Ex:

\[ j \omega M = j10 \, \Omega \]

\[ z_L \rightarrow \]

\[ j25 \, \Omega \]

\[ j6 \, \Omega \]

\[ \frac{1}{j \omega C} \]

\[ \text{linear} \]

a) Given \( C = 2\text{mF} \) and \( z_L = j50 \, \Omega \), find \( \omega \) in rad/s. Note that \( z_L \) is the equivalent impedance of the entire circuit.

\[ V_{gA'c'} = 213 \angle 0^\circ \, V \]
\[ z_{gA} = j1.5 \, \Omega \]
\[ V_{gA'b'} = 213 \angle -120^\circ \, V \]
\[ z_{line} = j4.5 \, \Omega \]
\[ V_{gAb'c'} = 213 \angle +120^\circ \, V \]
\[ z_{LY} = -j0.9 \, \Omega \]

b) Find the numerical value of the phasor current \( I_{ab} \). Note that \( I_{ab} \) is on the generator side of the circuit.
**SOL’N:** a) We model the secondary circuit as a reflected impedance in the primary circuit:

\[ z_L = j50 \, \Omega = j25 \, \Omega + z_R \]

or

\[ z_R = j25 \, \Omega \]

For the reflected impedance, we use the standard formula for linear transformers:

\[ z_R = j25 \, \Omega = \frac{(\omega M)^2}{z_{\text{sec tot}}} = \frac{10^2 \, \Omega^2}{6 \, \Omega + \frac{1}{\omega C}} \]

or

\[ 6 \, \Omega + \frac{1}{\omega C} = \frac{10^2 \, \Omega^2}{j25 \, \Omega} = -j4 \]

or

\[ \frac{1}{\omega C} = -j10 \, \Omega \]

or

\[ \omega C = \frac{1}{-j10 \, \Omega} = j \frac{1}{10} \]

or

\[ \omega = \frac{1}{10C} = \frac{1}{10 \, \Omega \cdot 2 \, \text{mF}} \]

or

\[ \omega = 50 \, \text{r/s} \]

b) We compute the current as a voltage drop across \( z_{gA} \):

\[ I_{ab} = \frac{V_{ab} - V_{gAa'b'}}{z_{gA}} \]
To find $V_{ab}$, we first find the single-phase equivalent circuit so we can compute $V_{an}$. We divide the generator impedance by three, and we use a vector diagram to relate $V_{a'n}$ to $V_{gDelta a'b'}$.

![Vector Diagram]

From the diagram, we have the following formula:

$$V_{gDelta a'b'} = V_{a'n} \sqrt{3} \angle 30^\circ$$

or

$$V_{a'n} = V_{gDelta a'b'} \frac{1}{\sqrt{3}} \angle -30^\circ = 213 \angle 0^\circ V \cdot \frac{1}{\sqrt{3}} \angle -30^\circ \approx 123 \angle -30^\circ V$$

This yields the following single-phase equivalent circuit:

$$V_{gYa'n} \approx 123 \angle -30^\circ V \quad z_{gY} = j0.5 \ \Omega \\
z_{line} = j4.5 \ \Omega \\
z_{LY} = -j0.9 \ \Omega$$
From the single-phase circuit, we have a voltage-divider formula for $V_{an}$:

$$V_{an} = V_{an}' \frac{z_{line} + z_{LY}}{z_{gY} + z_{line} + z_{LY}} \approx 123\angle -30^\circ V \frac{j4.5 - j0.9}{j0.5 + j4.5 - j0.9}$$

or

$$V_{an} \approx 123\angle -30^\circ V \frac{j3.6}{j4.1} = 108\angle -30^\circ V$$

Using our earlier relationship for the delta-to-Y conversion, which also applies to the voltages across the a-b phase of the generator including the generator impedance, we have the following result:

$$V_{ab} = V_{an} \sqrt{3} \angle 0^\circ = 108\angle -30^\circ V \cdot \sqrt{3} \angle 0^\circ = 187\angle 0^\circ V$$

Now we compute the voltage drop across $z_{g\Delta}$:

$$I_{ab} = \frac{V_{ab} - V_{g\Delta a'b'}}{z_{g\Delta}} = \frac{187 - 213}{j1.5} A = -26j1.5 A$$

or

$$I_{ab} = j\frac{52}{3} A = j17.33 A$$