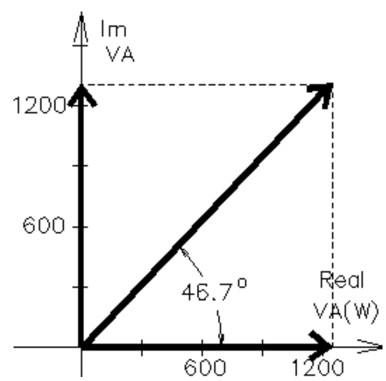
$S := \sqrt{P^2 + Q^2} = |\mathbf{S}| = 1.764 \cdot kVA = |\mathbf{V_{in}}| \cdot |\mathbf{I}| = 1.764 \cdot kVA$

OR, since we know that the voltage across each element of the load is V_{in} ...
Real power is dissipated only by resistors

$P := \frac{\left(|\mathbf{V_{in}}|\right)^2}{R}$ $\quad P = 1.21 \cdot kW$ $\quad Q := \frac{\left(|\mathbf{V_{in}}|\right)^2}{\omega \cdot L}$ $\quad Q = 1.284 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$

$S = |\mathbf{S}| = \sqrt{P^2 + Q^2} = 1.764 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.686$



What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q_C := -Q = \frac{(110 \cdot V)^2}{X_C}$ $\quad X_C := \frac{(110 \cdot V)^2}{-Q}$ $\quad X_C = -9.425 \cdot \Omega$

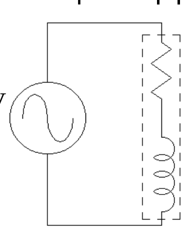
$\frac{1}{|X_C| \cdot \omega} = 281 \cdot \mu F$ OR.. $Q_C := -Q = (110 \cdot V)^2 \cdot \omega \cdot C$ $\quad C = \frac{-Q}{(110 \cdot V)^2 \cdot \omega} = -281.434 \cdot \mu F$

Ex. 2 R & L together are the load. Find the real power P, the reactive power Q, the complex power **S**, the apparent power |**S**|, & the power factor pf. Draw phasor diagram for the power.

Series R & L

$\mathbf{V_{in}} := 110 \cdot V$

$\omega := 377 \cdot \frac{rad}{sec}$



$R := 10 \cdot \Omega$ $\quad \mathbf{Z} := R + j \cdot \omega \cdot L$

load $\quad \mathbf{Z} = 10 + 9.425j \;\cdot \Omega$ $\quad |\mathbf{Z}| = 13.742 \cdot \Omega$

$L := 25 \cdot mH$ $\quad \theta := \arg(\mathbf{Z})$ $\quad \theta = 43.304 \cdot deg$ $\quad pf := \cos(\theta)$ $\quad pf = 0.728$

$\mathbf{I} := \frac{\mathbf{V_{in}}}{\mathbf{Z}}$ $\quad \mathbf{I} = 5.825 - 5.49j \;\cdot A$

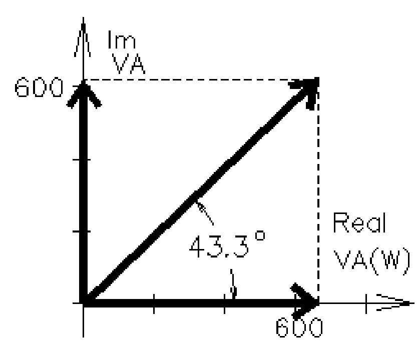
$|\mathbf{I}| = 8.005 \cdot A$ $\quad \arg(\mathbf{I}) = -43.304 \cdot deg$

$P := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot pf$ $\quad P = 0.641 \cdot kW$

$Q := |\mathbf{V_{in}}| \cdot |\mathbf{I}| \cdot \sin(\theta)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := \mathbf{V_{in}} \cdot \overline{\mathbf{I}}$ $\quad \mathbf{S} = 0.641 + 0.604j \;\cdot kVA$

$|\mathbf{S}| = 0.881 \cdot kVA$ $\quad \arg(\mathbf{S}) = 43.304 \cdot deg$ $\quad \mathbf{S} = 881VA\underline{/43.3^o}$



OR, if we first find the magnitude of the current which flows through each element of the load...

$$|\mathbf{I}| = \frac{\mathbf{V_{in}}}{\sqrt{R^2 + (\omega \cdot L)^2}} = 8.005 \cdot A$$

