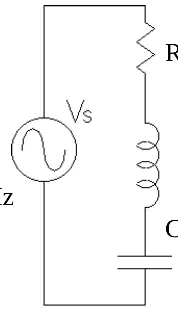
ECE 2210 / 00 Phasor Examples

Ex 1. Find $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ in polar phasor form. $f := 2\cdot kHz$



$\mathbf{V}(j\omega)$

$6\cdot V\cdot e^{j0}$

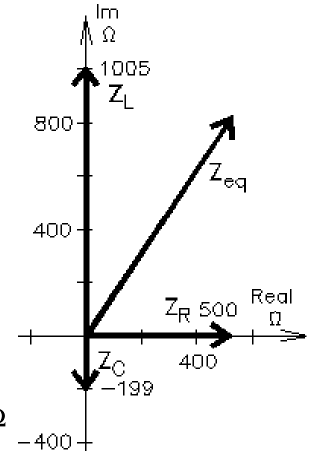
$f := 2\cdot kHz$

$R := 500\cdot\Omega$ $\qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 1.257\cdot 10^{4}\ \cdot\frac{rad}{sec}$

$L := 80\cdot mH \qquad \mathbf{Z_L} := j\cdot\omega\cdot L \qquad \mathbf{Z_L} = 1.005j\ \cdot k\Omega$

$C := 0.4\cdot\mu F \qquad \mathbf{Z_C} := \frac{1}{j\cdot\omega\cdot C} \qquad \mathbf{Z_C} = -0.199j\ \cdot k\Omega$

$\mathbf{Z_{eq}} := R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C} \qquad \mathbf{Z_{eq}} = 500 + 806.366j\ \cdot\Omega$



$\sqrt{500^2 + 806^2} = 948.491 \qquad \mathrm{atan}\left(\frac{806}{500}\right) = 58.187\ \cdot deg \qquad \mathbf{Z_{eq}} = 948.5\Omega\ \underline{/\ 58.}2^{o}$

find the current: $\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6\cdot V}{948.5\cdot\Omega} = 6.326\ \cdot mA$ angle: $0\cdot deg - 58.2\cdot deg = -58.2\ \cdot deg$

$\mathbf{I} = 6.326mA\ \underline{/\text{-}58.}2^{o}$

	<u>find the magnitude</u>	<u>find the angle</u>	
$\mathbf{V_R} := \mathbf{I}\cdot R$	$6.326\cdot mA\cdot 500\cdot\Omega = 3.163\ \cdot V$	$-58.2\cdot deg + 0\cdot deg = -58.2\ \cdot deg$	$\mathbf{V_R} = 3.163V\ \underline{/\text{-}58.}2^{o}$
$\mathbf{V_L} := \mathbf{I}\cdot\mathbf{Z_L}$	$6.326\cdot mA\cdot 1005\cdot\Omega = 6.358\ \cdot V$	$-58.2\cdot deg + 90\cdot deg = 31.8\ \cdot deg$	$\mathbf{V_L} = 6.358V\ \underline{/\ 31.8}^{o}$
$\mathbf{V_C} := \mathbf{I}\cdot\mathbf{Z_C}$	$6.326\cdot mA\cdot(-199)\cdot\Omega = -1.259\ \cdot V$	$-58.2\cdot deg + (90)\cdot deg = 31.8\ \cdot deg$	$\mathbf{V_C} = \text{-}1.259V\ \underline{/\ 31.8}^{o}$
OR:	$6.326\cdot mA\cdot(199)\cdot\Omega = 1.259\ \cdot V$	$-58.2\cdot deg + (-90)\cdot deg = -148.2\ \cdot deg$	$\mathbf{V_C} = 1.259V\ \underline{/\text{-}148.2}^{o}$

