Ex: Find the Laplace transforms of the following waveform:

\[ \frac{d}{dt}[t \cos(\omega t)] \]

Sol’n: First, we apply the derivative identity:

\[ \mathcal{L}\left[\frac{d}{dt}v(t)\right] = sV(s) - v(t=0^-) \]

This translates into the following equation:

\[ \mathcal{L}\left[\frac{d}{dt}[t \cos(\omega t)]\right] = s\mathcal{L}[t \cos(\omega t)] - \left[t \cos(\omega t)\right]_{t=0^-} = s\mathcal{L}[t \cos(\omega t)] \]

Second, we apply the "multiply by \( t \)" identity to the remaining Laplace transform:

\[ \mathcal{L}[tv(t)] = -\frac{d}{ds}V(s) \]

This translates into the following equation:

\[ \mathcal{L}[t \cos(\omega t)] = -\frac{d}{ds}\mathcal{L}[\cos(\omega t)] \]

Third, we lookup the remaining Laplace transform:

\[ \mathcal{L}[\cos(\omega t)] = \frac{s}{s^2 + \omega^2} \]

Fourth, we substitute later results into earlier equations.

\[ \mathcal{L}\left[\frac{d}{dt}[t \cos(\omega t)]\right] = s\left(-\frac{d}{ds}\frac{s}{s^2 + \omega^2}\right) = -s\frac{1}{s^2 + \omega^2} - s\frac{2s}{(s^2 + \omega^2)^2} \]

or

\[ \mathcal{L}\left[\frac{d}{dt}[t \cos(\omega t)]\right] = -s\frac{s^2 + \omega^2 + 2s^3}{(s^2 + \omega^2)^2} = \frac{s(s^2 - \omega^2)}{(s^2 + \omega^2)^2} \]