$P := (|\mathbf{I}|)^2 \cdot R$ $\quad P = 0.641 \cdot kW$ $\quad Q := (|\mathbf{I}|)^2 \cdot (\omega \cdot L)$ $\quad Q = 0.604 \cdot kVAR$

$\mathbf{S} := P + j \cdot Q$ $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 0.881 \cdot kVA$ $\quad pf = \frac{P}{|\mathbf{S}|} = 0.728$

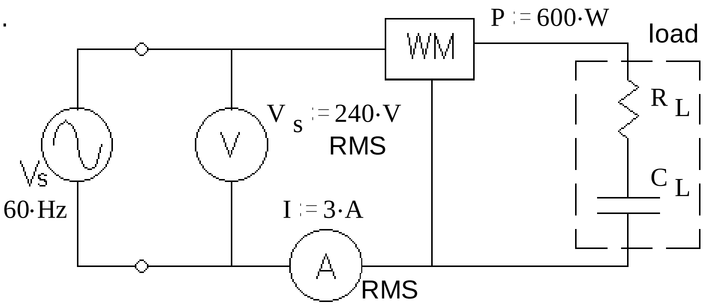
What value of C in parallel with R & L would make pf = 1 (Q = 0) ?

$Q = 603.9 \cdot VAR$ so we need: $Q_C := -Q$ $\quad Q_C = -603.9 \cdot VAR = \frac{V_{in}^2}{X_C}$

$X_C := \frac{\mathbf{V_{in}}^2}{Q_C}$ $\quad X_C = -20.035 \cdot \Omega = \frac{-1}{\omega \cdot C}$ $\quad C := \frac{1}{|X_C| \cdot \omega}$ $\quad C = 132 \cdot \mu F$

Check: $\frac{1}{\frac{1}{R + j \cdot \omega \cdot L} + j \cdot \omega \cdot C} = 18.883 \cdot \Omega$ No j term, so $\theta = 0^o$

Ex. 3 R, & C together are the load in the circuit shown. The RMS voltmeter measures 240 V, the RMS ammeter measures 3 A, and the wattmeter measures 600 W. Find the following: Be sure to show the correct units for each value.



a) The value of the load resistor. R_L = ?

$$P = I^2 \cdot R_L$$

$R_L := \frac{P}{I^2}$ $\quad R_L = 66.7 \cdot \Omega$

b) The apparent power. |**S**| = ? $\quad S := V_s \cdot I$ $\quad S = 720 \cdot VA$

c) The reactive power. Q = ? $\quad Q := -\sqrt{S^2 - P^2}$ $\quad Q = -398 \cdot VAR$

d) The complex power. **S** = ? $\quad \mathbf{S} := P + j \cdot Q$ $\quad \mathbf{S} = 600 - 398i \cdot VA$

e) The power factor. pf = ? $\quad pf := \frac{P}{V_s \cdot I}$ $\quad pf = 0.833$

f) The power factor is leading or lagging? leading (load is capacitive, Q is negative)

g) The two components of the load are in a box which cannot be opened. Add (draw it) another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

$f = 60 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$ $\quad \omega = 376.991 \cdot \frac{rad}{sec}$

Add an inductor in parallel with load

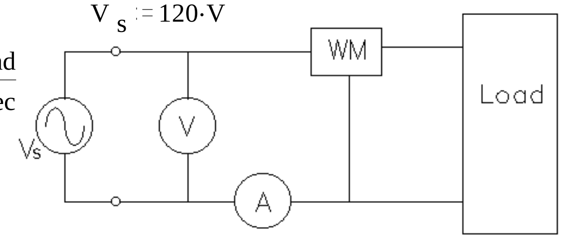
$Q = -398 \cdot VAR$ so we need: $Q_L := -Q$ $\quad Q_L = 398 \cdot VAR = \frac{V_s^2}{X_L}$

$X_L := \frac{V_s^2}{Q_L}$ $\quad X_L = 144.725 \cdot \Omega = \omega \cdot L$ $\quad L := \frac{|X_L|}{\omega}$ $\quad L = 384 \cdot mH$

Ex. 4 For the 60 Hz load shown in the figure, the RMS voltmeter measures 120 V. The phasor diagram for the power is also shown. Find the following:

$V_s := 120 \cdot V$

$\omega := 377 \cdot \frac{rad}{sec}$



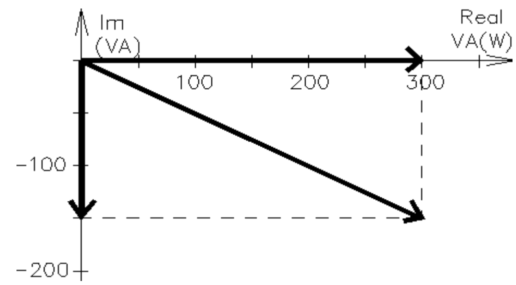
a) The complex power. **S** = ?