OR, you can also find these voltages directly, using a voltage divider. I.E. to find $\mathbf{V_C}$ directly:

$$\mathbf{V_C} := \frac{\frac{1}{j\cdot\omega\cdot C}}{R + j\cdot\omega\cdot L + \frac{1}{j\cdot\omega\cdot C}}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) + j\cdot\omega\cdot L\cdot(j\cdot\omega\cdot C) + 1}\cdot 6\cdot V = \frac{1}{R\cdot(j\cdot\omega\cdot C) - \omega^2\cdot L\cdot C + 1}\cdot 6\cdot V$$

$$= \frac{1}{\left(1 - \omega^2\cdot L\cdot C\right) + j\cdot\omega\cdot R\cdot C}\cdot 6\cdot V \qquad \left(1 - \omega^2\cdot L\cdot C\right) = -4.053 \qquad j\cdot\omega\cdot R\cdot C = 2.513j$$

$$= \frac{6\cdot V}{-4.053 + 2.513\cdot j}\cdot\frac{(-4.053 - 2.513\cdot j)}{(-4.053 - 2.513\cdot j)} = \frac{6\cdot V\cdot(-4.053 - 2.513\cdot j)}{(-4.053)^2 + 2.513^2}$$

$6\cdot V\cdot(-4.053 - 2.513\cdot j) = -24.318 - 15.078j\ \cdot V$

$(-4.053)^2 + 2.513^2 = 22.742$

$$= \left(\frac{-24.318}{22.742} - \frac{15.078\cdot j}{22.742}\right)\cdot V = -1.069 - 0.663j\ \cdot V$$

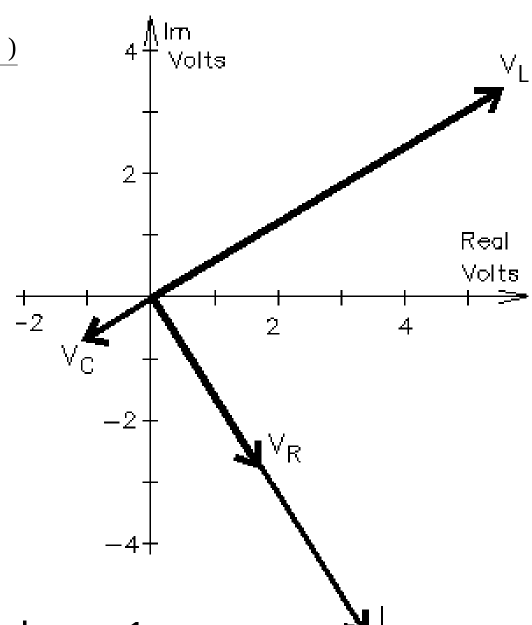
magnitude: $\sqrt{1.069^2 + 0.663^2} = 1.258$

angle: $\mathrm{atan}\left(\frac{-0.663}{-1.069}\right) = 31.81\ \cdot deg$

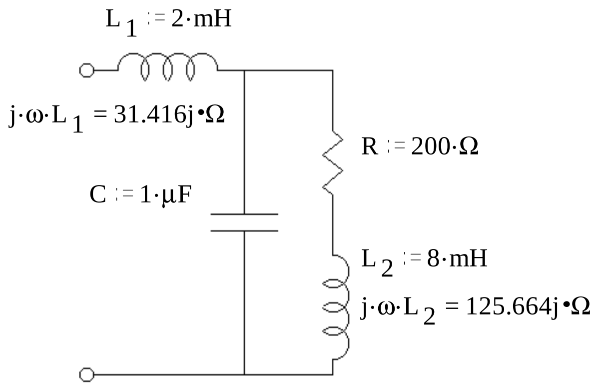
but this is actually in the third quadrant, so modify your calculator's results:

$31.81\cdot deg - 180\cdot deg = -148.19\ \cdot deg$

$= 1.258V\ \underline{/\text{-}148.2}^{o}$



Ex 2. a) Find $\mathbf{Z_{eq}}$. $f := 2.5 \cdot \text{kHz}$ $\omega := 2 \cdot \pi \cdot f$ $\omega = 1.571 \cdot 10^4 \cdot \frac{\text{rad}}{\text{sec}}$



$$\mathbf{Z_{eq}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} + \frac{1}{R + j \cdot \omega \cdot L_2}}$$

