Ex: Find the inverse Laplace transform for the following expression:

\[ F(s) = \frac{2s - 20}{(s^2 + 4)} \]

SOL'N: A simple approach for finding the inverse transform is to recognize that this form corresponds to the sum of transformed cosine and sine functions with a denominator of \( s^2 + 4 = s^2 + \omega^2 \). That is, \( \omega = 2 \).

\[ F(s) = \frac{2s - 20}{s^2 + 4} = A \frac{s}{s^2 + \omega^2} + B \frac{\omega}{s^2 + \omega^2} \]

If we equate the numerators on the left and right, we have the following result:

\[ 2s - 20 = As + B\omega = As + B \cdot 2 \]

The coefficients of each power of \( s \) must match (including \( s^0 \), the constant term). Starting with the highest power of \( s \), we match the coefficients on the left and right to determine the value of \( A \) and \( B \).

\[ A = 2 \text{ and } B = -10 \]

The inverse transform follows directly from the above results:

\[ \mathcal{L}^{-1} \left[ \frac{2s - 20}{s^2 + 4} \right] = \mathcal{L}^{-1} \left[ A \frac{s}{s^2 + \omega^2} + B \frac{\omega}{s^2 + \omega^2} \right] = A \cos 2t + B \sin 2t \]

or

\[ \mathcal{L}^{-1} \left[ \frac{2s - 20}{s^2 + 4} \right] = 2 \cos 2t - 10 \sin 2t \]