2. (40 points)

One period, $T$, of a function $v(t)$ is shown above. The formula for $v(t)$ is

$$v(t) = \begin{cases} 
-16V & 0 < t < T/4 \\
8V & T/4 < t < 3T/4 \\
0V & 3T/4 < t < T 
\end{cases}$$

Find the numerical value of the following coefficients of the Fourier series for $v(t)$:

10 pts a) $a_v$

10 pts b) $a_1$

10 pts c) $a_2$

10 pts d) $b_1$
soln a) We split $v(t)$ into even and odd parts.

$$v_e(t) = \frac{v(t) + v(-t)}{2}, \quad v_o(t) = \frac{v(t) - v(-t)}{2}$$

Add $v(t)$ and $v(-t)$ graphically.

Result is $v_e(t) \cdot 2$.

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Add $v(t)$ and $-v(-t)$ graphically.

Result is $v_o(t) \cdot 2$. 
Now we find $a_\nu$. We use only $v_e(t)$ since the DC offset is an even function.

By inspection, we observe that $v_e(t)$ has equal area under it above and below the time axis. Thus, $a_\nu = \text{ave of } v_e(t) = 0$


ea_\nu = 0

b) $a_1$ is the coeff of even function $\cos(\omega_0 t)$.

Thus, we use only $v_e(t)$.

Draw a picture showing $\int v_e(t) \cdot \cos(\omega_0 t) = \text{area}$ since

$$a_1 = \frac{2}{T} \int_0^T v(t) \cos(\omega_0 t) dt = \int_0^T v_e(t) \cos(\omega_0 t) dt$$

The total area = 4 $\cdot$ area between 0 and $T/4$.

$\therefore a_1 = 4 \cdot \frac{2}{T} \int_0^{T/4} v_e(t) \cos(\omega_0 t) dt$, $\omega_0 = \frac{2\pi}{T}$

$$= \frac{8}{T} \int_0^{T/4} -16V \cdot \cos \left( \frac{2\pi}{T} t \right) dt$$ (always)
$$a_1 = \frac{8}{T} (-16) \frac{1}{2} \sin \left( \frac{2\pi}{T} - \frac{t}{T} \right)$$

$$\left. \frac{z}{T} \right|_0^{T/4}$$

$$a_1 = -8 \frac{(16) V}{2 \cdot 2\pi} \left[ \sin \left( \frac{2\pi}{16} - \frac{0}{8} \right) - 0 \right]$$

$$a_1 = -\frac{32}{\pi} V$$

d) $a_2$ is coefficient of even function $\cos(2\omega_0 t)$. Thus, we use only $v_e(t)$.

But $v_e(t)$ has shift-flip symmetry. (If we shift $v_e(t)$ right by $\frac{T}{2}$ and then flip it upside down, we get $v(t)$ back again.)

Shift-flip symmetry implies $a_k = 0$ for $k$ even.

$$\therefore a_2 = 0 V$$

d) $b_1$ is coefficient of odd function $\sin(\omega_0 t)$.

Thus, we use only $v_o(t)$.

We draw a picture of $v_o(t) \cdot \sin(\omega_0 t)$:
We see that we need only compute the first area and double it.

\[ b_1 = \frac{2}{T} \int_0^T u(t) \sin(\omega_0 t) \, dt \]

\[ b_1 = 2 \cdot \frac{2}{T} \int_0^{T/4} u_0(t) \sin(\omega_0 t) \, dt \]

\[ b_1 = \frac{4}{T} \int_0^{T/4} \left( -\frac{16V}{2} \right) \sin \left( \frac{2\pi}{T} t \right) \, dt \]

\[ b_1 = \frac{4 \left( -\frac{16V}{2} \right)}{2 \cdot \frac{2\pi}{T}} \left[ -\cos \left( \frac{2\pi}{T} t \right) \right]_0^{T/4} \]

\[ b_1 = \frac{32V}{2} \left[ \cos \left( \frac{2\pi}{T} \times \frac{T}{4} \right) - \cos(0) \right] \]

\[ b_1 = \frac{32V}{2} \left[ 0 - 1 \right] \]

\[ b_1 = -16V \quad \text{Check: } b_1 < 0 \text{ from diagram} \]