But it's easier to split the problem up

Left branch

$$\mathbf{Z_l} := \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_l} = -63.662j \cdot \Omega$$

$$\frac{1}{\left(\frac{1}{j \cdot \omega \cdot C}\right)} = j \cdot \omega \cdot C = 0.01571i \cdot \frac{1}{\Omega}$$

Right branch

$$\mathbf{Z_r} := j \cdot \omega \cdot L_2 + R \qquad \mathbf{Z_r} = 200 + 125.664j \cdot \Omega$$

$$\frac{1}{200 + 125.664 \cdot j} = 3.585 \cdot 10^{-3} - 2.252 \cdot 10^{-3} j$$

denominator: $$j \cdot \omega \cdot C + \frac{1}{R + j \cdot \omega \cdot L_2} = 0.01571 \cdot j + \left(3.585 \cdot 10^{-3} - 2.252 \cdot 10^{-3} \cdot j\right) = 3.585 \cdot 10^{-3} + 1.346 \cdot 10^{-2} i \qquad \frac{1}{\Omega}$$

rectangular division:

$$\frac{1}{\left(3.585 \cdot 10^{-3} + 1.346 \cdot 10^{-2} \cdot j\right)} \cdot \frac{\left(3.585 \cdot 10^{-3} - 1.346 \cdot 10^{-2} \cdot j\right)}{\left(3.585 \cdot 10^{-3} - 1.346 \cdot 10^{-2} \cdot j\right)} = \frac{3.585 \cdot 10^{-3} - 1.346 \cdot 10^{-2} \cdot j}{1.94 \cdot 10^{-4}} = 18.479 - 69.381j \qquad \Omega$$

$$\left(3.585 \cdot 10^{-3}\right)^2 + \left(1.346 \cdot 10^{-2}\right)^2 = 1.94 \cdot 10^{-4}$$

add: $j \cdot \omega \cdot L_1 = 31.416j \cdot \Omega$ $\qquad 31.416 \cdot j + (18.479 - 69.381 \cdot j) = 18.479 - 37.965j \quad \Omega$

convert to polar (if needed): $\sqrt{18.48^2 + 37.97^2} = 42.228$ $\qquad \text{atan}\left(\frac{-37.97}{18.48}\right) = -64.048 \cdot \text{deg}$ $\qquad \mathbf{Z_{eq}} = 42.23\Omega\ \underline{/-64.05^\circ}$

Another Way

Sometimes you might simplify a little before putting in numbers.

$$\mathbf{Z_{eq}} := j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + \frac{1}{\frac{1}{j \cdot \omega \cdot C}}} = j \cdot \omega \cdot L_1 + \frac{1}{\frac{1}{R + j \cdot \omega \cdot L_2} + j \cdot \omega \cdot C} = j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 + j \cdot \omega \cdot C \cdot \left(R + j \cdot \omega \cdot L_2\right)}$$

$$= j \cdot \omega \cdot L_1 + \frac{R + j \cdot \omega \cdot L_2}{1 - \omega^2 \cdot C \cdot L_2 + j \cdot \omega \cdot C \cdot R}$$

$$\mathbf{Z_{eq}} = 31.416 \cdot j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot \Omega}{-0.974 + 3.142 \cdot j} \cdot \left(\frac{-0.974 - 3.142 \cdot j}{-0.974 - 3.142 \cdot j}\right) = 31.416 \cdot j \cdot \Omega + \frac{(200 + 125.664 \cdot j) \cdot (-0.974 - 3.142 \cdot j)}{0.974^2 + 3.142^2}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{((200 \cdot (-0.974) - 125.664 \cdot (-3.142)) + (125.664 \cdot (-0.974) - 200 \cdot 3.142) \cdot j) \cdot \Omega}{0.974^2 + 3.142^2}$$

$$= 31.416 \cdot j \cdot \Omega + \frac{(200.036288 - 750.796736 \cdot j) \cdot \Omega}{10.82084} = 31.416 \cdot j \cdot \Omega + 18.486 \cdot \Omega - 69.384 \cdot j \cdot \Omega = 18.486 - 37.968j \cdot \Omega$$

