Ex:

![RLC Circuit Diagram]

a) Find the characteristic roots, $s_1$ and $s_2$, for the above circuit.

b) If C is reduced to 100 nF, which will the circuit be: overdamped or underdamped?

c) Using the L and C values in the circuit diagram, what value of R yields an underdamped circuit with damping frequency $\omega_d = 150$ k rad/s?

**Sol’n:** a) For a parallel RLC circuit (or a series RLC circuit), we have the following formula for the characteristic roots:

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$$

For a parallel RLC circuit, the value of $\alpha$ is one-half the inverse RC time constant:

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 2 \Omega \cdot 1 \mu F} = \frac{1}{4} \text{ M/s} = 250 \text{ k/s}$$

For both parallel and series RLC circuits, the resonant frequency, $\omega_o$, is the inverse of the square root of the product of L and C:

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \frac{\omega_o^2}{L} = \frac{1}{LC} = \frac{1}{16 \mu H \cdot 1 \mu F} = \left( \frac{1}{4 \mu} \frac{r}{s} \right)^2 = (250 \text{ kr/s})^2$$

We find that, since $\alpha = \omega_o$, the two roots are the same:

$$s_{1,2} = -250 \text{ kr/s} \pm \sqrt{(250 \text{ kr/s})^2 - (250 \text{ kr/s})^2} = -250 \text{ kr/s}$$

When the roots are the same, the circuit is critically damped.
**SOL’N:** b) For \( C = 100 \text{ nF} \), we repeat the above calculations:
\[
\alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 2 \Omega \cdot 100 \text{nF}} = \frac{10 \text{ M/s}}{4} = 2.5 \text{ M/s}
\]
\[
\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad \omega_o^2 = \frac{1}{LC} = \frac{1}{16 \mu \text{H} \cdot 100 \text{ nF}} = 0.625 \text{ M}^2 (\text{r/s})^2
\]
\[
s_{1,2} = -2.5 \text{ M/s} \pm \sqrt{(2.5 \text{ M/s})^2 - 0.625 \text{ M}^2 (\text{r/s})^2}
\]
or
\[
s_{1,2} = -2.5 \text{ M/s} \pm \sqrt{6.25 \text{ M}^2 (\text{r/s})^2 - 0.625 \text{ M}^2 (\text{r/s})^2}
\]
or
\[
s_{1,2} = -2.5 \text{ M/s} \pm 3\sqrt{0.625 \text{ M}^2 (\text{r/s})^2} = -2.5 \text{ M/s} \pm 3\sqrt{0.625} \text{ M/s}
\]
or
\[
s_1 = -4.87 \text{ M/s} \quad \text{and} \quad s_2 = -128 \text{ kr/s}
\]

**SOL’N:** c) To find the value of \( R \) that yields \( \omega_d = 150 \text{ kr/s} \), we have
\[
\omega_d = \sqrt{\omega_o^2 - \alpha^2} = \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2} = 150 \text{ kr/s}
\]
or
\[
\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2 = (150 \text{ kr/s})^2
\]
or
\[
\left(\frac{1}{2RC}\right)^2 = -(150 \text{ kr/s})^2 + \frac{1}{LC} = -(150 \text{ kr/s})^2 + (250 \text{ kr/s})^2
\]
or, since \( 150^2 + 200^2 = 250^2 \), we have
\[
\left(\frac{1}{2RC}\right)^2 = (200 \text{ kr/s})^2
\]
or

\[ 2RC = \frac{1}{200 \text{ kr/s}} \]

\[ R = \frac{1}{2C \cdot 200 \text{ kr/s}} = \frac{1}{2 \cdot 1 \mu \text{F} \cdot 200 \text{ kr/s}} = 2.5 \ \Omega \]