Ex:

\[ C = 200 \text{ nF} \quad R_1 = 1.5 \text{ k\Omega} \quad R_3 = 2 \text{ k\Omega} \]

a) Determine the transfer function \( V_o/V_i \). **Hint:** Consider how \( R_3 \) affects the answer. Also, find \( V'/V_i \) and use a voltage-divider.

b) Plot \(|V_o/V_i|\) versus \( \omega \).

c) Find the cutoff frequency, \( \omega_c \).

**Sol’n:** a) We convert to the frequency domain and use a voltage divider in terms of phasors and impedances.

\( R_3 \) has no current flowing through it and has no effect on the transfer function. We also first consider the response from \( v_i \) to \( v' \):

\[
H'(j\omega) = \frac{V'}{V_i} = \frac{2R_1}{2R_1 + \frac{1}{j\omega C}} = \frac{1}{1 + \frac{1}{j\omega \cdot 2R_1 C}} = \frac{1}{1 - j \frac{1}{\omega \cdot 2R_1 C}}
\]

We use a voltage-divider to find \( v_o \) in terms of \( v' \):

\[
\frac{V_o}{V'} = \frac{R_1}{2R_1} = \frac{1}{2}
\]

The response of the entire circuit is the product of the responses:

\[
H(j\omega) = \frac{V_o}{V_i} = \frac{1}{2} \cdot \frac{1}{1 - j \frac{1}{\omega \cdot 2R_1 C}} = \frac{1}{2} \cdot \frac{1}{1 - j \frac{1}{\omega \cdot 2 \cdot 1.5 \text{ k\Omega} \cdot 200 \text{ nF}}}
\]

or

\[
H(j\omega) = \frac{1}{2} \cdot \frac{1}{1 - j \frac{1.67 \text{ kr/s}}{\omega}}
\]
b) We use the following Matlab code:

```matlab
% ECE2260F07_HW3p2Matlab.m

% Plot of filter's frequency response curve

figure(1)
omega = 1:5e1:5e3;
s = j * omega;

FilterResp = 0.5 ./ (1 + 10e3/6 ./ s);

plot(omega,abs(FilterResp))
axis([0, max(omega), 0, 1])
xlabel('omega')
ylabel('|H|')
```

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![Plot of filter's frequency response curve](image)

The cutoff frequency is, by definition, the frequency where the magnitude response is equal to the maximum of the magnitude response divided by the square root of two.

Since the maximum of $|H(j\omega)| = 0.5$, we have the following equation to be solved for $\omega$:
\[
\frac{|V_\omega|}{|V_i|} = \left| \frac{1}{\frac{1}{2} \left( 1 - j \frac{1.67 \text{ kr/s}}{\omega} \right)} \right| = \frac{1}{2\sqrt{2}}
\]

The factor of one-half on both sides cancels out, and the solution of this equation occurs when the imaginary part of the denominator is equal to one, since \(|1 + j| = \sqrt{2}\).

\[
\frac{1.67 \text{ kr/s}}{\omega} = 1
\]

or

\[
\omega_C = 1.67 \text{ kr/s}
\]