$\sqrt{18.49^2 + 37.97^2} = 42.233$ $\qquad \text{atan}\left(\frac{-37.97}{18.49}\right) = -64.036 \cdot \text{deg}$ $\qquad \mathbf{Z_{eq}} = 42.23\Omega\ \underline{/-64.04^\circ}$

a little roundoff difference

b) $\mathbf{V_{in}} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$ Find $\mathbf{I_{L1}}$, $\mathbf{V_C}$

$$\mathbf{I_{L1}} := \frac{\mathbf{V_{in}}}{\mathbf{Z_{eq}}} \qquad \frac{12\cdot V}{42.23\cdot\Omega} = 284.16\cdot mA \qquad 20\cdot deg - (-64.04)\cdot deg = 84.04\cdot deg$$

$$\mathbf{I_{L1}} = 284mA \underline{/ 84.04^o}$$

$$\mathbf{V_C} := \mathbf{I_{L1}}\cdot(18.479 - 69.381\cdot j)\cdot\Omega \qquad 284\cdot mA\cdot\sqrt{18.479^2 + 69.381^2}\cdot\Omega = 20.391\cdot V$$

$$84.04\cdot deg + atan\left(\frac{-69.381}{18.479}\right) = 8.954\cdot deg \qquad \mathbf{V_C} = 20.4V \underline{/ 8.95^o}$$

You could then use another voltage divider to find $\mathbf{V_R}$ or $\mathbf{V_{L2}}$.

convert to rectangular (if needed): $20.391\cdot V\cdot\cos(8.954\cdot deg) = 20.143\cdot V$

$$20.391\cdot V\cdot\sin(8.954\cdot deg) = 3.174\cdot V \qquad \mathbf{V_C} = 20.14 + 3.174\cdot j \quad V$$

Another Way

To find V_C directly:

$$\mathbf{V_C} := \frac{\dfrac{1}{\dfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}{j\cdot\omega\cdot L_1 + \dfrac{1}{\dfrac{1}{R + j\cdot\omega\cdot L_2} + j\cdot\omega\cdot C}}\cdot\mathbf{V_{in}}$$

--> math --> $\mathbf{V_C} = 20.153 + 3.178j \cdot V$ Same but for a little roundoff difference

c) Let's find $\mathbf{I_{L2}}$. $\mathbf{Z_r} = 200 + 125.664j \cdot\Omega$ $\qquad \sqrt{200^2 + 125.664^2} = 236.202$ $\qquad atan\left(\frac{125.664}{200}\right) = 32.142\cdot deg$

$$\mathbf{I_{L2}} := \frac{\mathbf{V_C}}{\mathbf{Z_r}} = \frac{20.4\cdot V\cdot e^{j\cdot 8.95\cdot deg}}{236.202\cdot\Omega\cdot e^{j\cdot 32.142\cdot deg}} = \frac{20.4\cdot V}{236.202\cdot\Omega} \underline{/ 8.95 - 32.142^o} = 86.4mA \underline{/-23.19}$$

Another Way

Directly by Current divider:

$$\mathbf{I_{L2}} := \frac{\dfrac{1}{R + j\cdot\omega\cdot L_2}}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = \frac{1}{j\cdot\omega\cdot C\cdot\left(R + j\cdot\omega\cdot L_2\right) + 1}\cdot\mathbf{I_{L1}} = \frac{\mathbf{I_{L1}}}{1 - \omega^2\cdot C\cdot L_2 + j\cdot\omega\cdot C\cdot R}$$

real part is negative

denominator: $\sqrt{\left(1 - \omega^2\cdot C\cdot L_2\right)^2 + (\omega\cdot C\cdot R)^2} = 3.289$ $\qquad atan\left(\frac{\omega\cdot C\cdot R}{1 - \omega^2\cdot C\cdot L_2}\right) + 180\cdot deg = 107.224\cdot deg$

$$\mathbf{I_{L2}} = \frac{284\cdot mA\cdot e^{j\cdot 84.04\cdot deg}}{3.289\cdot e^{j\cdot 107.224\cdot deg}} = \frac{284\cdot mA}{3.289} \underline{/ 84.04 - 107.224^o} = 86.4mA \underline{/-23.18^o}$$

d) How about $\mathbf{I_C}$?

