Ex: Find the inverse Laplace transform for the following expression:

\[ F(s) = \frac{7}{s^2 + 6s + 58} \]

Sol'n: Because the constant term of the quadratic in the denominator is larger than the square of half the middle coefficient, the quadratic has complex roots.

\[ s^2 + 6s + 58 = (s + 3 + j7)(s + 3 - j7) = (s + a + j\omega)(s + a - j\omega) \]

or

\[ s^2 + 6s + 58 = (s + 3)^2 + 7^2 = (s + a)^2 + \omega^2 \]

We observe that \( F(s) \) has the form of a transformed decaying sine:

\[ F(s) = \frac{7}{s^2 + 6s + 58} = \frac{\omega}{(s + a)^2 + \omega^2} = \mathcal{L}\left[ e^{-at} \sin(\omega t) \right] \]

Thus, we can write down our answer directly:

\[ f(t) = e^{-3t} \sin(7t)u(t) \]

Note: We add \( u(t) \) to indicate that the value in the time domain is unknown.