ECE 2210 / 00   Max power & Nodal Analysis Notes

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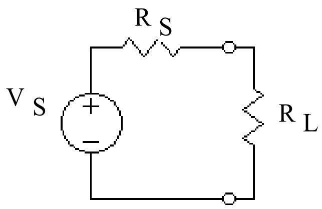
Maximum power transfer

If I wanted to maximize the power at the load, what R_L would I choose?



$$P_L = \frac{V_L^2}{R_L} = \left(\frac{R_L}{R_S + R_L}\cdot V_S\right)^2 \cdot \frac{1}{R_L} = \frac{R_L^2}{\left(R_S + R_L\right)^2}\cdot V_S^2 \cdot \frac{1}{R_L}$$

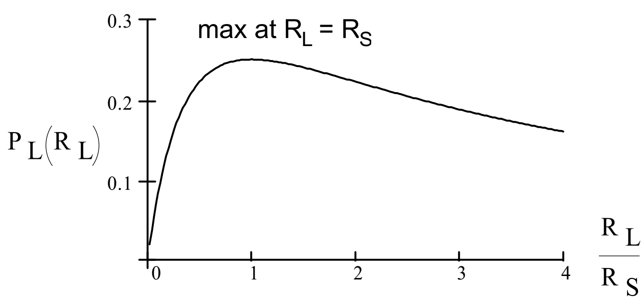
$$= \frac{R_L^2}{R_S^2 + 2\cdot R_S\cdot R_L + R_L^2}\cdot V_S^2\cdot\frac{1}{R_L} = \frac{R_L}{R_S^2 + 2\cdot R_S\cdot R_L + R_L^2}\cdot V_S^2$$

$$= \frac{1}{\frac{R_S^2}{R_L} + 2\cdot R_S + R_L}\cdot V_S^2$$

Next step would be to differentiate $\frac{d}{dR_L}P_L\left(R_L\right)$, set this equal to 0 and solve for R_L to find the maximum

Unfortunately this function is a pain to differentiate. What if we just differentiate the denominator and find its minimum, wouldn't that work just as well?

$$\frac{d}{dR_L}\left(\frac{R_S^2}{R_L} + 2\cdot R_S + R_L\right) = -1\cdot\frac{R_S^2}{R_L^2} + 0 + 1 = 0$$



Maximum power transfer happens when: $R_L = R_S$

This is rarely important in power circuitry, where there should be plenty of power and R_S should be small. It is much more likely to be important in signal circuitry where the voltages can be very small and the source resistance may be significant -- say a microphone or a radio antenna.

General Network Analysis

In many cases you have multiple unknowns in a circuit, say the voltages across multiple resistors. Network analysis is a systematic way to generate multiple equations which can be solved to find the multiple unknowns. These equations are based on basic Kirchoff's and Ohm's laws.

Loop or Mesh Analysis You may have used these methods in previous classes, particularly in Physics. The best thing to do now is to forget all that. Loop analysis is rarely the easiest way to analyze a circuit and is inherently confusing. Hopefully I've brought you to a stage where you have some intuitive feeling for how currents flow in circuits. I don't want to ruin that now by screwing around with loop currents that don't really exist.

Nodal analysis This is a much better method. It's just as powerful, usually easier, and helps you develop your intuitive feeling for how circuits work.

Nodal Analysis

A **node** is any part of a circuit which is connected by wire.

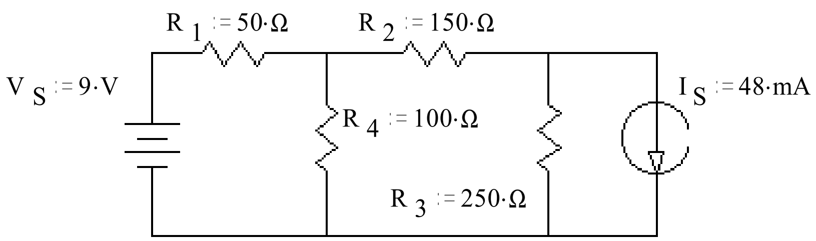
Ground: One node in the circuit which will be our reference node. Ground, by definition, will be the zero voltage node. All other node voltages will be referenced to ground and may be positive or negative. Think of gage pressure in a fluid system. In that case atmospheric pressure is considered zero. If there is no ground in the circuit, define one for yourself. Try to chose a node which is hooked to one side of a voltage source.

