AC Notes

AC = Alternating Current

This term is used for any time-varying voltage or current waveform

Periodic waveforms: Waveshape repeats

\[ T = \text{Period} = \text{repeat time} \]
\[ f = \text{frequency, cycles / second} \]
\[ \omega = \text{radian frequency, radians/sec} \]
\[ A = \text{amplitude} \]

DC = average

\[ y(t) = A \cdot \cos(\omega t + \phi) \]

Voltage: \[ v(t) = V_p \cdot \cos(\omega t + \phi) \]

Current: \[ i(t) = I_p \cdot \cos(\omega t + \phi) \]

Phase: \[ \phi = \frac{\Delta t}{T} \cdot 360 \text{ deg} \]

Adding and subtracting Sinusoidal AC voltages or currents

Voltages and currents are represented by phasors draw on a real - imaginary "phasor" diagram. These phasors add and subtract just like vectors.

\[ v_1(t) = V_{p1} \cos(\omega t + \phi) \]
\[ v_2(t) = V_{p2} \cos(\omega t + \phi) \]

The problem: \[ v_3(t) = v_1(t) + v_2(t) \]

Draw phasors as shown, \[ V_{v1}(j\omega) = V_{p1}/\psi \]
\[ V_{v2}(j\omega) = V_{p2}/\psi \]

Add the phasors just as you would vectors to get: \[ V_{v3}(j\omega) = V_{p3}/\psi \]

Convert phasor back to time-domain voltage: \[ v_3(t) = v_1(t) + v_2(t) = V_{p3} \cos(\omega t + \psi) \]

Phasor analysis. For steady-state sinusoidal response ONLY

Phasors are used for much more than just adding and subtracting sinusoidal waveforms and are drawn on a complex plane for a very good reason. The math is all based on the Euler's equation:

Euler's equation \[ e^{j\alpha} = \cos(\alpha) + j \cdot \sin(\alpha) \]

\[ e^{j(\omega t + \Theta)} = \cos(\omega t + \Theta) + j \cdot \sin(\omega t + \Theta) \]

If we freeze this at time \( t=0 \), then we can represent \( \cos(\omega t + \Theta) \) by \( e^{j\Theta} \) That's the phasor

Capacitors and Inductors in AC circuits cause 90° phase shifts between voltages and currents because they integrate and differentiate. But... integration and differentiation is a piece-of-cake in phasors.

Calculus \[ \frac{d}{dt} [A \cdot e^{j(\omega t + \Theta)}] = j \cdot \omega \cdot A \cdot e^{j(\omega t + \Theta)} = \omega \cdot A \cdot e^{j(\omega t + \Theta + 90°)} \]

Drop the \( \Theta \) at \( t=0 \) to get:

\[ \int A \cdot e^{j(\omega t + \Theta)} \, dt = \frac{1}{j \cdot \omega} \cdot A \cdot e^{j(\omega t + \Theta)} = \frac{1}{\omega} \cdot A \cdot e^{j(\omega t + 90°)} = \frac{1}{\omega} \cdot A \cdot e^{j(\theta - 90°)} \]
**Phasor analysis with impedances**, For steady-state sinusoidal response ONLY

<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
<th>AC impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitor</td>
<td>$i_C = C \frac{dv_C}{dt}$</td>
<td>$Z_C = \frac{1}{j \omega C} = \frac{j}{\omega C}$, $V_C(j\omega) = \frac{1}{j \omega C} \cdot I(j\omega)$</td>
</tr>
<tr>
<td>Inductor</td>
<td>$v_L = L \frac{di_L}{dt}$</td>
<td>$Z_L = j \omega L$, $V_L(j\omega) = j \omega L \cdot I(j\omega)$</td>
</tr>
<tr>
<td>Resistor</td>
<td>$v_R = i_R \cdot R$</td>
<td>$Z_R = R$, $V_R(j\omega) = R \cdot I(j\omega)$</td>
</tr>
</tbody>
</table>

You can use impedances just like resistances as long as you deal with the complex arithmetic. ALL the DC circuit analysis techniques will work with AC.

### Series

- **$Z_{eq} = Z_1 + Z_2 + Z_3 + \ldots$**

**Example:**

- $Z_{eq} = R + \frac{1}{j \omega C} + j \omega L$

### Parallel

- **$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$**

**Example:**

- $Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{j \omega C}} + \frac{1}{j \omega L}}$

**Example**

Find $V_O$ in the circuit shown. Express it as a magnitude and phase angle (the way $V_S$ is expressed).

$V_O := \frac{Z_2}{Z_1 + Z_2} \cdot V_S$  
Simple voltage divider

- $|Z_2| \cos(-60\text{deg}) = 40 \cdot \Omega$
- $|Z_2| \sin(-60\text{deg}) = -69.282 \cdot \Omega$

$Z_2 = 40 - 69.282j \cdot \Omega$

- $Z_1 + Z_2 = 25 \cdot \Omega + 35j \cdot \Omega + 40 \cdot \Omega - 69.282j \cdot \Omega = 65 - 34.282j \cdot \Omega = 73.486 \cdot \Omega \cdot e^{j27.81\text{deg}}$

- $|V_O| = \frac{Z_2}{Z_1 + Z_2} \cdot |V_S| = \frac{80 \cdot \Omega \cdot e^{j60\text{deg}}}{73.486 \cdot \Omega \cdot e^{j27.81\text{deg}}} \cdot (6 \cdot V \cdot e^{j18\text{deg}}) = \frac{80 \cdot \Omega}{73.486 \cdot \Omega} \cdot 6 \cdot V \cdot e^{j(60\text{deg} - 27.81\text{deg} + 18\text{deg})} = 6.53 \cdot V \cdot e^{j14.2\text{deg}}$