$$\mathbf{I_C} := \frac{\mathbf{V_C}}{\left(\dfrac{1}{j\cdot\omega\cdot C}\right)} = \mathbf{V_C}\cdot j\cdot\omega\cdot C = 20.4V \underline{/ 8.95^o}\ 0.015708 \underline{/ 90^o}\ \frac{1}{\Omega} = 320mA \underline{/ 98.95^o}$$

Another Way Could also be found directly by current divider:

$$\mathbf{I_C} := \frac{j\cdot\omega\cdot C}{j\cdot\omega\cdot C + \dfrac{1}{R + j\cdot\omega\cdot L_2}}\cdot\mathbf{I_{L1}} = 320mA \underline{/ 98.95^o}$$

<u>Something Weird</u>

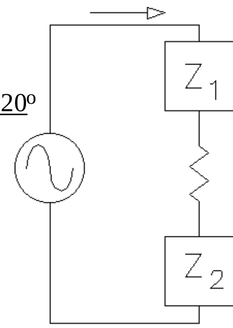
$\mathbf{I_C}$ is greater than the input current ($\mathbf{I_{L1}}$) . What's going on?

The angle between $\mathbf{I_C}$ & $\mathbf{I_{L2}}$ is big enough that they somewhat cancel each other out (partially resonate).

Check Kirchoff's Current Law: $\mathbf{I_C} + \mathbf{I_{L2}} = 29.485 + 282.569j \cdot mA \overset{?}{=} \mathbf{I_{L1}} = 29.485 + 282.569j \cdot mA$ yes

Ex 3. a) Find $\mathbf{Z_2}$.

$i(t) = 25\cdot mA\cdot\cos\left(377\cdot\frac{rad}{sec}\cdot t + 10\cdot deg\right)$



$\mathbf{Z_1} := (120 - 60\cdot j)\cdot\Omega$

$\mathbf{V_{in}}$ = 10V /-20°

$R := 50\cdot\Omega$

$\mathbf{Z_2}$ = ?

$\mathbf{I} := 25\cdot mA\cdot e^{j\cdot 10\cdot deg}$

$\mathbf{V_{in}} := 10\cdot V\cdot e^{-j\cdot 20\cdot deg}$

$\mathbf{Z_T} := \frac{\mathbf{V_{in}}}{\mathbf{I}} = \frac{10\cdot V}{25\cdot mA}$ $\underline{/-20 - 10^o}$ = 400Ω $\underline{/-30^o}$

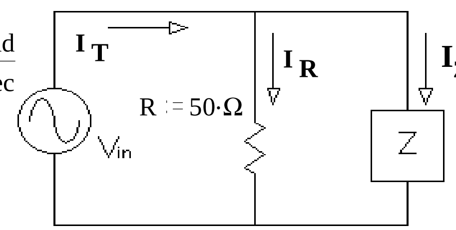
$\mathbf{Z_T} = 346.41 - 200j \;\cdot\Omega$

$\mathbf{Z_2} := \mathbf{Z_T} - R - \mathbf{Z_1} = (346.41 - 200\cdot j)\cdot\Omega - 50\cdot\Omega - (120 - 60\cdot j)\cdot\Omega = 176.41 - 140j \;\cdot\Omega$

b) Circle 1: i) The source current leads the source voltage <--- answer, because 10° > -20°.

ii) The source voltage leads the source current

Ex 4. a) Find $\mathbf{V_{in}}$ in polar form.