$P := 300 \cdot W$ $\qquad Q := -150 \cdot VA$

$\mathbf{S} := P + j \cdot Q$ $\qquad \mathbf{S} = 300 - 150j \cdot VA$

b) The apparent power. |**S**| = ? $\quad |\mathbf{S}| = \sqrt{P^2 + Q^2} = 335.4 \cdot VA$

c) The power factor. pf = ? $\quad pf := \frac{P}{|\mathbf{S}|}$ $\qquad pf = 0.894$



d) The item marked "WM" in the figure is a wattmeter, what does it read? (give a number) $\quad P = 300 \cdot W$

e) The item marked "A" in the figure is an RMS ammeter, what does it read? (give a number)

$I := \frac{|\mathbf{S}|}{V_s}$ $\qquad I = 2.795 \cdot A$ $\qquad I = 2.8 \cdot A$

f) The power factor is leading or lagging? $\quad$ leading (Q is negative)

g) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and <u>find its value</u>. This component should not affect the real power consumption of the load.

Add an inductor in parallel with load

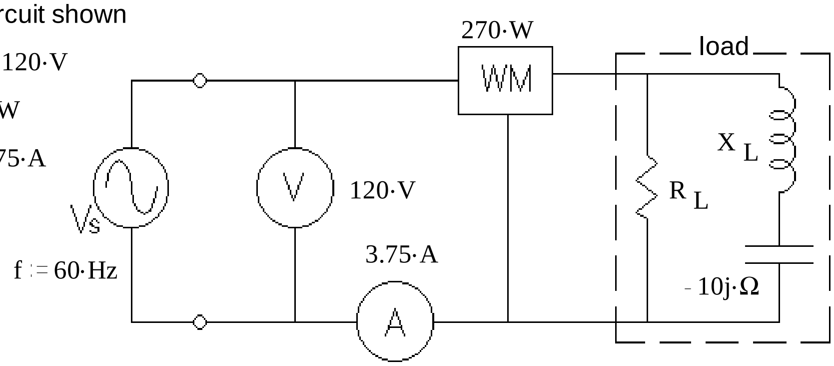
$Q = -150 \cdot VAR$ $\quad$ need: $\quad Q_L := -Q$ $\qquad Q_L = 150 \cdot VAR = \frac{V_s^2}{\omega \cdot L}$ $\qquad L := \frac{V_s^2}{\omega \cdot Q_L}$ $\qquad L = 255 \cdot mH$

Ex. 5 R, L, & C together are the load in the circuit shown

The RMS voltmeter measures 120 V. $\quad V_s := 120 \cdot V$

The wattmeter measures 270 W. $\quad P := 270 \cdot W$

The RMS ammeter measures 3.75 A. $\quad I := 3.75 \cdot A$



Find the following: Be sure to show the correct units for each value.

a) The value of the load resistor. R_L = ?

$P = \frac{V_s^2}{R_L}$ $\qquad R_L := \frac{V_s^2}{P}$ $\qquad R_L = 53.3 \cdot \Omega$

b) The magnitude of the impedance of the load inductor (reactance) . $|\mathbf{Z_L}| = X_L$ = ?

$I_R := \frac{V_s}{R_L}$ $\qquad I_R = 2.25 \cdot A$ $\qquad I_L := \sqrt{I^2 - I_R^2}$ $\qquad I_L = 3 \cdot A$ $\qquad X := \frac{V_s}{I_L}$ $\qquad X = 40 \cdot \Omega$

OR: $S := V_s \cdot I$ $\qquad S = 450 \cdot VA$ $\qquad Q := \sqrt{S^2 - P^2}$ $\qquad Q = 360 \cdot VAR = \frac{V_s^2}{X}$ $\qquad X = \frac{V_s^2}{Q} = 40 \cdot \Omega$

either way: $\quad X_C := -10 \cdot \Omega$ $\qquad X_L := X - X_C$ $\qquad X_L = 50 \cdot \Omega$

c) The reactive power. Q = ? $\quad Q := \sqrt{(V_s \cdot I)^2 - P^2}$ $\qquad Q = 360 \cdot VAR$ $\quad$ positive, because the load is primarily inductive

d) The power factor is leading or lagging? $\quad$ lagging (load is inductive, Q is positive)

e) The 3 components of the load are in a box which cannot be opened. Add another component to the circuit above which can correct the power factor (make pf = 1). Show the correct component in the correct place and find its value. This component should not affect the real power consumption of the load.

