Ex:

After being closed for a long time, the switch opens at $t = 0$.

Find $v(t)$ for $t > 0$.

**SOL'N:** We may perform the following initial steps in any order:

1) Find characteristic roots for the parallel RLC circuit (for $t > 0$)

2) Find the final value of $v(t)$ as $t \to \infty$, which is the $A_3$ (constant) term in the solution.

3) Find the initial values of energy variables: $i_L(0^+) = i_L(0^-)$ and $v_C(0^+) = v_C(0^-)$

We will perform them in the order listed. First, we find the characteristic roots.

\[ s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \]

For a parallel RLC circuit, the value of $\alpha$ is one-half the inverse RC time constant:

\[ \alpha = \frac{1}{2RC} = \frac{1}{2 \cdot 100\, \text{m}\Omega \cdot 5\, \mu\text{F}} = 1 \, \text{M/s} \]

For both parallel and series RLC circuits, the resonant frequency, $\omega_0$, is the inverse of the square root of the product of $L$ and $C$:

\[ \omega_0^2 = \frac{1}{LC} = \frac{1}{1 \, \mu\text{H} \cdot 5 \, \mu\text{F}} = 0.2 \, \text{M}^2(\text{r/s})^2 \]

Substituting into the equation for the roots yields the following:
The roots are real, so the circuit is overdamped, and we use the exponential form of general solution:

\[ v(t) = A_1 e^{s_{1}t} + A_2 e^{s_{2}t} + A_3 \]

Second, we find the \( A_3 \) value from \( v(t) \) as \( t \to \infty \). (The exponential terms decay as \( t \to \infty \), leaving only \( A_3 \).) We assume the circuit values become constant as \( t \to \infty \), causing the L to act like a wire and the C to act like an open circuit. The switch is also open, detaching the voltage source and resistor on the left.

Since the L acts like a wire, we have \( A_3 = v(t \to \infty) = 0 \). Third, we find the initial values of energy variables: \( i_L(0^+) = i_L(0^-) \) and \( v_C(0^+) = v_C(0^-) \). At \( t = 0^- \), we assume circuit values are constant, causing the L to act like a wire and the C to act like an open circuit. The switch is closed, connecting the parallel RLC to the voltage source and resistor on the left.

The L, acting like a wire, shorts the 100 mΩ resistor and the open C. Thus, the initial C voltage is zero:

\[ v_C(0^+) = v_C(0^-) = 0 \]

Also, all the current will flow through the L and will be limited only by the 1 kΩ resistor.

\[ i_L(0^+) = i_L(0^-) = \frac{2.3 \text{ V}}{1 \text{ kΩ}} = 2.3 \text{ mA} \]

At time \( t = 0^+ \), we treat the C as a voltage source of 0V and the L as a current source of 2.3 mA. (The switch is also open, removing the 2.3 V source and 1 kΩ resistor from the circuit.) We can solve the circuit for any voltage or current at \( t = 0^+ \). Using Kirchhoff’s laws for voltage loops and current sums at nodes, we have the following results, (with voltages measured with plus on top, and currents measured flowing in the down direction):
\[ v_R(0^+) = v_L(0^+) = v_C(0^+) = 0 \text{ V} \]
\[ i_R(0^+) = \frac{v_C(0^+)}{R} = \frac{0 \text{ V}}{100 \text{ m}\Omega} = 0 \text{ A} \]
\[ i_C(0^+) = -i_R(0^+) - i_L(0^+) = 0 \text{ A} - 2.3 \text{ mA} = -2.3 \text{ mA} \]

Now we are ready to find \( A_1 \) and \( A_2 \) by matching our symbolic solution to circuit values for \( v(0^+) \) and \( \left. \frac{dv(t)}{dt} \right|_{t=0^+} \). We have already found \( v(0^+) \) to be 0 V. Matching this to the symbolic solution for \( t = 0^+ \), we have the following:

\[ A_1 + A_2 = 0 \text{ V} \text{ or } A_1 = -A_2 \]

For the value of the derivative in the circuit, we always try to first write our variable, \( v \) in this case), in terms of energy (or state) variables, \( i_L \) and/or \( v_C \). Here, this is a simple matter:

\[ v(t) = v_C(t) \]

Then we differentiate, and use the component equations involving \( d/dt \) for \( L \) and/or \( C \):

\[ \left. \frac{d}{dt} v(t) \right|_{t=0^+} = \frac{d}{dt} v_C(t) \left|_{t=0^+} \right. = \frac{i_C(0^+)}{C} = \frac{-2.3 \text{ mA}}{5 \mu\text{F}} \]

We equate this to the symbolic derivative:

\[ \left. \frac{d}{dt} v(t) \right|_{t=0^+} = A_1 s_1 + A_2 s_2 = A_1 (s_1 - s_2) = A_1 2\sqrt{\alpha^2 - \omega_0^2} = \frac{-2.3 \text{ mA}}{5 \mu\text{F}} \]

or

\[ A_1 = \frac{-2.3 \text{ mA}}{2\sqrt{\alpha^2 - \omega_0^2} \cdot 5 \mu\text{F}} = \frac{-2.3 \text{ mA}}{2 \frac{2}{\sqrt{5}} \text{ Mr/s} \cdot 5 \mu\text{F}} = \frac{-2.3 \text{ mV}}{4\sqrt{5}} \approx -257 \mu\text{V} \]

Plugging in values gives the solution for \( v(t > 0) \):

\[ v(t) = -257\mu\text{V} \cdot e^{-1+\frac{2}{\sqrt{5}} \text{ Mr/s} \cdot t} + 257\mu\text{V} \cdot e^{-1-\frac{2}{\sqrt{5}} \text{ Mr/s} \cdot t} \]