$\mathbf{I_Z}$ = 100 $\underline{/\,0^o}$ mA

$\mathbf{Z} := (80 - 60\cdot j)\cdot\Omega$

$\mathbf{I_Z} := 100\cdot mA$ $\mathbf{Z} := (80 - 60\cdot j)\cdot\Omega$ $\omega := 1000\cdot\frac{rad}{sec}$

$\mathbf{V_{in}} := \mathbf{I_Z}\cdot\mathbf{Z}$ $\mathbf{V_{in}} = 8 - 6j \;\cdot V$

$\sqrt{8^2 + 6^2} = 10$ $\quad \operatorname{atan}\left(\frac{-6}{8}\right) = -36.87\cdot deg$

$\mathbf{V_{in}}$ = 10V $\underline{/-36.9^o}$

b) Find $\mathbf{I_T}$ in polar form. $\mathbf{I_R} := \frac{\mathbf{V_{in}}}{R} = \frac{10\cdot V}{50\cdot\Omega}$ $\underline{/-36.9^o}$ $= \frac{10\cdot V}{50\cdot\Omega}\cdot\cos(-36.9\cdot deg) + j\cdot\frac{10\cdot V}{50\cdot\Omega}\cdot\sin(-36.9\cdot deg) = 160 - 120i \;\cdot mA$

$\mathbf{I_T} := \mathbf{I_R} + \mathbf{I_Z} = (160 - 120\cdot j)\cdot mA + 100\cdot mA = 260 - 120j \;\cdot mA$

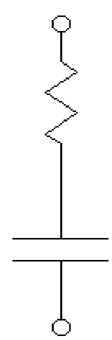
$\sqrt{260^2 + 120^2} = 286.356$ $\quad \operatorname{atan}\left(\frac{-120}{260}\right) = -24.78\cdot deg$ $\quad \mathbf{I_T}$ = 286mA $\underline{/-24.8^o}$

c) Circle 1: i) The source current leads the source voltage answer i), -24.8° > -36.9°

ii) The source voltage leads the source current

d) The impedance **Z** (above) is made of two components in series. What are they and what are their values?

$\mathbf{Z} = 80 - 60j \;\cdot\Omega$



Must have a resistor because there is a real part.

$R := \mathrm{Re}(\mathbf{Z})$ $\quad R = 80\cdot\Omega$

Must have a capacitor because the imaginary part is negative.

$\mathrm{Im}(\mathbf{Z}) = -60\cdot\Omega = \frac{-1}{\omega\cdot C}$ $\quad C := \frac{-1}{\omega\cdot\mathrm{Im}(\mathbf{Z})}$ $\quad C = 16.667\cdot\mu F$

Ex 5. The impedance $\mathbf{Z} = 80 - 60j \cdot \Omega$ is made of two components in parallel. What are they and what are their values?

Must have a resistor because there is a real part.

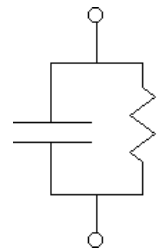
Must have an capacitor because the imaginary part is negative.

$$\mathbf{Z} = \frac{1}{\frac{1}{R} + j\cdot\omega\cdot C} \qquad \frac{1}{\mathbf{Z}} = \frac{1}{(80 - 60\cdot j)\cdot\Omega}\cdot\left(\frac{80+60\cdot j}{80+60\cdot j}\right) = \frac{80+60\cdot j}{80^2+60^2} = \frac{80+60\cdot j}{10{,}000}\cdot\frac{1}{\Omega}$$

$$\frac{1}{\mathbf{Z}} = 0.008 + 0.006i \cdot\Omega^{-1} = \frac{1}{R} + j\cdot\omega\cdot C$$

$$\frac{1}{R} = .008\cdot\frac{1}{\Omega} \qquad R := \frac{1}{.008\cdot\Omega^{-1}} \qquad R = 125\cdot\Omega$$

$$\omega\cdot C = .006\cdot\frac{1}{\Omega} \qquad C := \frac{.006\cdot\Omega^{-1}}{\omega} \qquad C = 6\cdot\mu F$$