Add a capacitor in parallel with load

$f = 60 \cdot \text{Hz} \qquad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}}$

$Q = 360 \cdot \text{VAR}$ so we need: $Q_C := -Q \qquad Q_C = -360 \cdot \text{VAR} = -\frac{V_s^2}{\frac{1}{\omega \cdot C}} = -\omega \cdot C \cdot V_s^2$

$$C := \frac{Q_C}{-\omega \cdot V_s^2} \qquad C = 66.3 \cdot \mu\text{F}$$

Ex. 6 A step-down transformer has an output voltage of 220 V (rms) when the primary is connected across a 560 V (rms) source.

a) If there are 280 turns on the primary winding, how many turns are required on the secondary?

$280 \cdot \frac{220 \cdot \text{volt}}{560 \cdot \text{volt}} = 110$ turns

b) If the current in the primary is 2.4 A, what current flows in the load connected to the secondary?

$2.4 \cdot \text{amp} \cdot \frac{280}{110} = 6.11 \cdot \text{A}$

c) If the transformer is rated at 700/275 V, 2.1 kVA, what are the rated primary and secondary currents?

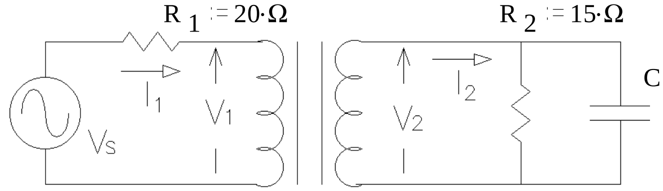
pri: $\frac{2.1 \cdot \text{kVA}}{700 \cdot \text{V}} = 3 \cdot \text{A}$ sec: $\frac{2.1 \cdot \text{kVA}}{275 \cdot \text{V}} = 7.636 \cdot \text{A}$

Ex. 7 The transformer shown in the circuit below is ideal. Find the following:

a) $|\mathbf{I_1}|$ = ?

$V_s := 120 \cdot \text{V} \qquad \omega := 377 \cdot \frac{\text{rad}}{\text{sec}}$

$R_1 := 20 \cdot \Omega \qquad R_2 := 15 \cdot \Omega \qquad C := 40 \cdot \mu\text{F}$



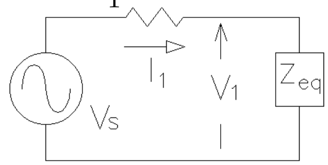
$N_1 := 150 \cdot \text{turns} \qquad N_2 := 50 \cdot \text{turns}$

$$\mathbf{Z_L} := \frac{1}{\frac{1}{R_2} + j \cdot \omega \cdot C}$$

$$\mathbf{Z_L} = 14.27 - 3.228j \ \cdot \Omega$$

Make an equivalent circuit:

$R_1 := 20 \cdot \Omega$



$$\mathbf{Z_{eq}} := \left(\frac{N_1}{N_2}\right)^2 \cdot \mathbf{Z_L} \qquad \mathbf{Z_{eq}} = 128.429 - 29.051j \ \cdot \Omega$$

$$R_1 + \mathbf{Z_{eq}} = 148.429 - 29.051j \ \cdot \Omega \qquad \sqrt{148.429^2 + 29.051^2} = 151.245$$

$$|\mathbf{I_1}| = \frac{V_s}{|R_1 + \mathbf{Z_{eq}}|} = \frac{V_s}{151.245 \cdot \Omega} = 0.793 \cdot \text{A}$$

b) $|\mathbf{I_2}|$ = ? $= \left(\frac{N_1}{N_2}\right) \cdot |\mathbf{I_1}| = \frac{150}{50} \cdot .793 \cdot \text{A} = 2.379 \cdot \text{A}$

c) $|\mathbf{V_1}|$ = ? $= V_s \cdot \left|\frac{\mathbf{Z_{eq}}}{R_1 + \mathbf{Z_{eq}}}\right|$ OR.. $|\mathbf{V_1}| = |\mathbf{I_1}| \cdot |\mathbf{Z_{eq}}| = .793 \cdot \text{A} \cdot \sqrt{128.429^2 + 29.051^2} \cdot \Omega = 104.417 \cdot \text{V}$