Nodal Voltage: The voltage of a node referenced to ground. The objective of nodal analysis is to find all the nodal voltages. If you know the voltage at a node then it's a "known" node. Ground is a known node (duh, it's zero). If one end of a known voltage source hooked to ground, then the node on the other end is also known (another duh).

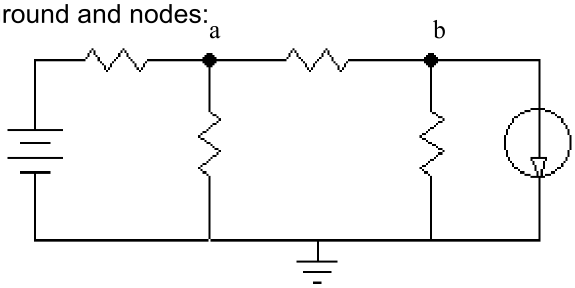
Method: Label all the unknown nodes as; "a", "b", "c", etc. Then the unknown nodal voltages become; V_a, V_b, V_c, etc. Write a KCL equation for each unknown node, defining currents as necessary. Replace each unknown current with an Ohm's law relationship using the nodal voltages. Now you have just as many equations as unknowns. Solve.

ECE 2210 / 00   Max power & Nodal Analysis Notes

Nodal Analysis Example



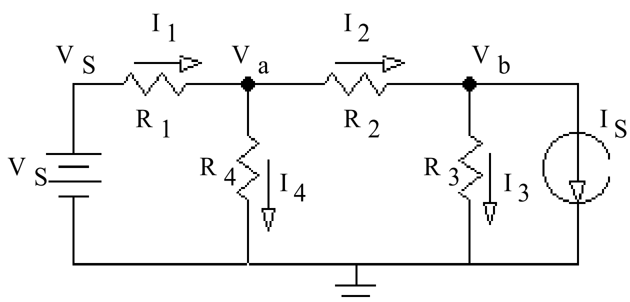
Define ground and nodes:



2 unknown node ---> will need 2 equation

Define currents for KCL equations:

It doesn't matter if these currents are in the correct directions.



Write the KCL equations at each unknown node:

node a $\quad I_1 = I_2 + I_4$

node b $\quad I_2 = I_3 + I_S$

Replace each unknown current with an Ohm's law relationship using the nodal voltages

node a $\quad I_1 = I_2 + I_4$

$$\frac{V_S - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a - 0\cdot V}{R_4}$$

node b $\quad I_2 = I_3 + I_S$

$$\frac{V_a - V_b}{R_2} = \frac{V_b - 0\cdot V}{R_3} + I_S$$

Now you have just as many equations as unknowns.

Solve by any method you like:

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{V_b}{R_2} + \frac{V_a}{R_4}$$

$$\frac{V_a}{R_2} - \frac{V_b}{R_2} = \frac{V_b}{R_3} + I_S \qquad V_b = \frac{\frac{V_a}{R_2} - I_S}{\frac{1}{R_2} + \frac{1}{R_3}}$$

$$\frac{V_S}{R_1} - \frac{V_a}{R_1} = \frac{V_a}{R_2} - \frac{\frac{V_a}{R_2} - I_S}{R_2\cdot\left(\frac{1}{R_2} + \frac{1}{R_3}\right)} + \frac{V_a}{R_4}$$

$$V_a := \frac{\left[\frac{V_S}{R_1} - \frac{1}{\left[R_2\cdot\left(\frac{1}{R_2} + \frac{1}{R_3}\right)\right]}\cdot I_S\right]}{\left[\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2{}^2\cdot\left(\frac{1}{R_2} + \frac{1}{R_3}\right)} + \frac{1}{R_4}\right]} \qquad V_a = 4.615\cdot V$$

$$V_b := \frac{\frac{V_a}{R_2} - I_S}{\frac{1}{R_2} + \frac{1}{R_3}} \qquad V_b = -1.615\cdot V$$

Check with Ex 3 of Thevenin examples:

$$V_L := \frac{R_L}{R_{Th} + R_L}\cdot V_{Th} \qquad V_L = 4.615\cdot V$$