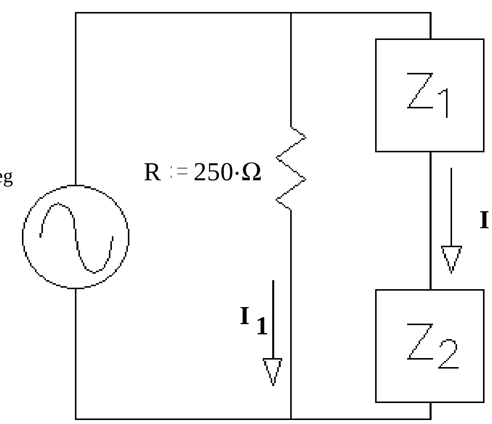
$R = 125\cdot\Omega$

Positive imaginary parts would require inductors

Ex 6. a) Find $\mathbf{I_1}$

$$\omega := 20000\cdot\frac{rad}{sec}$$

$$\mathbf{V_{in}} := 20\cdot V\cdot e^{j\cdot 30\cdot deg}$$



voltage $\mathbf{V_{Z1}} := (8 - 5\cdot j)\cdot V$

$\mathbf{I_2} := 20\cdot mA\cdot e^{j\cdot 20\cdot deg}$

$$\mathbf{I_1} := \frac{\mathbf{V_{in}}}{R} = \frac{20\cdot V}{250\cdot\Omega}\cdot e^{j\cdot 30\cdot deg} = 80\cdot mA\cdot e^{j\cdot 30\cdot deg}$$

polar division

b) Circle 1: i) $\mathbf{V_{in}}$ leads $\mathbf{I_2}$ ii) $\mathbf{V_{in}}$ lags $\mathbf{I_2}$

Why? Show numbers: __30__ > __20__ ______ < ______

c) Find $\mathbf{Z_2}$ in polar form

Convert $\mathbf{V_{in}}$ to rectangular coordinates

$20\cdot V\cdot\cos(30\cdot deg) = 17.321\cdot V$ $\quad 20\cdot V\cdot\sin(30\cdot deg) = 10\cdot V$ pol to rect

$\mathbf{V_{in}} = 17.321 + 10j \cdot V$

$\mathbf{V_{Z2}} := \mathbf{V_{in}} - \mathbf{V_{Z1}}$ $\quad \mathbf{V_{Z2}} = 9.321 + 15j \cdot V$ subtract

rect to pol $\sqrt{9.321^2 + 15^2} = \left|\mathbf{V_{Z2}}\right| = 17.66\cdot V$

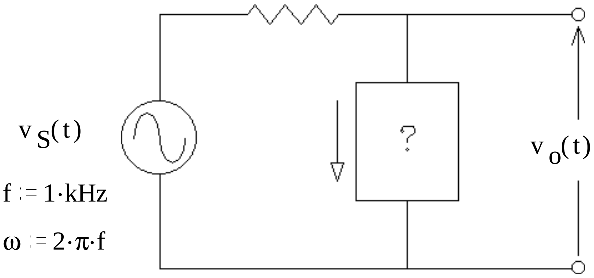
$$\operatorname{atan}\left(\frac{15}{9.321}\right) = \arg\left(\mathbf{V_{Z2}}\right) = 58.145\cdot deg$$

div $\mathbf{Z_2} := \frac{\mathbf{V_{Z2}}}{\mathbf{I_2}}$ $\quad \frac{17.66\cdot V}{20\cdot mA} = 883\cdot\Omega$ $\quad \angle\ 58.145\cdot deg - 20\cdot deg = 38.145\cdot deg$ $\quad \mathbf{Z_2} = 883\ \angle 38.15^\circ\ \Omega$

$\mathbf{Z_2} = 694.436 + 545.379j \cdot\Omega$

Ex 7. You need to design a circuit in which the "output" voltage leads the input voltage ($v_S(t)$) by 40° of phase.

$R := 400\cdot\Omega$



$f := 1\cdot kHz$

$\omega := 2\cdot\pi\cdot f$

$\omega = 6.283\cdot 10^{3}\cdot\frac{rad}{sec}$

a) What should go in the box: R, L, C?

$$\mathbf{V_o} = \frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}\cdot\mathbf{V_S}$$

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is 40°.

This can only happen if the angle of $\mathbf{Z_{box}}$ is positive, so $\mathbf{Z_{box}}$ is a inductor

b) Find its value. $\mathbf{V_o} =$ $\mathbf{V_o} = \frac{j\cdot\omega\cdot L}{R + j\cdot\omega\cdot L}\cdot\mathbf{V_S}$ angle $\frac{j\cdot\omega\cdot L}{R + j\cdot\omega\cdot L}$ is $90 - \operatorname{atan}\left(\frac{\omega\cdot L}{R}\right) = 40°$.

So: $\operatorname{atan}\left(\frac{\omega\cdot L}{R}\right) = 50°$ $\qquad \frac{\omega\cdot L}{R} = \tan(50\cdot deg) = 1.192$ $\qquad L = \frac{R\cdot 1.192}{\omega} = 75.9\cdot mH$

c) Repeat if the "output" voltage should lag the input voltage ($v_S(t)$) by 20° of phase.

angle of $\frac{\mathbf{Z_{box}}}{R + \mathbf{Z_{box}}}$ is -20°. This can only happen if the angle of $\mathbf{Z_{box}}$ is negative, so $\mathbf{Z_{box}}$ is a capacitor

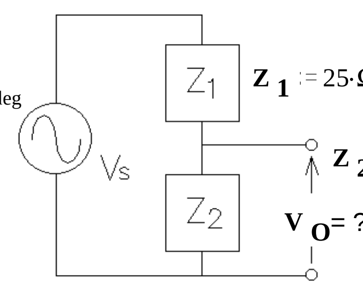
$$\mathbf{V_o} = \frac{\frac{1}{j\cdot\omega\cdot C}}{R + \frac{1}{j\cdot\omega\cdot C}}\cdot\mathbf{V_S}$$

angle $\frac{\frac{1}{j\cdot\omega\cdot C}}{R + \frac{1}{j\cdot\omega\cdot C}}$ is $-90 - \operatorname{atan}\left(\frac{-\frac{1}{\omega\cdot C}}{R}\right) = -90 - \operatorname{atan}\left(-\frac{1}{\omega\cdot C\cdot R}\right)$

$\operatorname{atan}\left(-\frac{1}{\omega\cdot C\cdot R}\right) = -70°.$ $\qquad -\frac{1}{\omega\cdot C\cdot R} = \tan(-70\cdot deg) = -2.747$ $\qquad C = \frac{1}{\omega\cdot R\cdot 2.747} = 0.145\cdot\mu F$

Ex 8. Find $\mathbf{V_O}$ in the circuit shown. Express it as a magnitude and phase angle (polar).

$\mathbf{V_S} := 6\cdot V\cdot e^{j\cdot 18\cdot deg}$



$\mathbf{Z_1} := 25\cdot\Omega + 35j\cdot\Omega$

$\mathbf{Z_2} := 80\cdot\Omega\cdot e^{-j\cdot 60\cdot deg}$

$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}}\cdot\mathbf{V_S}$ Simple voltage divider

$\left|\mathbf{Z_2}\right|\cdot\cos(-60\cdot deg) = 40\cdot\Omega$ $\qquad \left|\mathbf{Z_2}\right|\cdot\sin(-60\cdot deg) = -69.282\cdot\Omega$ $\qquad \mathbf{Z_2} = 40 - 69.282j\cdot\Omega$

$$\mathbf{Z_1} + \mathbf{Z_2} = 25\cdot\Omega + 35j\cdot\Omega + 40\cdot\Omega - 69.282\cdot j\cdot\Omega = 65 - 34.282j\cdot\Omega = 73.486\cdot\Omega\cdot e^{-j\cdot 27.81\cdot deg}$$

$$\mathbf{V_O} := \frac{\mathbf{Z_2}}{\mathbf{Z_1} + \mathbf{Z_2}}\cdot\mathbf{V_S} = \frac{80\cdot\Omega\cdot e^{-j\cdot 60\cdot deg}}{73.486\cdot\Omega\cdot e^{-j\cdot 27.81\cdot deg}}\cdot\left(6\cdot V\cdot e^{j\cdot 18\cdot deg}\right) = \frac{80\cdot\Omega}{73.486\cdot\Omega}\cdot 6\cdot V\cdot e^{j\cdot(-60-(-27.81)+18)\cdot deg} = 6.53\cdot V\cdot e^{-j\cdot 14.2\cdot deg}$$